Section 2: Static Equilibrium II- Balancing Torques

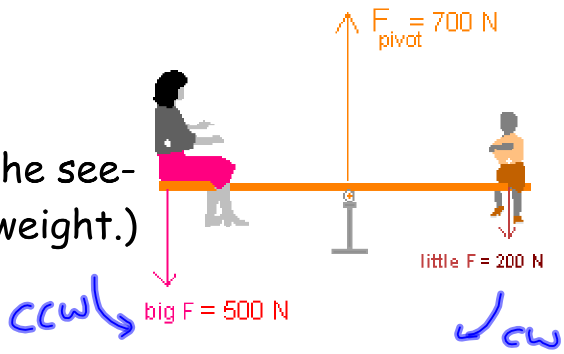
Last Section: If $\sum F_x = 0 \ and \sum F_y = 0$ (ie. Forces up = Forces down and Forces left = Forces right), then the object will have no translatory motion. In other words, the object will not move up/down or left/right. HOWEVER, it might rotate.

Example: Consider a see-saw.

As you can see,

$$F_{up} = F_{down}$$
$$700N = 500N + 200N$$

(Assuming the see-saw has no weight.)

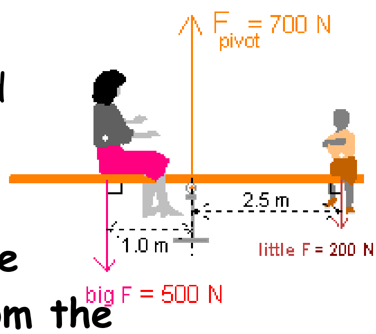


Thus, the see-saw will not move up into the air, nor fall flat to the ground. However, because the big person is much heavier than the little person, the left-hand part of the see-saw will fall and the right-hand part will rise. In other words, the see-saw will rotate.

In order to balance the see-saw, the big person has to move closer to the **fulcrum** or **pivot point**.

Example: Consider the following situation.

As you can see the Big person has moved closer to the pivot point.



What do you get when you multiply the forces by the associated distances from the pivot?

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(500 N) (1 m) = 500 Nm
(200 N) (2.5 m) = 500 Nm Notice that these are the same.

This special product of force times distance is called **torque**. The see-saw is balanced when the torques are equal to each other. A **torque** is a moment of force. You apply a torque when you open a bottle of water, tighten a screw, turn on the water tap, open a door, etc.

In the previous example, the big person produced a **counterclockwise torque** (τ_{ccw}) and the little person produced a clockwise torque (τ_{cw}) .

Using F to represent force and r to represent the distance between the force and the pivot point, we can define torque as follows:

$$\tau = F_{\perp} r$$

where F is the applied force, in Newtons, that is applied at 90° to the surface
r is the perpendicular distance, in meters, between the place where the force is applied and the pivot point (the point of rotation).
τ is the torque in Nm.

Now we have the **two conditions necessary for static equilibrium**.

Condition 1 (from last lesson): $\sum F_x = 0 \ and \sum F_y = 0$

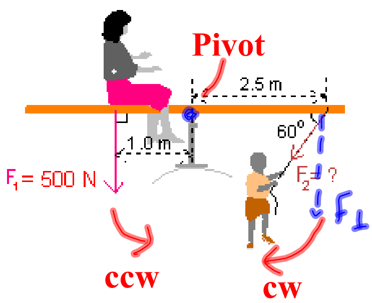
Condition 2 (from this lesson): $\sum \tau_{cw} = \sum \tau_{ccw}$

Or, $$\sum F_{cw} \times r_{cw} = \sum F_{ccw} \times r_{ccw}$$

The symbol **Σ** is used because there could be more than one person sitting in different locations on each side of the pivot.

Example: Now consider the following situation.

The picture shows that the boy is pulling downward on a rope attached at an angle of 60° to the see-saw. What force must the boy exert to balance the see-saw?



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To determine wether a torque is clockwise or counterclockwise perform the following. Hold a pencil between your fingers at the pivot point. F1 is a force acting downward. Using a finger on your other hand push down on the pencil where F1 is located. You should notice that your pencil rotates counterclockwise. Repeat for F2. You should notice that your pencil rotates clockwise. Hence F 1 produces a counterclockwise torque and F2 produces a clockwise torque.

