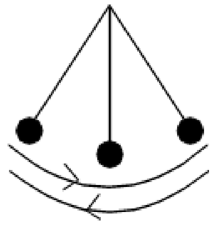
Circular Motion

Section 1: Terminology

Cycle: A complete path or oscillation of an object. In the case of a swinging mass, one cycle is the complete path or oscillation as the swinging mass goes through one complete swing and comes back to its starting point.



Period: The amount of time it takes for a vibrating object to go through one oscillation or one cycle.

$$Period = \frac{time}{cycles}$$

$$T = \frac{t}{cycles}$$ ← Know

where t is the time in seconds and T is the period in seconds

Frequency: the number of cycles an object can complete in one second.

$$frequency = \frac{cycles}{time}$$

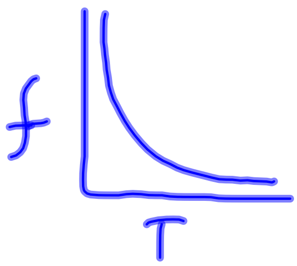
$$f = \frac{cycles}{t}$$ — Know

where t is the time in seconds and f is the frequency in (1/s) or s^{-1} or hertz (Hz)

Note: Frequency and period are <u>inverses</u> of each other.

$$f = \frac{cycles}{t} \quad and \quad T = \frac{t}{cycles}$$

$$f = \frac{1}{T} \quad and \quad T = \frac{1}{f}$$



Examples:

1. A pendulum completes 12 swings in 6.0 s. Find its frequency and period.

$$f = \frac{\text{cycles}}{\text{time}} = \frac{12\ \text{swings}}{6.0\,s} = 2.0\ Hz$$

$$T = \frac{1}{f} = \frac{1}{2.0\,Hz} = 0.50\ s$$

2. A tuning fork vibrates 512 times in 2.00 s. Find its frequency and its period.

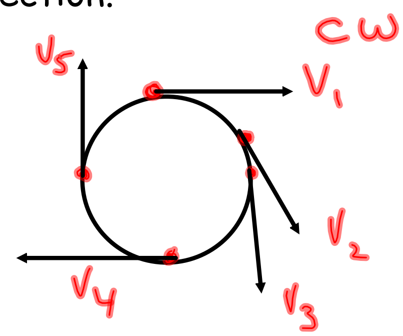
$$f = \frac{\text{cycles}}{\text{time}} = \frac{512}{2.0\,s} = 256\ Hz$$

$$T = \frac{1}{f} = \frac{1}{256\,Hz} = 3.91\times10^{-3}\ s$$

Uniform Circular Motion

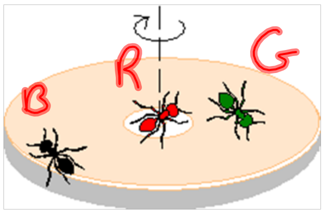
Uniform circular motion is the motion of an object in a circle with a constant or uniform speed.

An object moving in a circle of fixed radius will cover the same linear distance each second. When moving in a circle, the distance the object covers is actually the circumference of the circle. While the **speed** of the object remains **constant**, the MC **velocity** is continually **changing**. The velocity has a constant magnitude but a changing direction. The direction of the velocity is always directed tangent to the circle and as the object turns, the tangent line, and hence the velocity, is always pointed in a new direction.



Difference between Rotating and Revolving

Consider: The three ants on the following disc.
What is rotating and what is revolving?



The disc is rotating and the red ant is rotating. The black ant and green ant are revolving.

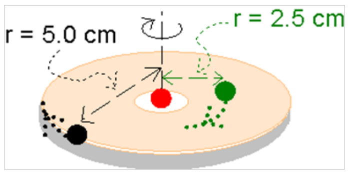
What's the difference?

The axis of rotation (the dashed line) passes through the disc and the red ant. The red ant and the disc are rotating about the axis. The axis of rotation is "outside of" the black ant and green ant. These two ants are revolving around the axis.

What rotates and revolves at the same time?

The earth rotates about its axis and revolves around the sun. (Tilt-a-world)

Look at the disc again. (Ants are replaced by dots.)



How far does each ant travel in $\frac{1}{2}$ second? (1 revolution)

Black Ant	Red Ant	Green Ant
$d = C = 2\pi r$ $= 2\pi(5.0\text{cm})$ $= 31.4\text{cm}$	$d = 0$	$d = 2\pi r$ $= 2\pi(2.5\text{cm})$ $= 15.7\text{cm}$

How fast is each ant traveling?

Black Ant	Red Ant	Green Ant
$V = \frac{d}{t} = \frac{31.4\text{cm}}{0.50\text{s}}$ $V = 62.8\text{cm/s}$	$V = 0\text{ m/s}$	$V = \frac{d}{t} = \frac{15.7\text{cm}}{0.50\text{s}}$ $= 31.4\text{cm/s}$

What is the relationship between the radius of revolution and the speed?

Pull

When the radius doubled (from 2.5 cm to 5.0 cm) the speed also doubled (from 31.4 cm/s to 62.8 cm/s).
That is, $v \propto r$

There may be something a little bothersome to you when you say that the black ant travels twice as fast as the green ant. After all, both ants made exactly one revolution in exactly the same time.

The reason for this is because there are two kinds of speeds:

1) angular or rotational speed - which refers to the number of rotations or revolutions per second

$$\omega = \frac{\theta}{t}$$ where θ is the angle in radians

not responsible for this - college or university level physics.

2) linear speed or tangential velocity - refers to the speed in the direction of a tangent drawn to the circular path. Linear speed depends on the distance an object is from the axis of rotation. $v = \frac{d}{t}$

We will concern ourselves with linear or tangential velocity.

$$v = \frac{d}{t}$$

But in a circle, d = c = 2πr and t = time for one period = T

Therefore, $v = \dfrac{2\pi r}{T}$ know

where, v - is the linear speed or tangential velocity in m/s
r - is the radius of the circle in m
T - is the period or time for one cycle in seconds

Recall: $f = \dfrac{1}{T}$

So, another formula for tangential velocity is $v = 2\pi r f$

where, v - is the linear speed or tangential velocity in m/s
r - is the radius of the circle in m
f - is the frequency in Hz

Examples:

1. On a simple playground merry-go-round Jack sits 1.5 m from the axis of rotation and Jill sits 2.5 m from the axis. Little Miss Muffet has a stop-watch and finds that the merry-go-round rotates 26 times in one minute. Determine the linear speed and ~~rotational speed~~ of Jack and Jill.

Jack: $r = 1.5\,m$

Jill: $r = 2.5\,m$

cycles = 26 times

time = 1 min

$$f = \frac{26\text{ times}}{60\,s} = 0.433\,Hz$$

Jack: $v = 2\pi r f = 2\pi(1.5\,m)(0.433\,Hz)$
$= 4.08\,m/s$
$= 4.1\,m/s$

Jill: $v = 2\pi(2.5\,m)(0.433\,Hz) = 6.8\,m/s$

2. A wheel of diameter 26 cm turns at 1500 rpm. How far will a point on the outer rim move in 2.0 s?

$r = 13\,cm$

$r = 0.13\,m$

$$f = \frac{1500\text{ rev}}{60\,s} = 25\,Hz$$

$d = ?$

$t = 2.0\,s$

$v = 2\pi r f$
$= 2\pi(0.13\,m)(25\,Hz)$
$= 20.42\,m/s$

$d = vt$
$= (20.42\,m/s)(2.0\,s)$
$= 41\,m$