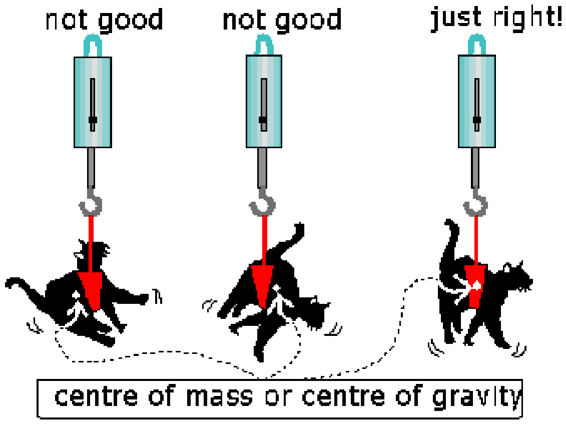
Static Equilibrium and Torque

Section 1: Static Equilibrium I - Balancing Forces

Statics - the physics of keeping objects still by applying forces on them in the appropriate places.
- An object is static equilibrium has no translational motion and no rotational motion.
Hence, all applied forces lead to zero velocity and zero acceleration.

To acquire **static equilibrium**, we need to know
- what forces must be applied and
- where the force or forces must be applied.

Every object has a point where its mass and weight appear to be concentrated. This point is called the **centre of mass (CM) or the centre of gravity (CG)**. The cat in the picture is "balanced" when the supporting string falls in line with the white dot, which is its CM and CG.

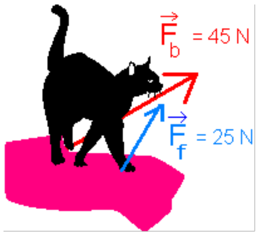


To achieve static equilibrium, 2 conditions must be met:

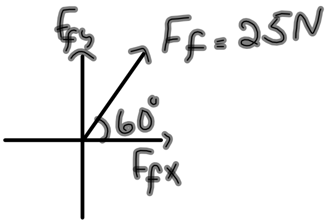
1. $F_{net} = 0$, ie å $F_x = 0$, and å $F_y = 0$, where F_{net} is the sum of all forces acting through the cente of mass. If $F_{net} = 0$, then there is no translational acceleration or motion.
2. $\tau_{net} = 0$. If $\tau_{net} = 0$, then there is no rotational motion.

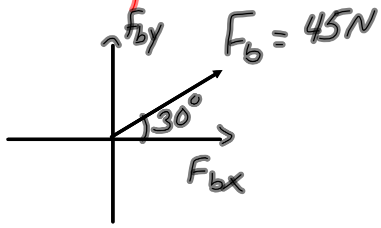
Example 1

In the picture to the right the cat is pushing with its front legs and back legs. The front legs cause a forward force of F_f = 25 N [60° above the horizontal], and the back legs cause a forward force of F_b = 45 N [30° above the horizontal]. What must be the magnitude and direction of the equilibrium force (F_e) applied by you in order to maintain static equilibrium?



equilibrium force => equal in magnitude but opposite in direction to the net force.





$F_{fx} = 25N\cos 60° = 12.5N$

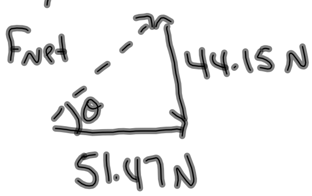
$F_{fy} = 25N\sin 60° = 21.65N$

$F_{bx} = 45N\cos 30° = 38.97N$

$F_{by} = 45N\sin 30° = 22.5N$

$\Sigma F_x = 51.47N$ (R)

$\Sigma F_y = 44.15N$ (U)



$F_{net}^2 = (44.15N)^2 + (51.47N)^2$

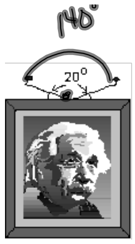
$F_{net} = 67.8N$ at 40.6° above horizontal

$\tan\theta = \frac{44.15N}{51.47N}$

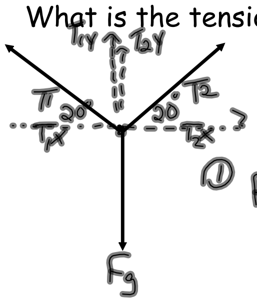
$\theta = 40.6°$

Answer: $F_e = 68$ N at 41° below the horizontal.

