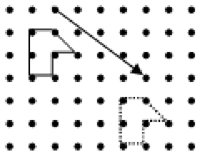
Transformational Geometry

Translations (Slides)

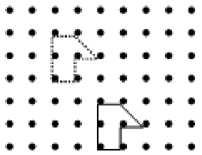
Translating (sliding) a figure causes it to move position without changing orientation, size or shape.

Visual instructions for these movements are given as an arrow, which indicates a direction and overall change in position. The number of spaces both sideways (left/right) and vertically (up/down) can be found by counting from the base of the arrow across and then vertically to the point of the arrow.



In this example, the figure is moved so that each point travels 4 spaces to the right and 3 spaces down. The new figure is drawn with dotted lines.

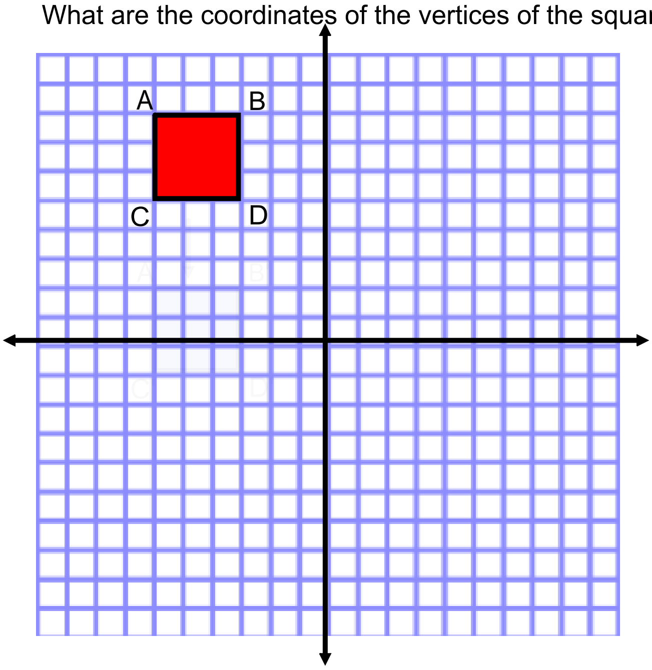
Numeric instructions are given in the following format [L2, U3]. This tells you that each point would move 2 spaces to the left and 3 spaces up. See the example below.



Translations

What are the coordinates of the vertices of the square below?

Answers



Translate the square 6 blocks down

what are the coordinates of the vertices of the new translated square?

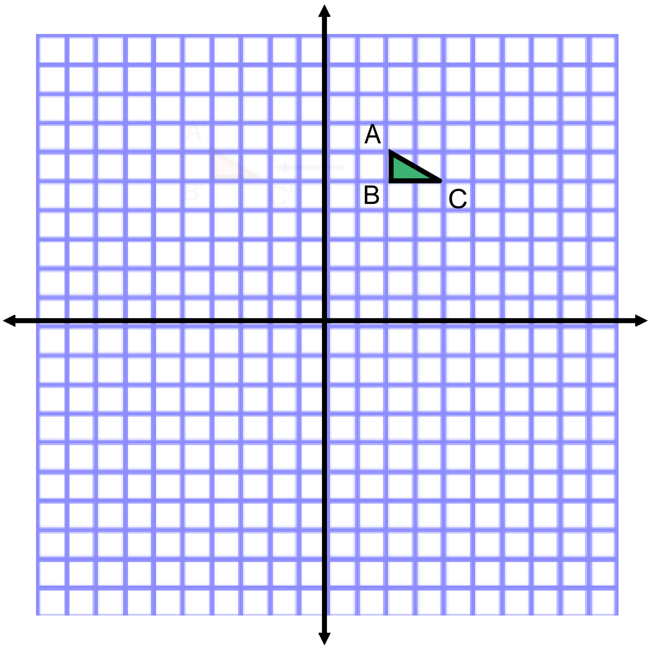
Answers

Click where you think the new translated square should be

Translations

What are the coordinates of the vertices of the triangle below?