Remember the rule: **the forces used must always be perpendicular to the see-saw.** This means that F_1 is OK but F_2 is not. Thus, we need to find the component of F_2 that is perpendicular to the see-saw.

The perpendicular component of F_2 is $F_{\perp} = F_2 \sin\theta$

$\Sigma\tau_{cw} = \Sigma\tau_{ccw}$
$(F_2 \sin\theta) r_2 = F_1 r_1$
$(F_2 \sin 60^{o})(2.5\ m) = (500\ N)(1\ m)$
$(2.17\ m)\ F_2 = 500\ Nm$
$F_2 = 230\ N$

A more general expression for torque is given by the equation:

$$\tau = (F_{\perp app})r$$

$$\tau = (F\sin\theta)r$$

$$\tau = Fr\sin\theta$$

Not given

where F is the applied force in Newtons
r is the perpendicular distance between the place where the force is applied and the pivot point
θ is the angle between the surface and the applied force.

Examples

1 How far must a 250 N person sit from the pivot of a see-saw to balance a 420 N person sitting 0.75 m from the pivot? (Assume that the see-saw itself is weightless).

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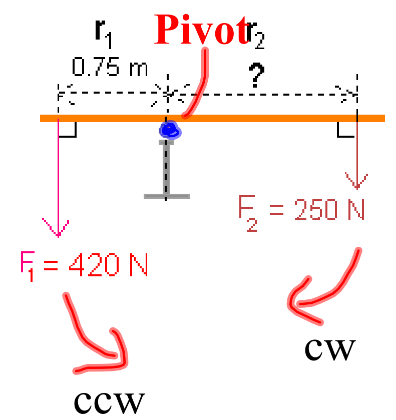


Copy Solution

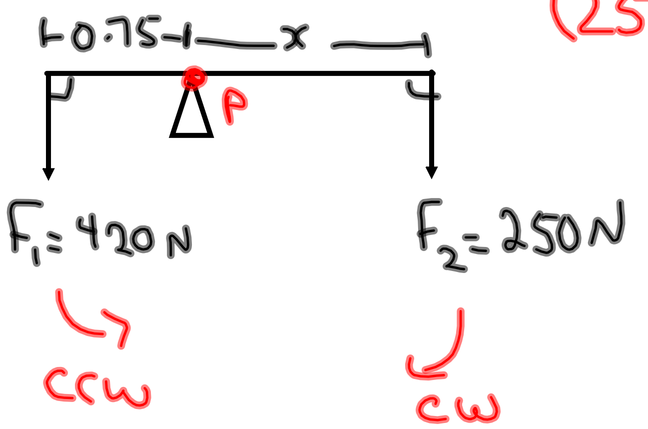
$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$

$$F_2 r_2 = F_1 r_1$$

$$(250N)x = (420N)(0.75m)$$

$$(250N)x = 315\ Nm$$

$$\boxed{x = 1.3m}$$



2 Repeat practice exercise 1 with the real pivot in its usual place, but with an imaginary pivot located at the point that the 420 N force is applied.

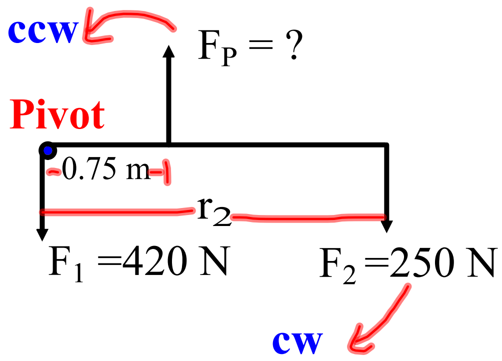
1st condition of static equilibrium: $\Sigma F = 0$

So, $F_{up} = F_{down}$

$F_P = F_1 + F_2$

$F_P = 420\text{ N} + 250\text{ N}$

$F_p = 670\text{ N}$



F_1 is included in the diagram but it will not be used in the calculations because it is 0 m away from the imaginary pivot.

$$\sum \tau_{cw} = \sum \tau_{ccw}$$

$$F_2 r_2 = F_1 r_1$$

$$(250N) r_2 = (670N)(0.75m)$$

$$(250N) r_2 = 502.5 Nm$$

$$r_2 = 2.01 m$$

This is the distance away from the imaginary pivot not the actual pivot.

