Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. What is the reference angle for 15° in standard position?
   A  255°  
   B  30°  
   C  345°  
   D  15°

2. What are the three other angles in standard position that have a reference angle of 54°?
   A  99°, 144°, 234°  
   B  108°, 162°, 216°  
   C  144°, 234°, 324°  
   D  126°, 234°, 306°

3. What is the exact sine of ∠A?

   A  \frac{1}{\sqrt{3}}  
   B  \frac{1}{3}  
   C  \frac{2}{\sqrt{3}}  
   D  \frac{1}{2}

4. Which set of angles has the same terminal arm as 40°?
   A  80°, 120°, 160°  
   B  130°, 220°, 310°  
   C  200°, 380°, 560°  
   D  400°, 760°, 1120°

5. The point (40, –9) is on the terminal arm of ∠A. Which is the set of exact primary trigonometric ratios for the angle?
   A  \sin A = \frac{41}{9}, \cos A = \frac{41}{40}, \tan A = -\frac{9}{40}  
   B  \sin A = \frac{40}{41}, \cos A = -\frac{9}{41}, \tan A = -\frac{40}{9}  
   C  \sin A = \frac{40}{41}, \cos A = \frac{9}{41}, \tan A = -\frac{9}{40}  
   D  \sin A = -\frac{9}{41}, \cos A = \frac{40}{41}, \tan A = -\frac{9}{40}

6. What is the exact value for \tan (240°)?
   A  \frac{1}{\sqrt{3}}  
   B  1  
   C  1  
   D  \frac{1}{2}
7. An angle is in standard position such that \( \cos \theta = \frac{1}{9} \). What are the possible values of \( \theta \), to the nearest degree, if \( 0^\circ \leq \theta \leq 360^\circ \)?
   A  6° and 174°
   B  6° and 276°
   C  84° and 264°
   D  84° and 276°

8. Solve to the nearest tenth of a unit for the unknown side in the ratio
\[
\frac{a}{\sin 30^\circ} = \frac{12}{\sin 115^\circ}
\]
   A  24
   B  21.8
   C  6.6
   D  24.6

9. Determine the length of \( x \), to the nearest tenth of a centimetre.

   [Diagram not drawn to scale]

   A  26.6
   B  36.5
   C  11.2
   D  17.1

10. Determine, to the nearest tenth of a centimetre, the two possible lengths of \( a \).

   [Diagram not drawn to scale]

   A  72.8 cm and 26.3 cm
   B  34.3 cm and 26.3 cm
   C  72.8 cm and 55.8 cm
   D  55.8 cm and 34.3 cm

11. Which of the following triangles cannot be solved using the sine law?
Diagrams not drawn to scale.

12. Which strategy would be best to solve for $x$ in the triangle shown?

A  cosine law  C  sine law
B  primary trigonometric ratios  D  none of the above

13. Determine the measure of $x$, to the nearest tenth of a degree.
14. Solve the following triangle, rounding side lengths to the nearest tenth of a unit and angle measures to the nearest degree.

\[
\begin{align*}
\angle A &= 152^\circ, \ b = 19, \ a = 23.5 \\
A & \ \angle B = 22^\circ, \ \angle C = 6^\circ, \ c = 5.0 \\
B & \ \angle B = 158^\circ, \ \angle C = 84^\circ, \ c = 5.0 \\
C & \ \angle B = 68^\circ, \ \angle C = 174^\circ, \ c = 28.7 \\
D & \ \angle B = 35^\circ, \ \angle C = 7^\circ, \ c = 28.2
\end{align*}
\]

**Completion**

*Complete each statement.*

1. The expression \( \cos 30^\circ \) is equivalent to \( \sin \) ________________.

2. The tangent ratio is positive in the first and ________________ quadrants.

**Matching**

*Match the correct term to its description below.*

A reference angle B cosine law C angle in standard position D ambiguous case E sine law F terminal arm

1. a problem with two or more solutions
2. a law used when two sides and an opposite angle are given
3. the acute angle between the terminal arm and the $x$-axis of an angle in standard position
4. an angle with the initial arm on the positive $x$-axis
5. a law used when two sides and a contained angle are given

Short Answer

1. The hypotenuse of a right isosceles triangle is 5 cm long.
   a) Write an exact expression for the base and the height of the right triangle, using primary trigonometric ratios.
   b) Use your expressions to determine the exact area of the triangle.