Answers



Translate the triangle 6 blocks left

what are the coordinates of the vertices of the new translated triangle?

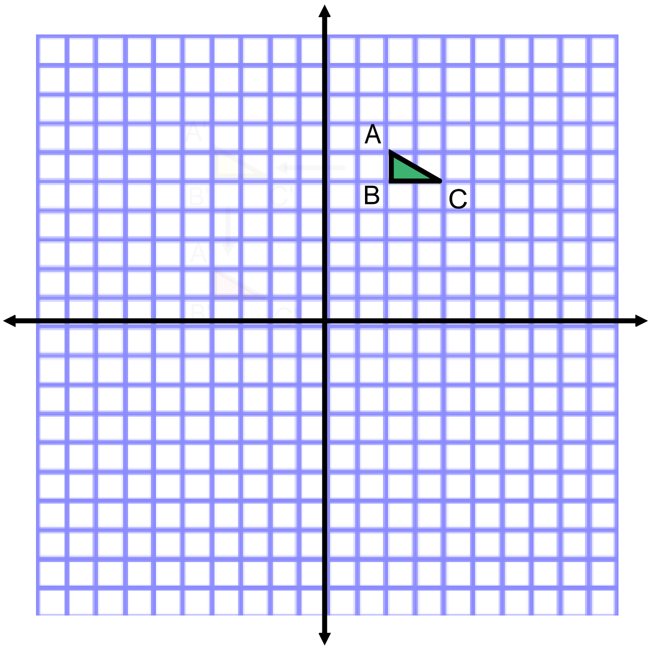
A' (-4, 6)
B' (-4, 5)
C' (-2, 5)
Answers

Click where you think the new translated triangle should be

Translations

What are the coordinates of the vertices of the triangle below?

Answers



Translate the triangle 6 block left and 4 blocks down

what are the coordinates of the vertices of the new translated triangle?

Answers

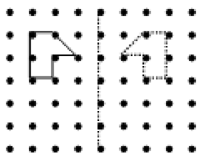
A' (-4, 2)
B' (-4, 1)
C' (-2, 1)

Click where you think the new translated triangle should be

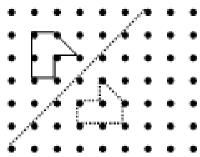
Reflections (Flips)

Reflecting (flipping) a figure causes a mirror image of the figure to be formed. It is the same size, but reversed in shape.

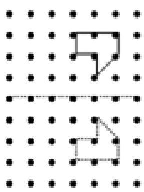
Reflections are usually indicated using a reflection line. This line provides information about the distance and angle a mirror might be set up to see the new image.



example 1: horizontal reflection



example 2: 45 degree angle reflection

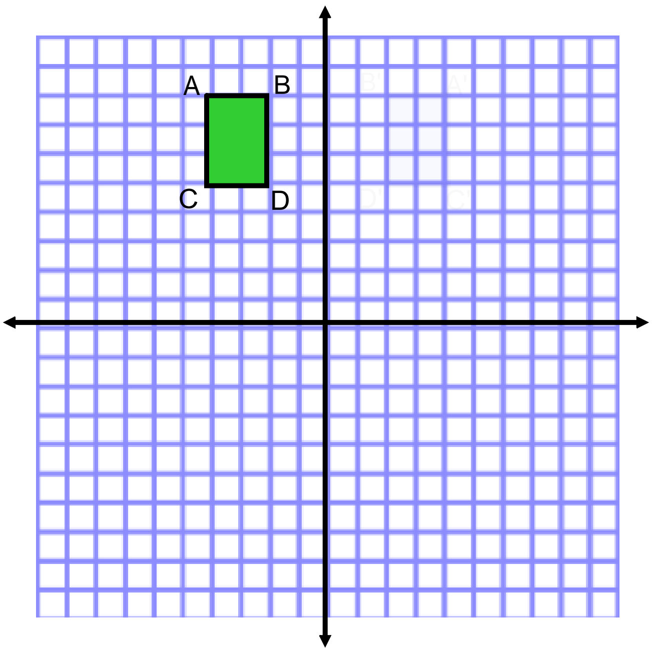


example 3: vertical reflection

Reflections

What are the coordinates of the vertices of the rectangle below?