Example 2



In the picture to the right the CG of the piece of art is directly under the point where the supporting strings are attached. The hanging picture weighs 72 N and each of the supporting strings makes an angle of 20° with the frame. What is the tension in each string?



Picture is in static equilibrium so $F_{Net} = 0$. ie $\Sigma F_{left} = \Sigma F_{right}$ & $\Sigma F_{up} = \Sigma F_{down}$.

① $F_{left} = F_{right}$

$T_{1x} = T_{2x}$

② $F_{up} = F_{down}$

$T_{1y} + T_{2y} = F_g$

$2T_y = F_g$

$2T\sin\theta = F_g$

$T = \frac{F_g}{2\sin\theta}$

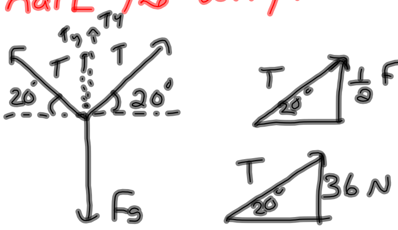
$T = \frac{72N}{2\sin 20°}$

$T = 110N$

* $T_{1y} = T_{2y}$ b/c both strings are acting at the same angle.

Second Method:

Since the strings make the same angle with the picture, the tensions in the strings are the same. Each string supports half the weight.



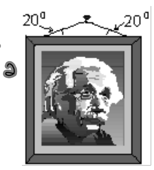
$\frac{1}{2}F_g = \frac{1}{2}(72N) = 36N$

$\sin 20° = \frac{36N}{T}$

$T = \frac{36N}{\sin 20°} = 110N$

Example 3

In the picture to the right the CG of the piece of art is directly under the nail that supports it. The hanging picture weighs 72 N and the supporting string forms an isosceles triangle with the frame as shown. What is the tension in the string?



$F = F_g$

$T_1 = T_2$

$F_g = T_{1y} + T_{2y}$

$F_g = 2T_y$

$F_g = 2T\sin\theta$

$72N = 2T\sin 20°$

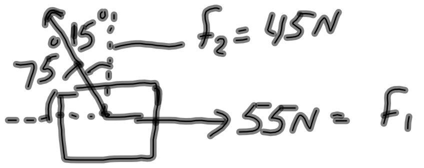
$110N = T$

Example 4

The following set of forces act on a common point: F_1 = 55 N [due E] and F_2 = 45 N [15° W of N]. What additional force is needed to maintain static equilibrium?

15 W of N

[N 15° W]

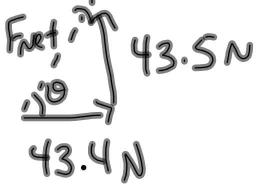


$F_{2x} = 45N\cos 75° = 11.6N\,[W]$

$F_{2y} = 45N\sin 75° = 43.5N\,[N]$

$\Sigma x = 11.6N\,[W] + 55N\,[E] = 43.4N\,[E]$

$\Sigma y = 43.5N\,[N]$



$F_{Net}^2 = (43.5N)^2 + (43.4N)^2$

$F_{Net} = 61N\,[E\,45°N]$

Ans

$F_{eq} = 61N\,[W\,45°S]$

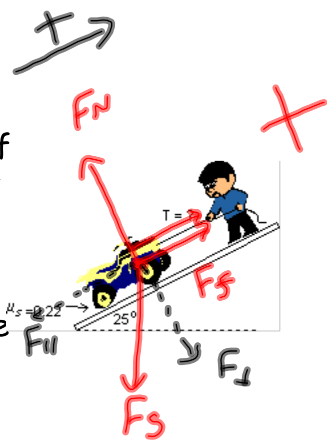
$\tan\theta = \frac{43.5N}{43.4N}$

$\theta = 45°$

Example 5

Our intrepid physics student has gotten himself in trouble again. He has manage to get his ATV halfway up a ramp and realizes that he needs help.