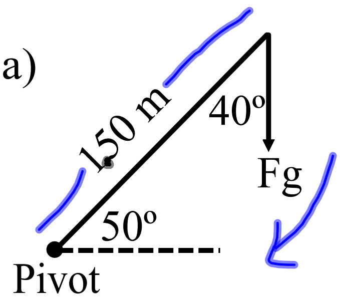
The distance to the real pivot is 2.01 m - 0.75 m = 1.26 m = 1.3 m

3. The trunk of an old Cadillac Eldorado, 1.50 m long and open to an angle of 50° above the horizontal, can only be closed by someone with a mass greater than 45.0 kg hanging vertically from the end of the open lid. (ignore mass of trunk lid itself)

a) Draw a labeled diagram of this situation. Be sure to note any forces applied.

b) What is the minimum amount of torque required to close this trunk?

a)



b) $F_g = mg$

$F_g = (45\ \text{kg})(9.81\ \text{m/s}^2)$

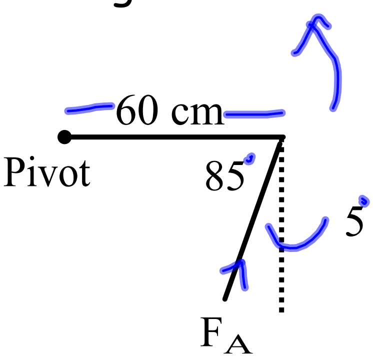
$F_g = 441\ \text{N}$

$\tau = Fr \sin\theta$

$\tau = (441\ \text{N})(150\ \text{m}) \sin 40^\circ$

$\tau = 425$ Nm clockwise

4. Hannah pushes a door with a force of 45 N at an angle of 5° from the perpendicular, 60 cm across from the hinges. What torque does she apply to the door?



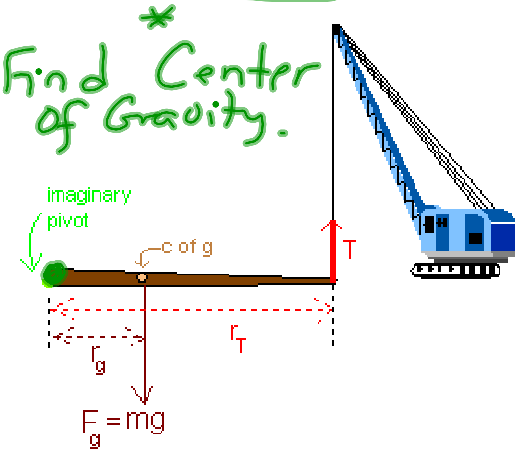
$\tau = Fr\sin\theta$

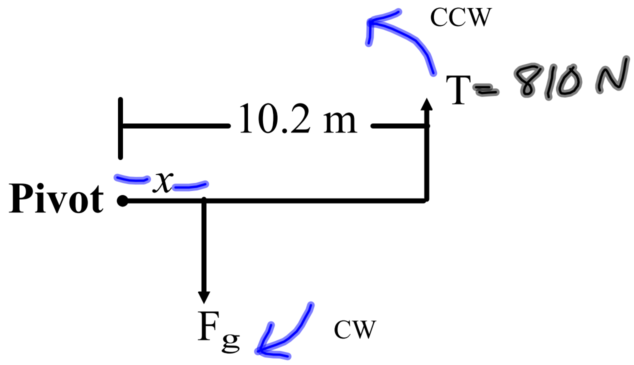
$\tau = (45\ \text{N})(0.60\ \text{m})\sin 85^\circ$

$\tau = 27$ N CCW

5 A tapered power pole has a mass of 250 kg and a length of 10.2 m. A cable attached to the smaller end has a tension of 810 N just as the end is about to rise from the ground. Where would the cable have to be attached in order for the pole to rise horizontally?

Find Center of Gravity.





$F_g = mg$
$F_g = (250\text{ kg})(9.8\text{ m/s}^2)$
$F_g = 2450\text{ N}$

In order for the pole to move horizontally, the cable will have to be attached to the pole at its center of gravity (F_g).