Answers



Reflect the rectangle over the Y - axis

what are the coordinates of the vertices of the new reflected rectangle?

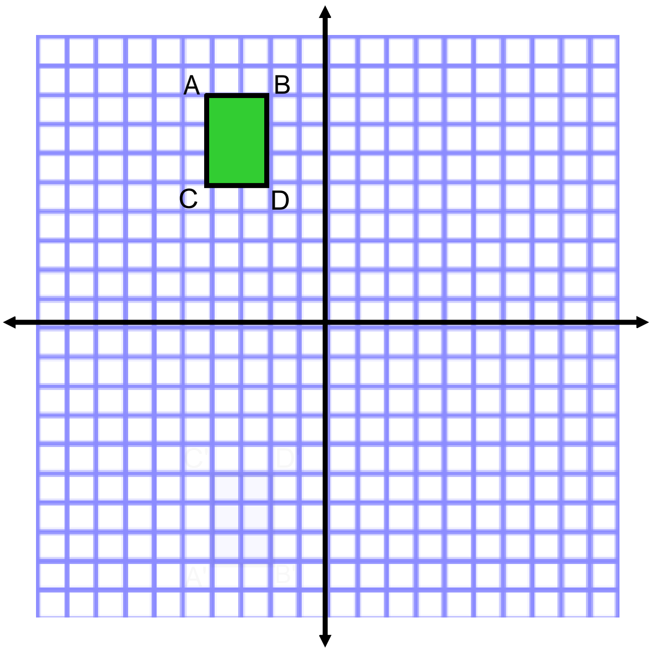
Answers

Click where you think the new reflected rectangle should be

Reflections

What are the coordinates of the vertices of the rectangle below?

Answers



Reflect the rectangle over the X - axis

what are the coordinates of the vertices of the new reflected rectangle?

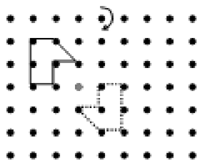
Answers

Click where you think the new reflected rectangle should be

Rotations (Turns)

Rotating a figure causes a change in orientation, but not in size or shape. This type of transformation is often very difficult to visualize.

Rotations are frequently described using turn arrows, which indicate the direction (clockwise or counter-clockwise) of the turn as well as the portion of a complete circle the figure is to be turned. The "turn center" must be identified also.



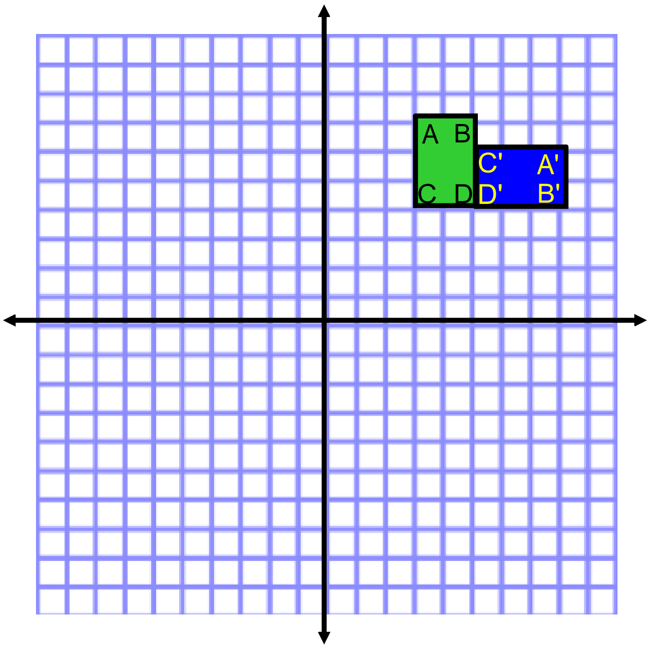
In this example, the figure turns 180 degrees clockwise, with the turn center marked in grey.