2. The point A(–3, –5) is on the terminal arm of an angle $\theta$. Determine exact expressions for the primary trigonometric ratios for the angle.

3. A survey of a plot of land is shown. The plot is to have a hedge along its border. How many linear metres of hedge are needed, to the nearest tenth of a metre?

4. Diana is designing a triangular race course for a sailing regatta. The course is triangular and has a $35^\circ$ angle between two sides of 7 km and 6 km. What is the length of the third side of the race course, to the nearest kilometre?

Problem

1. The point (–5, 7) is located on the terminal arm of $\angle A$ in standard position.
   a) Determine the primary trigonometric ratios for $\angle A$.
   b) Determine the primary trigonometric ratios for an $\angle B$ that has the same sine as $\angle A$, but different signs for the other two primary trigonometric ratios.
   c) Use a calculator to determine the measures of $\angle A$ and $\angle B$, to the nearest degree.

2. a) Without using a calculator, determine two angles between $0^\circ$ and $360^\circ$ that have a sine ratio of $-\frac{1}{2}$.
   b) Use a calculator and a diagram to verify your answers to part a).
3. Two wires are connected to a tower at the same point on the tower. Wire 1 makes an angle of 45° with the ground and wire 2 makes an angle of 60° with the ground.
   a) Represent this situation with a diagram.
   b) Which wire is longer? Explain.
   c) If the point where the two wires connect to the tower is 35 m above the ground, determine exact expressions for the lengths of the two wires.
   d) Determine the length of each wire, to the nearest tenth of a metre.
   e) How do your answers to parts b) and d) compare?

4. Gursant and Leo are both standing on the north side of a monument that is 6.0 m tall. Leo is standing 3.5 m closer to the monument than Gursant. Leo measures the angle from the ground to the top of the monument to be 41°. Determine the angle that Gursant would measure from the ground to the top of the monument, to the nearest degree.

5. In \( \triangle ABC \), \( c = 11 \text{ cm} \), \( b = 7 \text{ cm} \), and \( \angle B = 38^\circ \).
   a) Sketch possible diagrams for this situation.
   b) Determine the measure of \( \angle C \) in each diagram.
   c) Find the measure of \( \angle A \) in each diagram.
   d) Calculate the length of BC in each diagram.

6. A salvage vessel locates a sunken ship directly below it. The angle of depression from the salvage vessel to one end of the ship is 29.3° and to the other end is 47.5°. If the length of the ship is 143 m, determine how far beneath the water’s surface it is, to the nearest metre.

7. Two ranger stations, A and B, are located in a pine forest and are 10 km apart. A forest fire breaks out at point F and is spotted by both rangers. The angle formed at station A by the line of sight to the fire and the line to station B is 63.2°. The angle at station B formed by the line of sight to the fire and the line between the ranger stations is 57.9°.
   a) How far is station A from the fire, to the nearest tenth of a kilometre?
   b) How far is station B from the fire, to the nearest tenth of a kilometre?
   c) Which ranger is closer to the fire, and by how much?