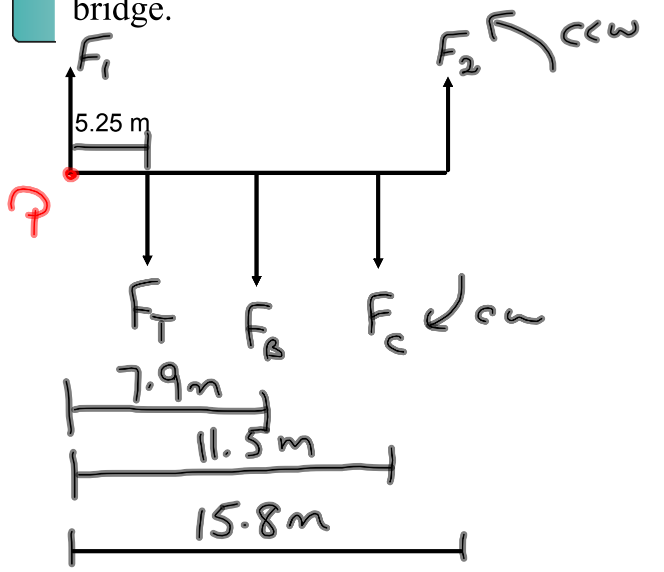
$$\sum \tau_{cw} = \sum \tau_{ccw}$$
$$F_g r_g = T r_T$$
$$(2450N)x = (810N)(10.2m)$$
$$x = 3.4\,m$$

The cable must be attached 3.4 m from the larger end.

6 A uniform wooden bridge is 15.8 m long and weighs 1.25×10^6 N. It is supported by a pillar at each end. How is the weight shared by the pillars when there is 2.22×10^4 N truck 5.25 m from one end and a 1.50×10^4 N car 4.30 m from the other end?

Pull

Uniform Bridge - this means that we can assume that all the mass/weight of the bridge is concentrated at the center of the bridge.



$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$

$$F_T r + F_B r + F_c r = F_2 r$$

$$(2.22 \times 10^4)(5.25) + (1.25 \times 10^6)(7.9) + (1.5 \times 10^4)(11.5) = (F_2)(15.8m)$$

$$643\,000N = F_2$$

$$6.43 \times 10^5 N = F_2$$

Use 1st condition of Static Equilibrium to get F_1.

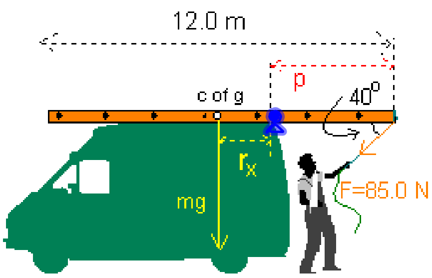
$$\Sigma F_{up} = \Sigma F_{down}$$

$$F_1 + F_2 = F_T + F_B + F_c$$

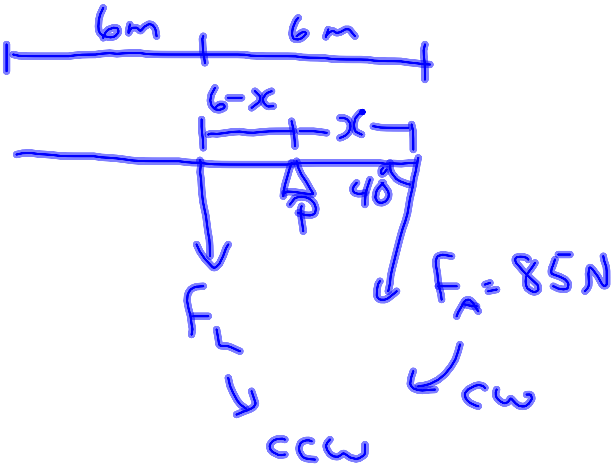
$$F_1 + 6.43 \times 10^5 N = 2.22 \times 10^4 + 1.25 \times 10^6 + 1.5 \times 10^4$$

$$F_1 = 6.44 \times 10^5 N$$

7 A worker with an injured arm in sling exerts a force of 85.0 N as shown in the picture to reach a ladder on the top of his van. The uniform ladder has a mass of 25.0 kg and is 12.0 m long. How much of the ladder must protrude from the end of the van if the worker is to successfully pull it down?



