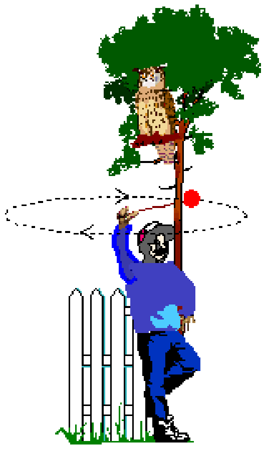
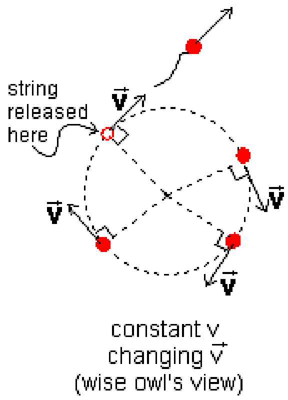
Section 2: Tangential Velocity and Centripetal Acceleration

Look at the two pictures below. On the left you see a boy twirling a ball on a string, which he later releases. On the right you see the circular path from the point of view of the wise old owl sitting in the tree.





What does the wise old owl see?

If the wise old owl is really wise in the ways of physics, she should notice at least three things:

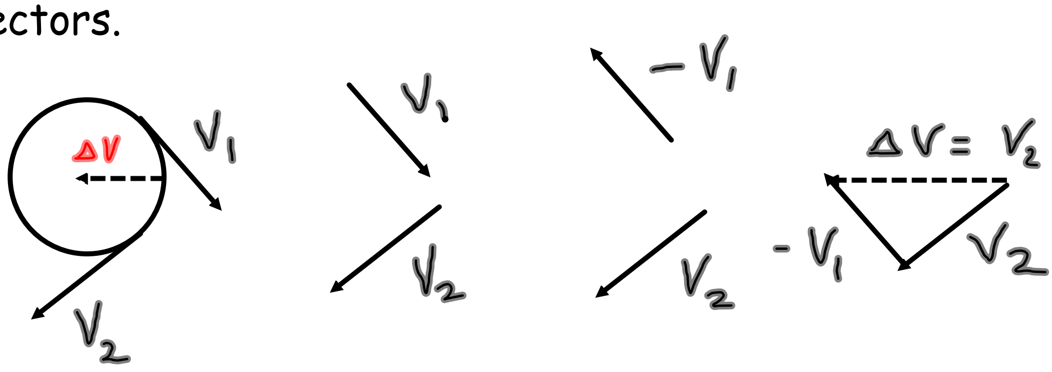
- in uniform circular motion the **speed of the object** is constant
- in uniform circular motion the **velocity is always** changing!
- the **velocity vector** is always **perpendicular** to the **radius** of the circular path
- if the object is released it will fly off in a direction perpendicular to the radius.

It's because of the ever present, 90° angle that the velocity is called **tangential velocity**.

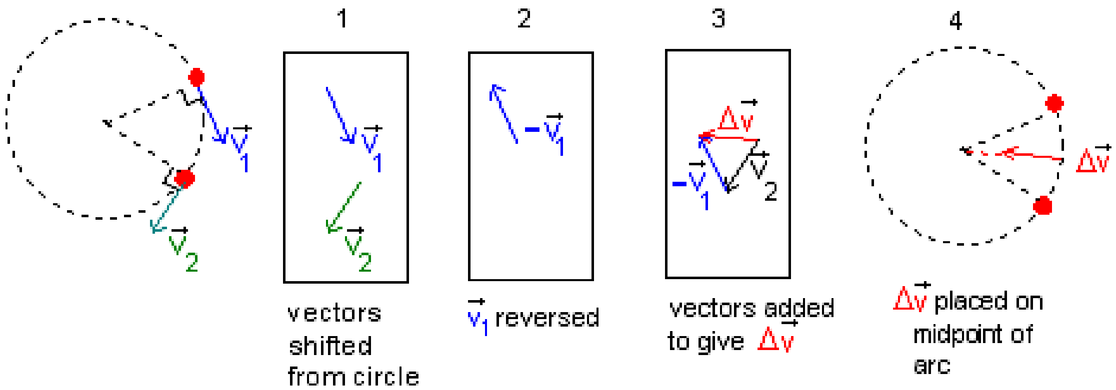
Now it's time to look more closely at the way that the tangential velocity is changing.

- To get "a change" a subtraction must be done.
- In other words, if a vector is to be subtracted, add its opposite instead. Obviously, the "opposite" for a vector means that it must be flipped through 180° so that it heads in the opposite direction.

How do we find the change in tangential velocity on a circle? First, we need to choose 2 positions and draw the velocity vectors at each position. Then we will subtract the velocity vectors.



$$\vec{\Delta V} = \vec{V_2} - \vec{V_1}$$
$$\vec{\Delta V} = \vec{V_2} + \vec{-V_1}$$



The vector, $\vec{\Delta v}$ is placed on the midpoint of the arc because $\vec{\Delta v}$ is the average change in the velocity over the whole arc. Since it is a uniform change, Δv is also the instantaneous change at the very centre of the arc.