8. A clock has two hands that are 12 cm and 15 cm long. What is the distance, to the nearest tenth of a centimetre, between the tips of the hands at 2 p.m.?
### Multiple Choice

1. **ANS:** D  
   **PTS:** 1  
   **DIF:** Easy  
   **OBJ:** Section 2.1  
   **NAT:** T 1  
   **TOP:** Angles in Standard Position  
   **KEY:** reference angle | < 180°

2. **ANS:** D  
   **PTS:** 1  
   **DIF:** Average  
   **OBJ:** Section 2.1  
   **NAT:** T 1  
   **TOP:** Angles in Standard Position  
   **KEY:** reference angle

3. **ANS:** D  
   **PTS:** 1  
   **DIF:** Average  
   **OBJ:** Section 2.1  
   **NAT:** T 1  
   **TOP:** Angles in Standard Position  
   **KEY:** special angles | sine

4. **ANS:** D  
   **PTS:** 1  
   **DIF:** Easy  
   **OBJ:** Section 2.1  
   **NAT:** T 1  
   **TOP:** Angles in Standard Position  
   **KEY:** co-terminal angles

5. **ANS:** D  
   **PTS:** 1  
   **DIF:** Average  
   **OBJ:** Section 2.2  
   **NAT:** T 1  
   **TOP:** Trigonometric Ratios of Any Angle  
   **KEY:** point on terminal arm | cosine | sine | tangent

6. **ANS:** D  
   **PTS:** 1  
   **DIF:** Average  
   **OBJ:** Section 2.2  
   **NAT:** T 1  
   **TOP:** Trigonometric Ratios of Any Angle  
   **KEY:** tangent | reference angle | related angles

7. **ANS:** D  
   **PTS:** 1  
   **DIF:** Difficult  
   **OBJ:** Section 2.2  
   **NAT:** T 1  
   **TOP:** Trigonometric Ratios of Any Angle  
   **KEY:** arccosine | reference angle | related angles

8. **ANS:** C  
   **PTS:** 1  
   **DIF:** Easy  
   **OBJ:** Section 2.3  
   **NAT:** T 3  
   **TOP:** The Sine Law  
   **KEY:** sine law | side length

9. **ANS:** B  
   **PTS:** 1  
   **DIF:** Easy  
   **OBJ:** Section 2.3  
   **NAT:** T 3  
   **TOP:** The Sine Law  
   **KEY:** sine law | side length

10. **ANS:** B  
    **PTS:** 1  
    **DIF:** Difficult  
    **OBJ:** Section 2.3  
    **NAT:** T 3  
    **TOP:** The Sine Law  
    **KEY:** sine law | side length | ambiguous case

11. **ANS:** A  
    **PTS:** 1  
    **DIF:** Average  
    **OBJ:** Section 2.3  
    **NAT:** T 3  
    **TOP:** The Sine Law  
    **KEY:** sine law | < 180°

12. **ANS:** C  
    **PTS:** 1  
    **DIF:** Easy  
    **OBJ:** Section 2.3  
    **NAT:** T 3  
    **TOP:** The Sine Law  
    **KEY:** sine law | solution method

13. **ANS:** B  
    **PTS:** 1  
    **DIF:** Average  
    **OBJ:** Section 2.4  
    **NAT:** T 3  
    **TOP:** The Cosine Law  
    **KEY:** cosine law | angle measure

14. **ANS:** A  
    **PTS:** 1  
    **DIF:** Difficult  
    **OBJ:** Section 2.3 | Section 2.4  
    **NAT:** T 3  
    **TOP:** The Sine Law | The Cosine Law  
    **KEY:** cosine law | sine law | solve a triangle

### Completion

1. **ANS:** 60°
   **PTS:** 1  
   **DIF:** Easy  
   **OBJ:** Section 2.1  
   **NAT:** T 1  
   **TOP:** Angles in Standard Position  
   **KEY:** cosine | sine | special angles

2. **ANS:** third or 3rd
PTS: 1  DIF: Easy  OBJ: Section 2.2  NAT: T 2
TOP: Trigonometric Ratios of Any Angle  KEY: ratio | quadrant