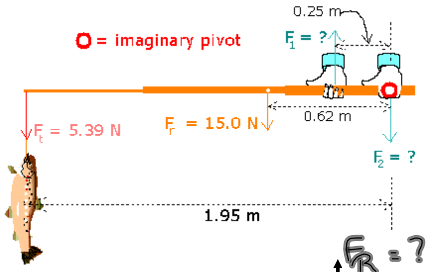
$$F_L = mg$$
$$= (25\,kg)(9.8\,m/s^2)$$
$$= 245\,N$$

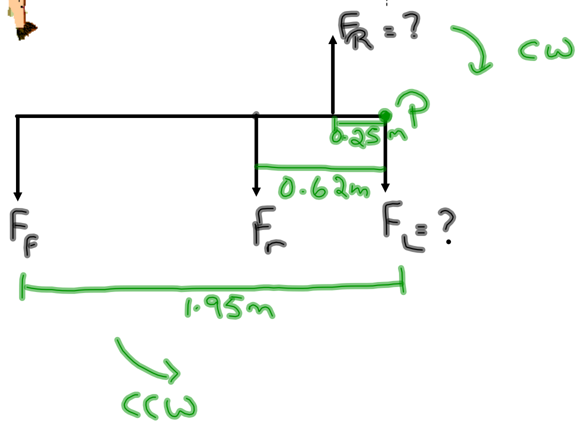


$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$
$$F_A r \sin\theta = F_L r$$
$$85N(x)\sin 40° = (245N)(6m - x)$$
$$54.6x = 1470 - 245x$$
$$299.6x = 1470$$
$$x = 4.91\,m$$

So, 4.91m of the ladder must protrude.

8 A trouter grips her fishing rod so that her hands are 25 cm apart and the rod is horizontal. The centre of gravity of the 1.53 kg rod is 62 cm from her left hand. A 550 g trout hangs from the end of the rod 1.95 m from the trouter's left hand and the whole system is in static equilibrium. What is the force exerted by each hand?





$F_F = mg$
$= (0.550\,kg)(9.8\,m/s^2)$
$= 5.39\,N$

$F_r = (1.53\,kg)(9.8\,m/s^2)$
$F_r = 15.0\,N$

$$\Sigma\tau_{cw} = \Sigma\tau_{ccw}$$
$$F_R r = F_F r + F_r r$$
$$F_R(0.25\,m) = (5.39\,N)(1.95\,m) + (15\,N)(0.62\,m)$$
$$F_R = 79.2\,N$$

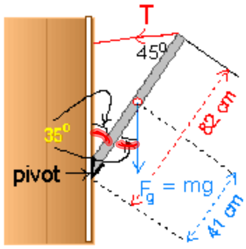
Use 1st condition of Static Equilibrium to find F_L.

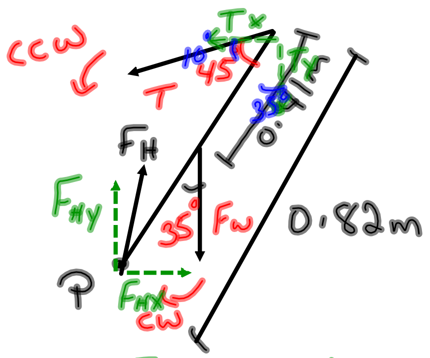
$$\Sigma F_{up} = \Sigma F_{down}$$
$$F_R = F_L + F_r + F_F$$
$$79.2\,N = F_L + 15\,N + 5.39\,N$$
$$58.8\,N = F_L$$

9 A window of which the centre of mass lies in its centre opens from the top and is hinged on the bottom. A cable which makes an angle of 45° with the window holds it stationary at an angle of 35° as shown in the picture.

A If the window has a mass of 12.0 kg and is 82 cm deep, what is the tension in the cable?

B Determine the horizontal and vertical force exerted on the window by the hinge at the bottom.



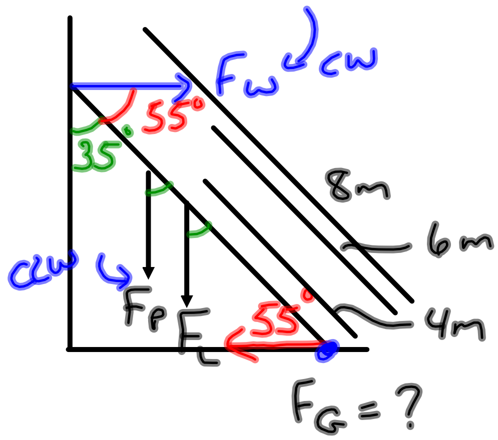


$$F_w = mg = (12\,kg)(9.8\,m/s^2) = 117.6\,N$$

A)
$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$
$$F_w r \sin\theta = T r \sin\theta$$
$$(117.6\,N)(0.41\,m)\sin 35° = T(0.82\,m)\sin 45°$$
$$\underline{47.7\,N = T}$$