An alternate method of describing this motion is 180° cw.

Rotations

What are the coordinates of the vertices of the rectangle below?

Answers



Rotate the rectangle 90^0 clockwise around point D

what are the coordinates of the vertices of the new rotated rectangle?

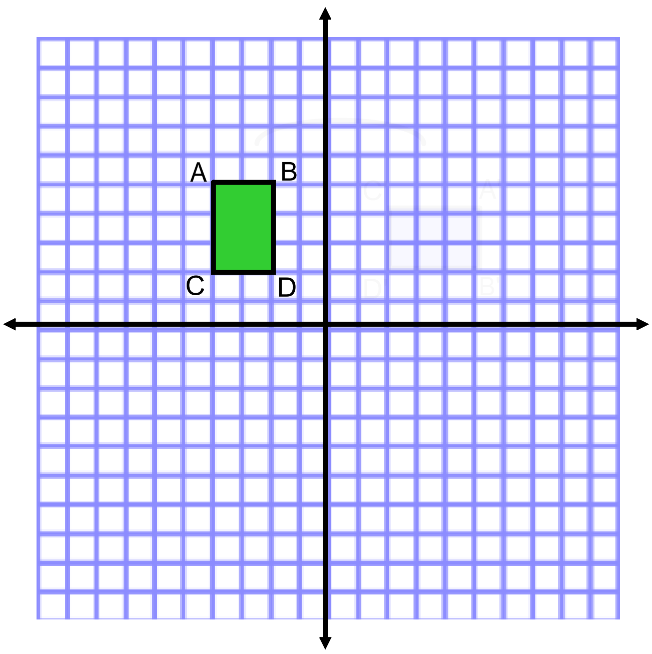
Answers

Click where you think the new rotated rectangle should be

Rotations

What are the coordinates of the vertices of the rectangle below?

Answers



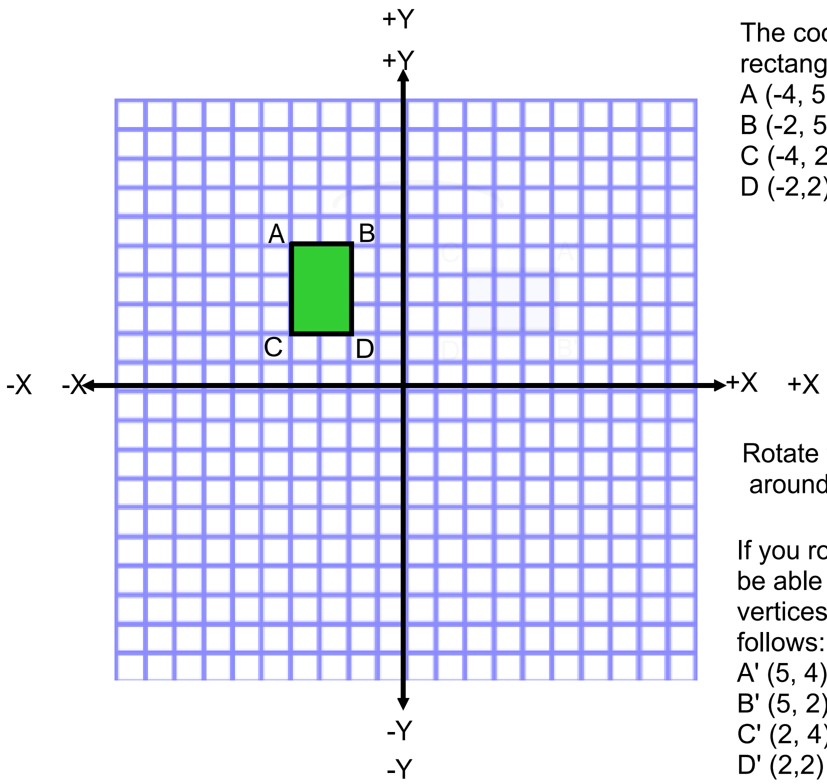
Rotate the rectangle 90^0 clockwise around the origin