If that last sentence bothered you, think about this: at the first dot in the picture, there is no change in the velocity. Then as the ball moves around to the second dot, its direction (and, therefore, the velocity) changes more and more. There is a smooth change like this: $\vec{\Delta v}$, $\vec{\Delta v}$, $\vec{\Delta v}$, $\vec{\Delta v}$, $\vec{\Delta v}$.

The average change, $\vec{\Delta v}$, occurs in the middle of the sequence. And that's why Δv is placed in the centre of the arc.

$\vec{\Delta v}$ is pointed towards the centre of the circle, no matter where we choose v_1 and v_2.

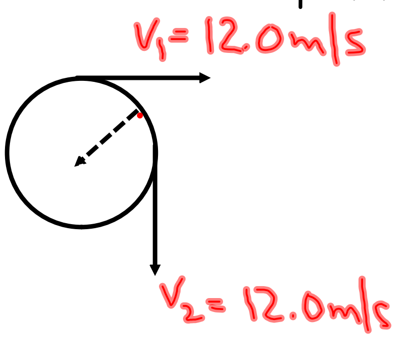
Since Δt has no direction, and since $\Delta\vec{v}/\Delta t$ is the acceleration, it follows that an object twirling on a circular path is forever accelerating toward the center of the circular path. The word for center-seeking is <u>centripetal</u>.

"Whenever an object is undergoing uniform circular motion, it has centripetal acceleration."

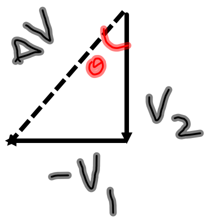
Example:

At Δt = 0.0 s the velocity of an object twirling in uniform circular motion is 12.0 m/s East. One-tenth of a second later its velocity is 12.0 m/s South. (Notice that its speed did not change). Determine the average centripetal acceleration.

A picture should help immensely:



$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t}$$



Pythagorean Theorem

$$\Delta v^2 = (12\,m/s)^2 + (12\,m/s)^2$$

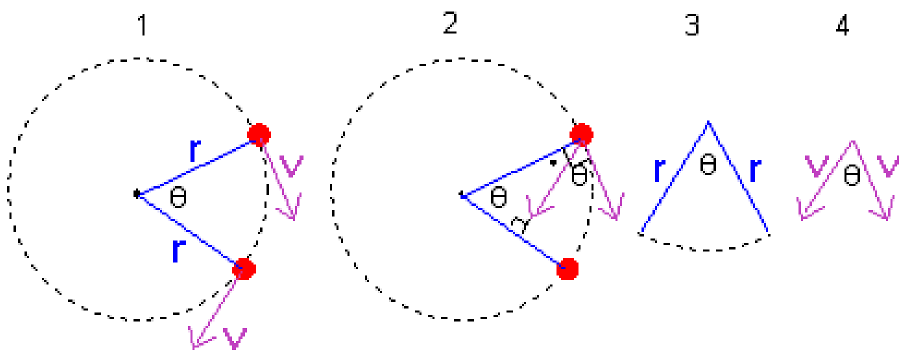
$$\Delta v = 17\,m/s\ [S\ 45°\ W]$$

$$a = \frac{\Delta v}{t} = \frac{17\,m/s\ [S\ 45°\ W]}{0.10\,s}$$

$$= 170\,m/s^2\ [S\ 45°\ W]$$

Another way to find centripetal acceleration (a_c)

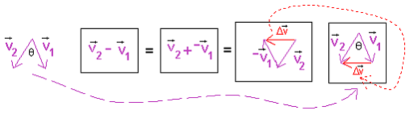
You must not rush this section. You may have to read it more than once. Look at the picture below and then read the material after it.



The vector symbol has been omitted to keep the picture as "clean" as possible. The two speeds (v) are identical because the ball has uniform circular motion. The two radii (r) are also equal.

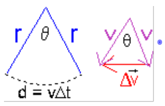
In part 2 of the picture, the second speed has been displaced so that its tail is at the tail of the first speed. Note that its direction did not change. It is still parallel to its original position. With a little thought you should also be able to see that the angles marked θ are equal to each other.

In parts 3 and 4 the two radii and the two speeds have been isolated to give two triangles (of sorts). In the case of the radi triangle one side is curved, but if θ were smaller, you would hardly notice the curve. This curved part is just the distance traveled by the ball: d = vΔt. In the case of the speed "triangle" one side is missing altogether! This is remedied in the next series of pictures.



Follow the picture from left to right and you will learn that the missing bit of the speed triangle is the change in the velocities, Δv.

We are now ready to consider the radii triangle and the speed triangle again. Note that both are isosceles triangles with the apex angles equal. This means the base angles in one triangle equal the base angles in the other.



