Section 1.3
Using Reasoning to Find a Counterexample to a Conjecture
In order to disprove or make a conjecture false we must find one example of the statement that does not satisfy the conjecture.
Example: (page 18)
Kerry created a series of circles. Each circle has points marked on its circumference and joined by chords.

As the number of points on the circumference increased, Kerry noticed a pattern for the number of regions created by the chords.
She made the following conjecture:
As the number of connected points on the circumference of a circle increases by 1, the number of regions created within the circle increases by a factor of 2.

How can Kerry test the validity of her conjecture?
Testing a Conjecture:
To the right is another diagram with five points on its circumference. The pairs of points are joined with chords. The diagram has 16 regions. This supported Kerry's conjecture because the pattern for the resulting regions was $2^1, 2^2, 2^3, 2^4$. 

![Diagram with five points on circumference]()}
Draw another circle with six points on its circumference. Join the pairs of points with chords and count the number of regions.

Does this support Kerry’s conjecture? Explain:
No it does not. There are only 31 regions and there should be 32 according to Kerry's conjecture. Since this does not increase by a factor of 2, it disproves Kerry's conjecture.

This is a counterexample, since it does not increase by a factor of 2.
Why is only one counterexample enough to disprove a conjecture?

One counterexample is enough to disprove a conjecture because the counterexample shows a case when the conjecture is not valid. Once a counterexample is found, the conjecture is no longer valid.
LOOK AT THE FOLLOWING CONJECTURES.

EX. 1

Conjecture: The sum of two prime numbers is an even number.

(a) Find two examples that support this conjecture.

(b) Find a example that falsifies the conjecture.
Ex. 2
Conjecture: The difference between consecutive perfect squares is always a prime number.

(a) Find two examples that support the conjecture.

(b) Find a example that falsifies the conjecture.
A **COUNTEREXAMPLE** is an example that invalidates or proves a conjecture to be false. Counterexamples are useful because they disprove conjectures.

If you can find ONE counter example then you have disproved the conjecture.
**Conjecture:**

All but one of the vowels (a, e, i, o, u, y) are used to spell numbers (zero, one, two, ....)

Is this conjecture true?

The conjecture appears to be true until "thousand"
**Counter Example:**
One example that proves a conjecture to be false, but maybe leads you to revise the conjecture.

**Revised Conjecture:**
The letter "a" is the only letter not used to spell words for numbers from 0 to 999.
Example: Matt found an interesting numeric pattern:

\[
\begin{align*}
1 \times 8 + 1 &= 9 \\
12 \times 8 + 2 &= 98 \\
123 \times 8 + 3 &= 987 \\
1234 \times 8 + 4 &= 9876
\end{align*}
\]

Matt thinks that this pattern will continue. Search for a counterexample to Matt's conjecture.
<table>
<thead>
<tr>
<th></th>
<th>1 x 8 + 1</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12 x 8 + 2</td>
<td>98</td>
</tr>
<tr>
<td>3</td>
<td>123 x 8 + 3</td>
<td>987</td>
</tr>
<tr>
<td>4</td>
<td>1234 x 8 + 4</td>
<td>9876</td>
</tr>
<tr>
<td>5</td>
<td>12345 x 8 + 5</td>
<td>98765</td>
</tr>
<tr>
<td>6</td>
<td>123456 x 8 + 6</td>
<td>987654</td>
</tr>
<tr>
<td>7</td>
<td>1234567 x 8 + 7</td>
<td>9876543</td>
</tr>
<tr>
<td>8</td>
<td>12345678 x 8 + 8</td>
<td>98765432</td>
</tr>
<tr>
<td>9</td>
<td>123456789 x 8 + 9</td>
<td>987654321</td>
</tr>
<tr>
<td>10</td>
<td>12345678910 x 8 + 10</td>
<td>98765431290</td>
</tr>
</tbody>
</table>
Is Matt’s conjecture valid? Why or why not?

No, because on our tenth trial a counterexample was found. Since the pattern did not continue, Matt’s conjecture is disproven.
Revise Matt’s Conjecture so it is valid:

When the value at the end/being added is from 1-9, the pattern will continue.
If we had not found a counterexample at the tenth step, should we have continues checking? When would it be reasonable to stop gathering evidence?

If we had not found a counterexample at the 10th step, we could have still stopped there. With the quantity of evidence found to support the conjecture, and a two-digit number further validating the conjecture, the conjecture could be considered strongly supported. If we wanted to do one more example, then it might have been logical to try a three-digit number to see if the conjecture was valid in that case.
Key Ideas:
• Once you have found a counterexample to a conjecture, you have disproved the conjecture. This means that the conjecture is invalid.
• You may be able to use a counterexample to help you revise a conjecture.
• Inductive reasoning can be used to make a conjecture; a conjecture is supported by evidence and can be invalidated by a counterexample.
1.3 Assignment:
Nelson Foundations of Mathematics 11, Sec 1.3, pg. 22-26
Questions: 1(b,d,f, g), 2, 4, 5, 7, 8, 9, 10, 12, 14, 15, 16, 27