what are the coordinates of the vertices of the new rotated rectangle?

Answers

Click where you think the new rotated rectangle should be

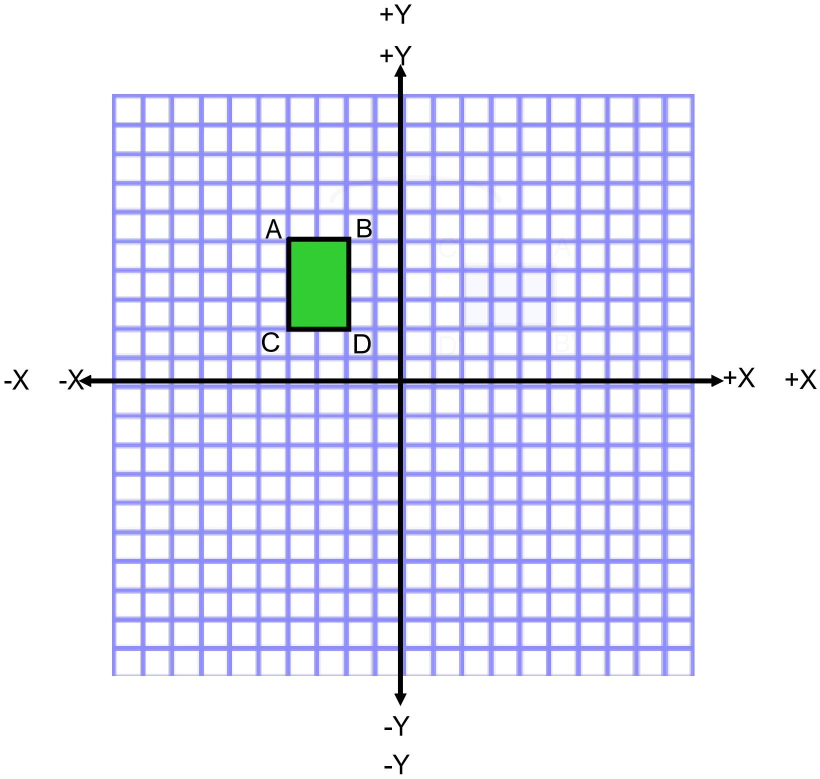
To complete questions involving rotations around the origin, visuals the entire grid being rotated.



The coordinates of the vertices of the rectangle are
A (-4, 5)
B (-2, 5)
C (-4, 2)
D (-2,2)

Rotate the rectangle 90^0 clockwise around the origin

If you rotate the entire image 90^0, you should be able to tell the the coordinates of the vertices of the new rotated rectangle is as follows:
A' (5, 4)
B' (5, 2)
C' (2, 4)
D' (2,2)

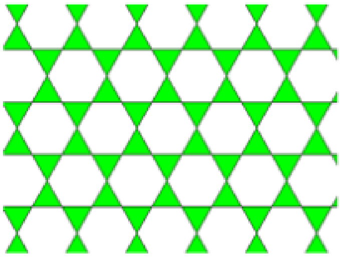

+Y
+Y
A
B
C
D
-X
-X
+X
+X
-Y
-Y

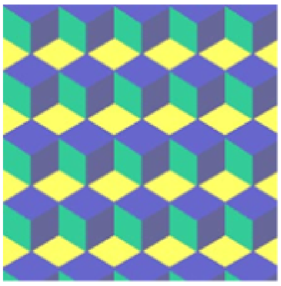
Section 8.5: Constructing Tessellations

A **tessellation** is created when a shape is repeated over and over again covering a plane without any gaps or overlaps. Another word for a tessellation is tiling.