Therefore the triangles are similar to each other, which means that ratios of corresponding sides are equal. It is useful to use

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v \times v}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r} \qquad but \ \frac{\Delta v}{\Delta t} = a$$

$$a = \frac{v^2}{r} \qquad a_c = \frac{v^2}{r}$$

In this case, a is the centripetal acceleration and therefore we will use the symbol a_c.

$a_c \propto v^2$ if "r" remains constant

(If v doubles, a will quadruple.)

$a \propto v^2$

$a \propto (2)^2$

$a \propto 4$

* $a_c \propto 1/r$ if and only if the velocity is constant which is seldom the case.

(If 'r' is doubled & velocity remains constant, then a is decreased by a factor of 2.)

$a \propto \frac{1}{r}$

$a \propto \frac{1}{2}$

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r is doubled a: ?

$a = \frac{v^2}{r}$ $v = 2\pi r f$

$v \propto r$

$v \propto (2)$

$a \propto \frac{(2)^2}{2}$

$a \propto 2$

Note: $v = 2\pi r f$ and $a = \frac{v^2}{r}$

so, $a_c = \frac{(2\pi r f)^2}{r}$

$a_c = \frac{4\pi^2 r^2 f^2}{r}$

$a_c = 4\pi^2 r f^2$

$\boxed{\text{So: } a_c \propto r}$

So, $a_c = \frac{v^2}{r}$

Example:

A good example of very rapid uniform circular motion is the way that your rear bicycle wheel spins when you turn your bike bottom up and crank the peddles. Imagine that you have done so, and have place two wads of modeling clay on the spokes. One wad is 12 cm from the axel, while the other is 24 cm from the axel. The wheel rotates at 120 RPM. Determine the centripetal acceleration for each wad of clay.

$r_1 = 0.12$ m $\qquad$ $r_2 = 0.24$ m

$$f = \frac{120}{60\,\text{sec}} = 2.0Hz$$

$$v_1 = 2\pi rf$$
$$v_1 = 2\pi(0.12)(2.0Hz)$$
$$v_1 = 1.51\,m/s$$

$$v_2 = 2\pi rf$$
$$v_2 = 2\pi(0.24)(2.0Hz)$$
$$v_2 = 3.0\,m/s$$

$$a_c = \frac{v^2}{r} = \frac{(1.51\,m/s)^2}{0.12\,m} = 19\,m/s^2$$

$$a_c = \frac{v^2}{r} = \frac{(3.0\,m/s)^2}{0.24\,m} = 38\,m/s^2$$

In the above example, the centripetal acceleration doubled from 19 m/s^2 to 38 m/s^2 when the radius doubled from 12 cm to 24 cm. Yet the expression for centripetal acceleration is $a_c = v^2/r$ which suggests that a_c should have decreased when r increased. How do you explain away the apparent contradiction?

$$a_c = \frac{v^2}{r}$$

$a_c \propto v^2$ if "r" constant.

[$a_c \propto \frac{1}{r}$ <u>if and only if</u> V is constant.]

Since $v = 2\pi r f$

$$a_c = \frac{v^2}{r} = \frac{(2\pi r f)^2}{r} = 4\pi^2 r f^2$$

From the above formula

$a_c \propto r$

Example

If a third wad of clay is placed 18 cm from the axle of the wheel of practice exercise 2 where the acceleration is 28.5 m/s^2, how fast will it be moving?

$r = 0.18\,m$

$a = 28.5\,m/s^2$

$v = ?$

$$a_c = \frac{v^2}{r}$$

$$v^2 = a_c r$$

$$v^2 = (28.5\,m/s^2)(0.18\,m)$$

$$v = 2.3\,m/s$$

Homework

1. A motor cycle traveling at 55 km/hr has a centripetal acceleration of 7.5 m/s^2 as it rounds a circular curve in the road. What is the radius of the curve?

2. What is the magnitude of the centripetal acceleration at a point 7.6 cm from the centre of a centrifuge that is spinning at 9.5 x 10^3 rpm? How many "g's" are represented?

Do questions 1 - 6, p. 206 and 17, p. 226

1\. $v = 55\,km/h = 15.3\,m/s$

$a = 7.5\,m/s^2$

$r = ?$

$$a_c = \frac{v^2}{r}$$

$$r = \frac{v^2}{a} = \frac{(15.3\,m/s)^2}{7.5\,m/s^2}$$

$$r = 31\,m$$

2\. $a = ?\ (v^2/r)$

$r = 0.076\,m$

$$f = \frac{9500}{60\,s} = 158.3\,Hz$$

$v = ?$

$$v = 2\pi r f$$

$$= 2\pi(0.076\,m)(158.3\,Hz)$$

$$= 75.6\,m/s$$

$$a_c = \frac{v^2}{r} = \frac{(75.6\,m/s)^2}{0.076\,m} = 75202\,m/s^2$$

$$\#g = \frac{75202\,m/s^2}{9.8\,m/s^2} = 7674\,g\text{'s} = 7700\,g\text{'s}$$