B) Use 1st Condition of SE
$$\Sigma F_{left} = \Sigma F_{right}$$
$$T_x = F_{HX}$$
$$T\cos\theta = F_{HX}$$
$$47.7\,N \cos 10° = F_{HX}$$
$$\Sigma F_{up} = \Sigma F_{down}$$
$$F_{HY} = F_w + T_y$$
$$F_{HY} = 117.6\,N + 47.7\,N \sin 10°$$
$$F_{HY} = 126\,N$$

10 A uniform 8.00 m ladder has a mass of 17.0 kg and makes an angle of 35° with a frictionless wall. A 75.0 kg painter stands in the ladder 2.00 m from the top. What horizontal force must the ground exert on the ladder to keep it from slipping?



$F_L = mg$
$= (17\,kg)(9.8\,m/s^2)$
$= 166.6\,N$

$F_P = mg$
$= (75\,kg)(9.8\,m/s^2)$
$= 735\,N$

Pull

If we take the top of the ladder as the pivot point, all the forces would produce clockwise torques making it impossible to set up the 2nd condition of static equilibrium, $\Sigma\tau_{cw} = \Sigma\tau_{ccw}$. Therefore, we choose the pivot point it be the ground and find F_W. Once we have F_W, we can use the first condition of static equilibrium to find F_G.

$$\Sigma\tau_{cw} = \Sigma\tau_{ccw}$$

$$F_W r \sin\theta = F_P r \sin\theta + F_L r \sin\theta$$

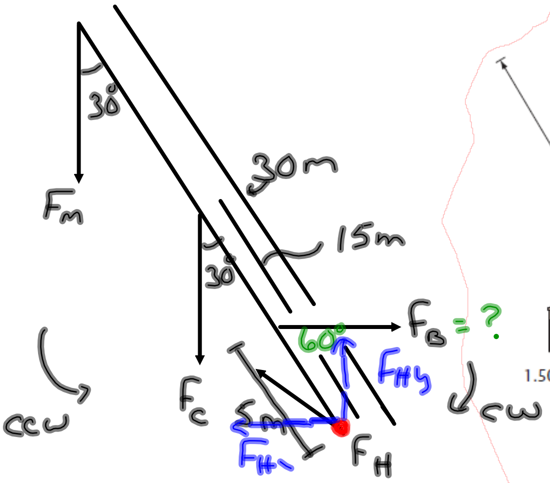
$$F_W(8m)\sin 55^\circ = (735N)(6m)\sin 35^\circ + (166.6N)(4m)\sin 35^\circ$$

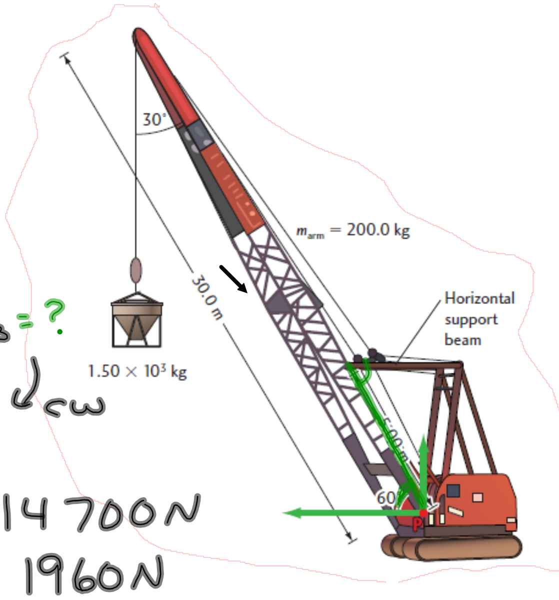
$$(6.5532m)F_W = 2911.7\,Nm$$

$$F_W = 444\,N$$

$\therefore F_G = 444N$ in opposite direction.

11 A 30.0 m long crane of mass 200.0 kg is supported at an angle of 60.0° above the horizontal by a support beam 5.00 m from its base. A mass of 1.50 x 10^3 kg hanging from the crane and the crane's arm are held in static equilibrium by a torque provided by the support beam and a force applied at the base. Find the tension in the support beam and the vertical and horizontal reaction forces at the base if the crane's arm.