With the aid of friction caused by a flat tire he just manages to hold the machine steady. The coefficient of static friction is 0.22 and the ATV has a mass of 250 kg. What is the tension in the rope?



$F_{Net} = T + F_f - F_{\parallel}$

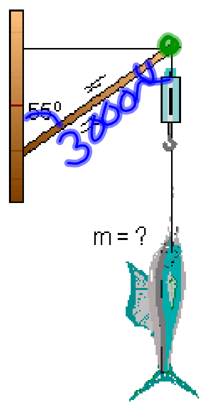
$0 = T + \mu mg\cos\theta - mg\sin\theta$

$0 = T + (0.22)(250)(9.8)\cos 25 - (250)(9.8)\sin 25°$

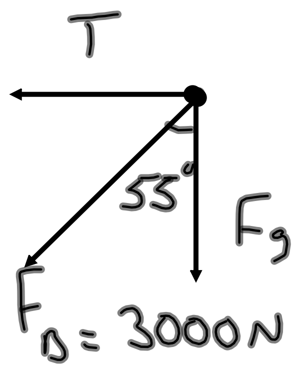
$T = 550N$

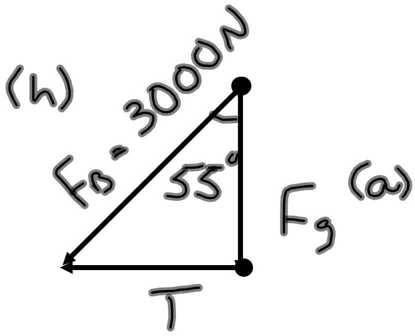
Example 6

A boom set-up for weighing very large fish is shown in the picture. The boom can withstand a compression force of 3.0×10^3 N. What is the mass of the largest fish that can be weighed?



Setting up an FBD for the boom will introduce unnecessary info.
It is sufficient to consider the force at the top of the boom.,





$$\cos 55^\circ = \frac{F_g}{3000N}$$

$$F_g = 1720.7N$$

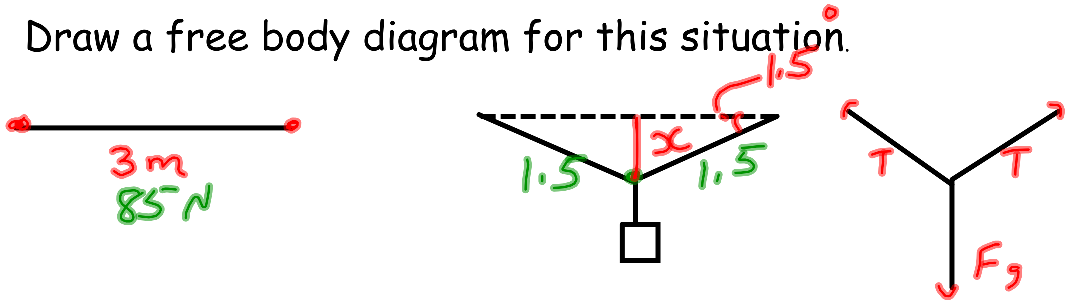
$$m = \frac{F_g}{g} = \frac{1720.7N}{9.8 m/s^2}$$

$$m = 176 kg$$

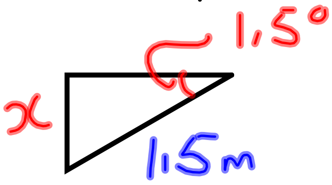
Example 7

A bag of clothespins hung in the middle of a 3.00 m clothesline causes the line to dip 1.5° below the horizontal at each end.

a) Draw a free body diagram for this situation.



b) How far does the center of the line dip when the bag of clothespins is hung on it? find "x" in (a)

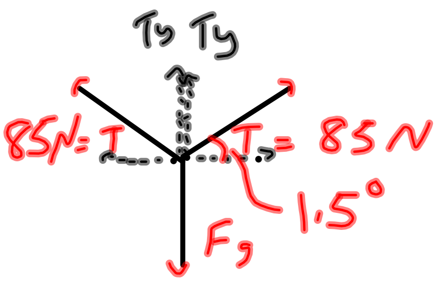


$$\sin 1.5^\circ = \frac{x}{1.5\,m}$$

$$x = 0.039\,m$$

$$x = 3.9\,cm$$

c) What is the mass of the bag of clothespins if the tension in the line is 85.0 N?