MATCHING

1. ANS: D  PTS: 1  DIF: Easy  OBJ: Section 2.3
   NAT: T 3  TOP: The Sine Law  KEY: ambiguous case
2. ANS: E  PTS: 1  DIF: Easy  OBJ: Section 2.3
   NAT: T 3  TOP: The Sine Law  KEY: sine law
3. ANS: A  PTS: 1  DIF: Easy  OBJ: Section 2.1
   NAT: T 1  TOP: Angles in Standard Position  KEY: reference angle
4. ANS: C  PTS: 1  DIF: Easy  OBJ: Section 2.1
   NAT: T 1  TOP: Angles in Standard Position  KEY: standard angle
5. ANS: B  PTS: 1  DIF: Easy  OBJ: Section 2.4
   NAT: T 3  TOP: The Cosine Law  KEY: cosine law

SHORT ANSWER

1. ANS:
   a) The acute angles in a right isosceles triangle both measure 45°.

   \[
   \cos 45^\circ = \frac{\text{base}}{5} \quad \text{and} \quad \sin 45^\circ = \frac{\text{height}}{5}
   \]

   \[
   \text{base} = 5 \cos 45^\circ \quad \text{height} = 5 \sin 45^\circ
   \]

   \[
   \text{base} = \frac{5}{\sqrt{2}} \quad \text{height} = \frac{5}{\sqrt{2}}
   \]

   b) \[ A = \frac{1}{2} \cdot \text{base} \cdot \text{height} \]

   \[
   = \frac{1}{2} \left( \frac{5}{\sqrt{2}} \right) \left( \frac{5}{\sqrt{2}} \right)
   \]

   \[
   = \frac{25}{4}
   \]
The area of the triangle is \( \frac{25}{4} \) cm².

PTS: 1  DIF: Average  OBJ: Section 2.1 | Section 2.2  
NAT: T 1 | T 2  TOP: Angles in Standard Position | Trigonometric Ratios of Any Angle  
KEY: special angles | primary trigonometric ratios | area

2. ANS:
The angle is in the third quadrant, so only the tangent ratio will be positive. 
From the given point, \( x = -3 \) and \( y = -5 \). 
\[ r^2 = x^2 + y^2 \] 
\[ = 9 + 25 \] 
\[ = 34 \] 
\[ r = \sqrt{34} \] 
Therefore, \( \sin \theta = -\frac{5}{\sqrt{34}} \), \( \cos \theta = -\frac{3}{\sqrt{34}} \), and \( \tan \theta = \frac{5}{3} \).

PTS: 1  DIF: Average  OBJ: Section 2.2  
NAT: T 2  TOP: Trigonometric Ratios of Any Angle  
KEY: primary trigonometric ratios | point on terminal arm

3. ANS:
Use the sine law. 
\[ \frac{AC}{\sin 65^\circ} = \frac{46}{\sin 68^\circ} \] 
\[ AC = \frac{46 \sin 65^\circ}{\sin 68^\circ} \approx 45.0 \] 
\[ \angle A = 180^\circ - (65^\circ + 68^\circ) \] 
\[ = 47^\circ \] 
Again, use the sine law. 
\[ \frac{BC}{\sin 47^\circ} = \frac{46}{\sin 68^\circ} \] 
\[ BC = \frac{46 \sin 47^\circ}{\sin 68^\circ} \approx 36.3 \] 
The total amount of hedge needed is approximately 46 + 45.0 + 36.3, or 127.3 m.

PTS: 1  DIF: Average  OBJ: Section 2.3  
NAT: T 3  TOP: The Sine Law  
KEY: sine law

4. ANS:
Let \( a = 6 \), \( b = 7 \), and \( \angle C = 35^\circ \). 
Use the cosine law.
\[ c^2 = a^2 + b^2 - 2ab \cos C \]
\[ c^2 = 6^2 + 7^2 - 2(6)(7) \cos 35^\circ \]
\[ c \approx 4 \]

The third side of the race course is approximately 4 km long.