$F_m = (1500kg)(9.8m/s^2) = 14\,700N$

$F_c = (200kg)(9.8m/s^2) = 1960N$

$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$

$F_B r \sin\theta = F_m r \sin\theta + F_c r \sin\theta$

$F_B (5)\sin 60° = (14700)(30)\sin 30° + (1960)(15)\sin 30°$

$F_B = 54\,300N$

B) $F_{left} = F_{Right}$

$F_{Hx} = F_B$

$F_{Hx} = 54\,300N$ (in opposite direction)

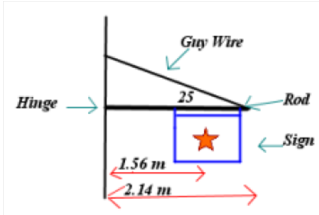
$F_{up} = F_{down}$

$F_{Hy} = F_m + F_c$

$F_{Hy} = 14700N + 1960N$

$F_{Hy} = 16\,700N$

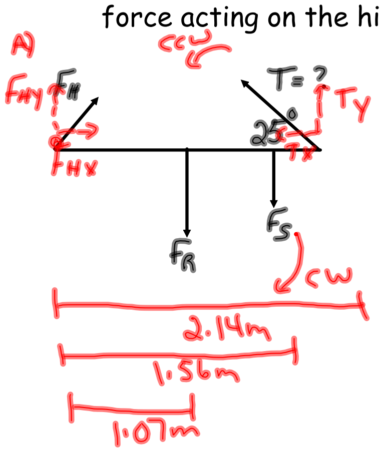
12. A 20.0 kg sign hangs from a 5.00 kg uniform rod, 2.14 m long as shown in the diagram.



A) What tension must the guy wire provide to prevent the sign from rotating about the hinge if the center of mass of the sign is 1.56 m from the hinge?

B) Find the horizontal and vertical components of the force acting on the hinge.

A)



$$F_R = (5\,kg)(9.8\,m/s^2)$$
$$= 49\,N$$
$$F_S = (20\,kg)(9.8\,m/s^2)$$
$$F_S = 196\,N$$

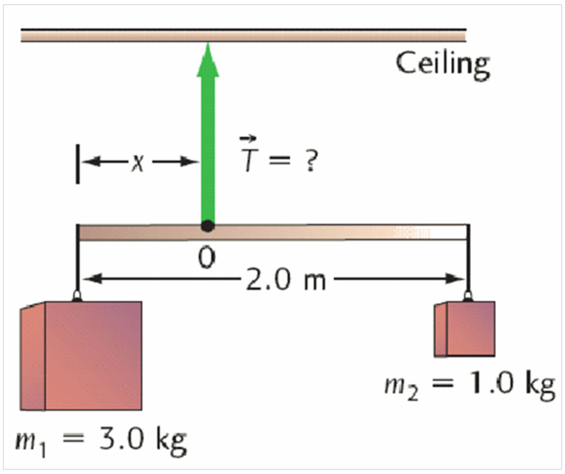
$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$
$$F_r r + F_S r = T r \sin\theta$$
$$(49N)(1.07m) + (196N)(1.56m) = T(2.14m)\sin 25^\circ$$
$$396N = T$$

B)

$$F_{Left} = F_{Right}$$
$$T_X = F_X$$
$$T\cos\theta = F_X$$
$$396N \cos 25^\circ = F_X$$
$$359N = F_X$$
$$F_{up} = F_{down}$$
$$F_{HY} + T_Y = F_S + F_R$$
$$F_{HY} + T\sin\theta = F_S + F_R$$
$$F_{HY} + 396N\sin 25^\circ = 196N + 49N$$
$$F_{HY} = 77.6N$$

13. Two children of masses 17 kg and 27 kg sit at opposite ends of a 3.8 m long teeter-totter that is pivoted at the centre. Where would a third child of mass 20 kg sit in order to balance the ride?

14. The diagram at the right shows a 2.0-m-long rod with a 1.0-kg mass at one end and a 3.0-kg mass at the other end.



a) If the mass of the rod is negligible and the system is balanced, where is the centre of gravity of the system?

b) What is the tension in the single support cable?