Real-Life Examples of Tessellations:

- Floor tiles
- Quilting
- Fencing patterns
- Wall paper patterns
- Bricklaying patterns
- Company logos



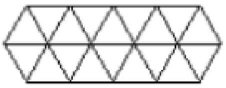


In order for a shape to tessellate, the sum of the angles at any point where vertices meet must be 360°. We say that the *polygons surround a point*.

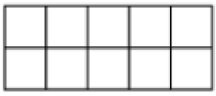
A regular polygon is a simple closed figure that has all sides congruent and all angles congruent. A **regular tessellation** means a tessellation made up of congruent regular polygons.

In a plane, only three regular polygons tessellate: triangles, squares or hexagons. This is because at any point where vertices meet, the sum of the angles is 360 degrees.

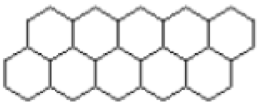
Triangles



Squares



Hexagons



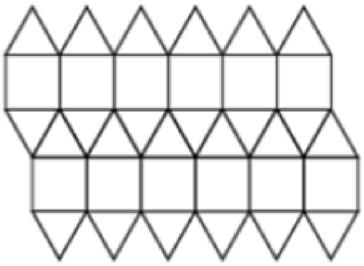
Here are the measures of the interior angles of several regular polygons:

Polygon	**Interior angle measure**
Triangle	60°
Square	90°
Pentagon	108°
Hexagon	120°
Octagon	135°
Decagon	144°
Dodecagon	150°

It is also possible to tessellate using combinations of regular polygons. When we combine two or more shapes together, we create a **composite shape.** Again, the sum of the angles at any point where vertices meet must still be 360 degrees.

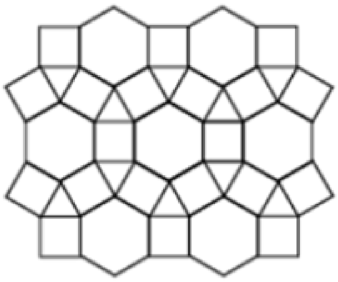
Examples:

- Three triangles and two squares at each vertex point



$$(60° + 60° + 60°) + (90° + 90°) = 360°$$

- Square, triangle, square, hexagon at each vertex point



$$90° + 60° + 90° + 120° = 360°$$

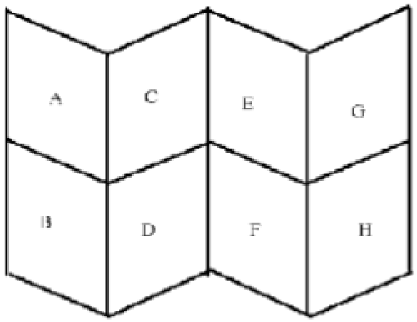
Notes:

- **All triangles tessellate**
- **All quadrilaterals (4-sided figures) tessellate**
- **A polygon with more than 6 sides will not tessellate**

Unit 8.6: Identifying Transformations in Tessellations

We can describe a tessellation in terms of transformations.

For example:



- Shape A can be transformed into shape E through translation to the right.
- Shape A can be transformed into shape C by reflection in the side that is shared between them.
- Shape A can be transformed into shape B by rotating 180 degrees around the midpoint of their shared side.
- Shape A can be transformed into shape F if it is translated to the right to position E and then rotated 180 degrees around the midpoint of the shared side between E and F.
- Shape A can be transformed into shape D by being rotated 180 degrees around the midpoint of the side shared between A and B and then reflected in the side shared between B and D.

***Note: There are multiple ways to describe these translations.**