

Directions: Complete the following assignment on your own on this paper using a pencil. Refer to your notes or textbook if you have any questions. All questions should be attempted.

Part I: Multiple Choice: Circle the best possible answer.

- Gary works at a bicycle store in Vancouver. For the start of spring, the manager of the store has ordered 50 mountain bikes and 10 racing bikes. Which conjecture is Gary most likely to make from this evidence?
 - Either type of bike will sell equally well.
 - Racing bikes will likely sell better than mountain bikes.
 - It will rain all summer and no one will ride bicycles.
 - Mountain bikes will likely sell better than racing bikes.
- Which conjecture, if any, could you make about the product of an odd Integer and an even integer?
 - The product will be an even integer.
 - The product will be an odd integer.
 - The product will be negative.
 - It is not possible to make a conjecture.
- Gina created the following table to show a pattern.

Multiples of 9	18	27	36	45	54
Sum of the Digits	9	9	9	9	9

Which conjecture could Gina make, based solely on this evidence?

Choose the best answer.

- The sum of the digits of a multiple of 9 is divisible by 9.
 - The sum of the digits of a multiple of 9 is an odd integer.
 - The sum of the digits of a multiple of 9 is equal to 9.
 - Gina could make any of the above conjectures, based on this evidence.
- Anne made the following conjecture:
The difference between two numbers always lies between the two numbers.
Is the following equation a counterexample to this conjecture? Explain.
$$6 - 2 = 4$$
 - No, it is not a counterexample, because 4 lies between 2 and 6.
 - Yes, it is a counterexample, because 4 does not lie between 2 and 6.
 - Yes, it is a counterexample, because 4 lies between 2 and 6.
 - No, it is not a counterexample, because 4 does not lie between 2 and 6.
 - Which of the following choices, if any, uses inductive reasoning to show that the sum of three even integers is even?
 - $2x + 2y + 2z = 2(x + y + z)$
 - $2 + 4 + 6 = 12$ and $4 + 6 + 8 = 18$
 - $x + y + z = 2(x + y + z)$
 - None of the above choices

6. Choose the numbers that are counterexamples for the following statement.
“If two odd numbers are added, then the sum is also an odd number.”

- a. 3, 8 b. 4, 6 c. 1, 7 d. 2, -1

Part II: Constructed Response:

Directions: Show all workings in the space provided.

Refer to your notes or textbook if you have any questions.

All questions should be attempted.

1. State whether each of the following statements are **true** or **false**.

If false, give a counterexample.

- a) If a triangle has two equal sides, then it has equal angles.
- b) If two triangles have equal perimeters, then they have equal sides.
- c) If $x^2 > 0$, then $x > 0$.
- d) A number is divisible by 4 if the last digit is divisible by 4.
- e) Every odd whole number can be written as the difference of two squares.
- f) The square of a number is larger than the number.

2. Try the following calculator trick with three different numbers. Make a conjecture about the trick.

- Start with your age.
- Multiply it by 3.
- Multiply it by 7.
- Multiply it by 37.
- Multiply it by 13.

3. Use **deductive** reasoning to reach the conclusion stated in each of the following examples.

- a) Choose a number, triple the number, add 6, subtract the original number, divide by 2, and subtract 3.

- b) Choose a positive number, square the number, add one more than twice the original number, take the square root of the number, and subtract one.

4. Edward gathered the following evidence.

$$4(33) = 132 \quad 5(33) = 165 \quad 6(33) = 198$$

From this evidence, Edward made the following conjecture.

When you multiply a one-digit number by 33, the first and last digits of the product form a number that is three times the original number.

Is his conjecture reasonable? Justify your decision.

5. Consider the following statement:

“Whenever you square an odd integer, the result is odd.”

a. Use inductive reasoning to support this statement. Provide at least four specific examples.

b. Use deductive reasoning to prove the statement is true.

6. The square of an even integer is added to the square of an odd integer. Develop a conjecture about whether the sum is odd or even. Provide evidence to support your conjecture.

7. Alexandra discovered a number trick in a book she was reading:
Choose a number.
Add 2.
Multiply by 4.
Add 4.
Divide by 4.
Subtract 3.
Try the trick several times. Make a conjecture about the relation between the number picked and the final result. Can you find a counterexample to your conjecture? What does this imply?

8. **Prove** that the difference between consecutive perfect squares is always odd.

9. Support inductively **and** then prove deductively:
"The square of an even number is always even."