PTS: 1  DIF: Easy  OBJ: Section 2.4  NAT: T 3
TOP: The Cosine Law  KEY: cosine law

PROBLEM

1. ANS:
   a) \( \angle A \) is in the second quadrant. Therefore, only the sine ratio is positive.
   Use the Pythagorean theorem.
   \[ r^2 = x^2 + y^2 \]
   \[ = (-5)^2 + 7^2 \]
   \[ = 25 + 49 \]
   \[ = 74 \]
   \[ r = \sqrt{74} \]
   Therefore, \( \sin A = \frac{7}{\sqrt{74}} \), \( \cos A = -\frac{5}{\sqrt{74}} \), and \( \tan A = -\frac{7}{5} \).

   b) The quadrant in which the sine ratio is still positive, but the cosine and tangent ratios change from negative to positive, is the first quadrant. In this quadrant, all three primary trigonometric ratios are positive.
   \[ \sin B = \frac{7}{\sqrt{74}} \], \( \cos B = \frac{5}{\sqrt{74}} \), and \( \tan B = \frac{7}{5} \).

   c) \( \sin B = \frac{7}{\sqrt{74}} \)

   \[ \angle B \approx 54^\circ \]
   Use the fact that \( \angle B \) is the reference angle for \( \angle A \).
   \[ \angle A = 180^\circ - 54^\circ \]
   \[ = 126^\circ \]

PTS: 1  DIF: Average  OBJ: Section 2.1 | Section 2.2
NAT: T 1 | T 2  TOP: Angles in Standard Position | Trigonometric Ratios of Any Angle
KEY: primary trigonometric ratios | reference angle

2. ANS:
   a) Since \( \sin 30^\circ = \frac{1}{2} \), the reference angle is 30\(^\circ\). The sine ratio is negative in the third and fourth quadrants.
   Look for reflections of the 30\(^\circ\) angle in these quadrants.
   third quadrant: \( 180^\circ + 30^\circ = 210^\circ \)
   fourth quadrant: \( 360^\circ - 30^\circ = 330^\circ \)

   b) Using a calculator, \( \sin 210^\circ = -\frac{1}{2} \) and \( \sin 330^\circ = -\frac{1}{2} \).
3. ANS:

a) 

b) Both wires are connected to the tower at the same height, which is the opposite side to the given angles. Each wire length represents the hypotenuse of its respective triangle. The longer hypotenuse is the wire that forms the smaller angle, as it will need to be longer to reach the tower.

c) Let \( x \) represent the length of wire 1 and \( y \) represent the length of wire 2.

Wire 1: \( \sin 45^\circ = \frac{35}{x} \) 
\[ x = \frac{35}{\sin 45^\circ} = \frac{35}{\frac{1}{\sqrt{2}}} = 35\sqrt{2} \]

Wire 2: \( \sin 60^\circ = \frac{35}{y} \) 
\[ y = \frac{35}{\sin 60^\circ} = \frac{35}{\frac{\sqrt{3}}{2}} = \frac{70}{\sqrt{3}} \]

d) The length of wire 1 is 49.5 m, and the length of wire 2 is 40.4 m.

e) The values calculated in part d) support the answers in part b).
Let $x$ represent the distance that Leo is from the base of the monument.

$$\tan 41^\circ = \frac{\delta}{x}$$

$$x = \frac{\delta}{\tan 41^\circ}$$

Let $\angle G$ represent the angle that Gursant would measure. The distance that Gursant is from the monument is given by the expression $3.5 + \frac{\delta}{\tan 41^\circ}$.

$$\tan G = \frac{\delta}{3.5 + \frac{\delta}{\tan 41^\circ}}$$

$$\angle G \approx 30^\circ$$

PTS: 1     DIF: Difficult     OBJ: Section 2.2     NAT: T 3     TOP: Trigonometric Ratios of Any Angle