$$F_{up} = F_{down}$$

$$2T_y = F_g$$

$$\frac{2T\sin\theta}{g} = \frac{mg}{g}$$

$$\frac{2(85N)\sin 1.5^\circ}{9.8\,m/s^2} = m$$

$$0.45\,kg = m$$

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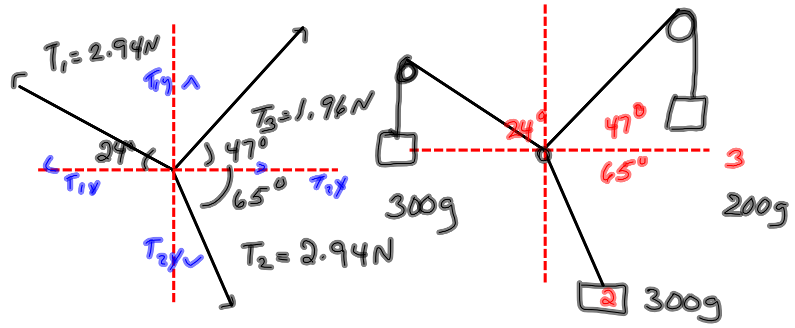


Tension in the strings is provided by the force of gravity.

*Purpose of Lab: Show that the masses are in static equilibrium.

Step 1: Find F_g for each mass. This is the tension in each string.

$F_{g1} = (0.3\,kg)(9.8\,m/s^2) = 2.94\,N$

$F_{g2} = 2.94\,N$

$F_{g3} = (0.20\,kg)(9.8\,m/s^2) = 1.96\,N$

To show static equilibrium show that

$\Sigma F_{left} = \Sigma F_{right}$ & $\Sigma F_{up} = \Sigma F_{down}$.

$T_{1x} = 2.94\,N \cos 24° = 2.69\,N$ (L)

$T_{1y} = 2.94\,N \sin 24° = 1.20\,N$ (U)

$T_{2x} = 2.94\,N \cos 65° = 1.24\,N$ (R)

$T_{2y} = 2.94\,N \sin 65° = 2.66\,N$ (D)

$T_{3x} = 1.96\,N \cos 47° = 1.34\,N$ (R)

$T_{3y} = 1.96\,N \sin 47° = 1.43\,N$ (U)

$\Sigma T_{left} = 2.69\,N$

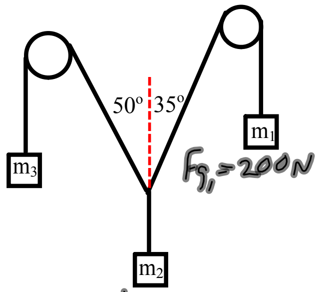
$\Sigma T_{right} = 1.24\,N + 1.34\,N = 2.58\,N$

$\Delta T = 0.11\,N$ (left) — source of errors
- should be zero.

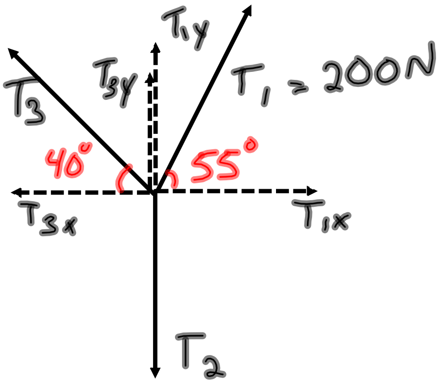
$\Sigma T_{up} = 1.20\,N + 1.43\,N = 2.63\,N$

Example 8

In the figure below, the pulleys are frictionless and the system is in static equilibrium. If Fg1 = 200.0 N, find the weight of object 2 and 3.