5. ANS:
   a) This is the ambiguous case, so there are two triangles.

**Triangle 1**

- $11 \text{ cm}$
- $38^\circ$
- $7 \text{ cm}$

For triangle 1,

$$\sin E = \frac{\sin C}{c}$$

$$\frac{\sin 38^\circ}{7} = \frac{\sin C}{11}$$

$$\sin C = \frac{11 \sin 38^\circ}{7}$$

$$\angle C \approx 75.3^\circ$$

e) For triangle 1,

**Triangle 2**

- $11 \text{ cm}$
- $38^\circ$
- $7 \text{ cm}$

For triangle 2,

$$\angle C = 180^\circ - 75.3^\circ$$

$$\approx 104.7^\circ$$

KEY: tangent
\[ \angle A = 180^\circ - (38^\circ + 75.3^\circ) = 66.7^\circ \]
\[ \angle A = 180^\circ - (38^\circ + 104.7^\circ) = 37.3^\circ \]

For triangle 1,
\[ \frac{BC}{\sin A} = \frac{AC}{\sin B} \]
\[ \frac{BC}{\sin 66.7^\circ} = \frac{7}{\sin 38^\circ} \]
\[ BC = \frac{7 \sin 66.7^\circ}{\sin 38^\circ} \approx 10.4 \]

For triangle 2,
\[ \frac{BC}{\sin A} = \frac{AC}{\sin B} \]
\[ \frac{BC}{\sin 37.3^\circ} = \frac{7}{\sin 38^\circ} \]
\[ BC = \frac{7 \sin 37.3^\circ}{\sin 38^\circ} \approx 6.9 \]

In triangle 1, the length of BC is 10.4 cm. In triangle 2, the length of BC is 6.9 cm.

PTS: 1  DIF: Average  OBJ: Section 2.3  NAT: T 3  TOP: The Sine Law  KEY: sine law | ambiguous case

6. ANS:

The angle made by the salvage vessel and the ends of the sunken ship is
\[ 180^\circ - 29.3^\circ - 47.5^\circ = 103.2^\circ \]

Use the sine law to determine the distance, SL, from the salvage vessel to one end of the sunken ship:
\[ \frac{143}{\sin 103.2^\circ} = \frac{SL}{\sin 29.3^\circ} \]
\[ SL \approx 71.88 \]

Now use the sine ratio to determine the water depth, SA.
\[ \sin 47.5^\circ = \frac{SA}{71.88} \]
\[ SA \approx 53.0 \]

The sunken ship is approximately 53 m beneath the water’s surface.

PTS: 1  DIF: Average  OBJ: Section 2.3  NAT: T 3  TOP: The Sine Law  KEY: sine law

7. ANS:
a) \[ \angle F = 180^\circ - 63.2^\circ - 57.9^\circ = 58.9^\circ \]
\[ \frac{b}{\sin 57.9^\circ} = \frac{10}{\sin 58.9^\circ} \]
\[ b \approx 9.9 \]

Station A is approximately 9.9 km from the fire.
b) \( \frac{a}{\sin 63.2^\circ} = \frac{10}{\sin 58.9^\circ} \)

\( a \approx 10.4 \)

Station B is approximately 10.4 km from the fire.

e) \( 10.4 - 9.9 = 0.5 \)

The ranger at Station A is approximately 0.5 km closer to the fire.

PTS: 1  DIF: Average  OBJ: Section 2.3  NAT: T 3

TOP: The Sine Law  KEY: sine law

8. ANS:

At 2 p.m., the hour hand is \( \frac{2}{12} = \frac{1}{6} \) of the distance around the clock.

\( \frac{1}{6} \times 360^\circ = 60^\circ \)

The angle formed by the two hands measures 60°.

Let \( x \) represent the distance between the tips of the hands, in centimetres, and use the cosine law.

\( x^2 = 12^2 + 15^2 - 2(12)(15) \cos 60^\circ \)

\( x \approx 13.7 \)

The distance between the tips of the hands at 2 p.m. is approximately 13.7 cm.

PTS: 1  DIF: Difficult  OBJ: Section 2.4  NAT: T 3

TOP: The Cosine Law  KEY: cosine law