*Tension in each string is being provided by the masses. $(T = F_g)$



$$\Sigma F_x = 0$$

$$T_{3x} = T_{1x}$$

$$T_3 \cos 40^\circ = T_1 \cos 55^\circ$$

$$\frac{T_3 \cos 40^\circ}{\cos 40^\circ} = \frac{200\,N \cos 55^\circ}{\cos 40^\circ}$$

$$T_3 = 150\,N$$

$$\Sigma F_y = 0$$

$$T_{3y} + T_{1y} = F_{g2}$$

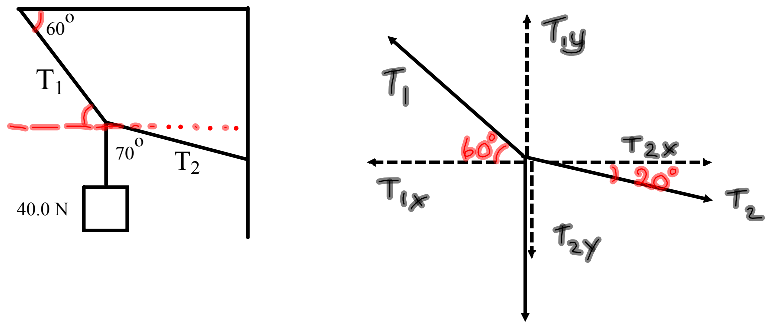
$$T_3 \sin\theta + T_1 \sin\theta = F_{g2}$$

$$150\,N \sin 40^\circ + 200\,N \sin 55^\circ = F_{g2}$$

$$260\,N = F_{g2}$$

Example 9

Find the tension in each string.



1st Condition of Static Equilibrium

$\Sigma F_x = 0$

$\Sigma F_{left} = \Sigma F_{right}$

$T_{1x} = T_{2x}$

$T_1 \cos 60° = T_2 \cos 20°$

$T_1 = \dfrac{T_2 \cos 20°}{\cos 60°}$

$\boxed{T_1 = 1.8794 T_2}$

$\Sigma F_y = 0$

$\Sigma F_{up} = \Sigma F_{down}$

$T_{1y} = T_{2y} + T_3$

$T_1 \sin 60° = T_2 \sin 20° + 40\,N$

$(1.8794 T_2) \sin 60° = T_2 \sin 20° + 40\,N$

$1.6276 T_2 = 0.3420 T_2 + 40\,N$

$1.2856 T_2 = 40\,N$

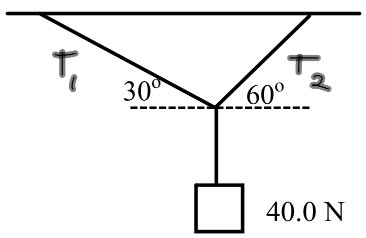
$\boxed{T_2 = 31.1\,N}$

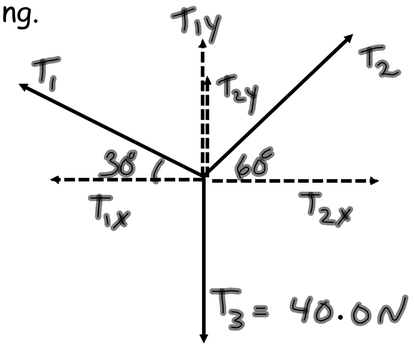
$T_1 = 1.8794 T_2$

$= 1.8794(31.1\,N)$

Example 10

Find the tension in each string.





$$\Sigma F_x = 0$$

$$\Sigma F_{left} = \Sigma F_{right}$$

$$T_{1x} = T_{2x}$$

$$T_1 \cos 30° = T_2 \cos 60°$$

$$T_1 = \frac{T_2 \cos 60°}{\cos 30°}$$

$$\boxed{T_1 = 0.5774\,T_2}$$

$$\Sigma F_y = 0$$

$$\Sigma F_{up} = \Sigma F_{down}$$

$$T_{1y} + T_{2y} = T_3$$

$$T_1 \sin 30° + T_2 \sin 60° = 40N$$

$$(0.5774\,T_2)\sin 30° + T_2 \sin 60 = 40N$$

$$0.2887\,T_2 + 0.8660\,T_2 = 40N$$

$$1.1547\,T_2 = 40N$$

$$\boxed{T_2 = 34.6N}$$

$$T_1 = 0.5774\,T_2$$

$$= 0.5774(34.6N)$$

$$= 20.0N$$