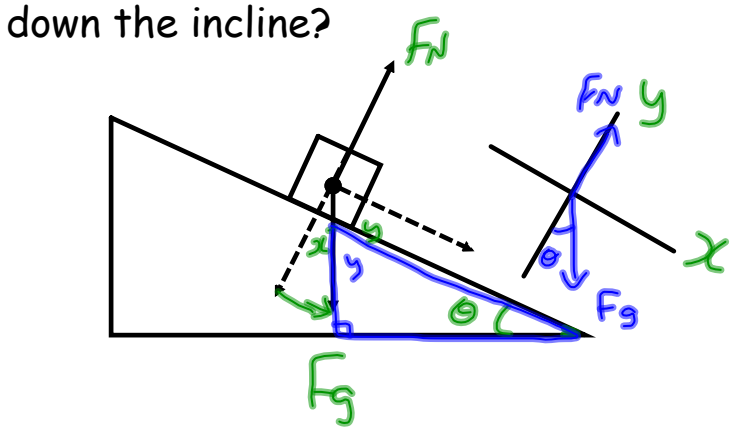
Section 2.4: Newton's Laws on an Incline

An object on a tilted surface will often slide down the surface. The rate at which it slides down the surface is dependent upon how tilted the surface is; the **greater the tilt (angle of inclination), the faster the object will slide.**

A **tilted surface** is called an **incline plane**. Objects accelerate down an incline plane because of an unbalanced force. What force is responsible for the object sliding down the incline?



$x + y = 90°$
$y + \theta = 90°$
$x + y = y + \theta$
$x = \theta$

There are **at least two forces** acting on an object on an incline plane:

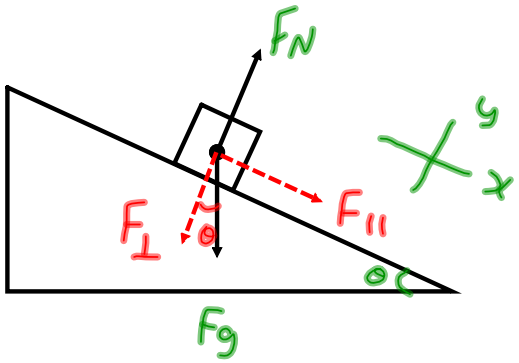
1) F_g - downward direction
2) F_N - acts in a direction perpendicular to the surface.

If we use the x- and y-axis to analyze the motion of an object on an incline plane, the object will move in both the x and y directions and therefore, the problem may be difficult to solve.

SO, we rotate the coordinate axes to make problems easier to solve.

- x-axis is parallel to the incline plane
- y-axis is perpendicular to the incline plane.

Now the object will be moving in one direction and it becomes much easier to analyze its motion.



As you know any force directed at an angle to the horizontal must be resolved into x- and y-components.
In this case, we will have to resolve the force of gravity into components - one directed parallel to the inclined surface and one perpendicular to the inclined surface.

$\sin\theta = \frac{F_{||}}{F_g}$ $\quad$ $\cos\theta = \frac{F_\perp}{F_g}$

$F_{||} = F_g \sin\theta$ $\quad$ $F_\perp = F_g \cos\theta$

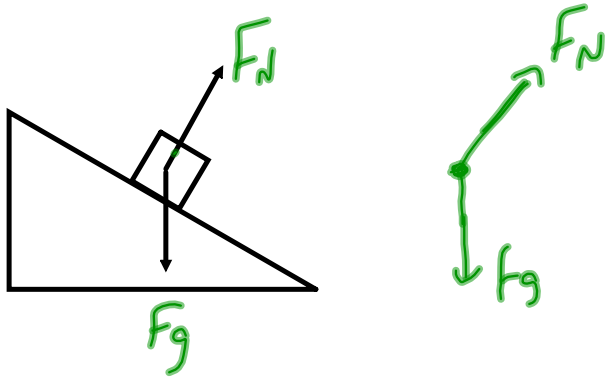
$\boxed{F_{||} = mg\sin\theta}$ $\quad$ $\boxed{F_\perp = mg\cos\theta}$

Note: $F_N = F_\perp$ mc

Always

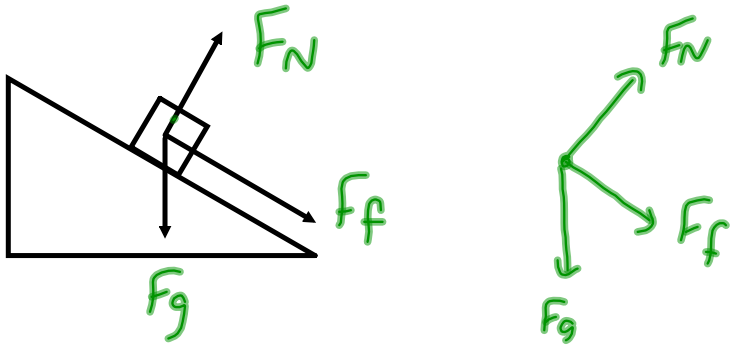
Free Body Diagrams for Objects on an Incline

Frictionless Incline:

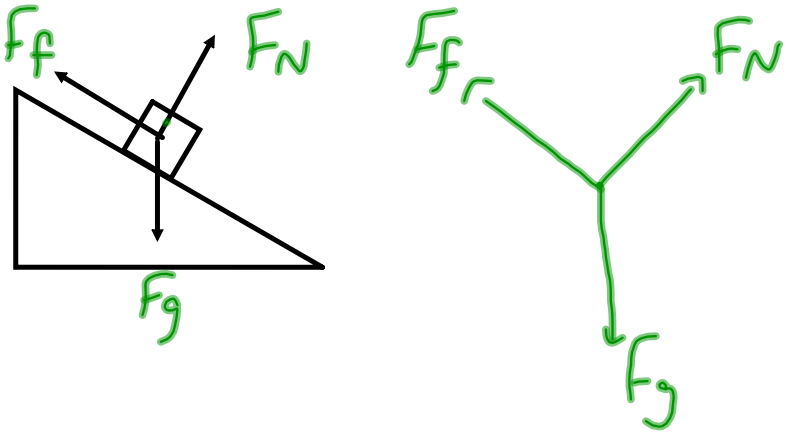


Incline with friction

up the incline

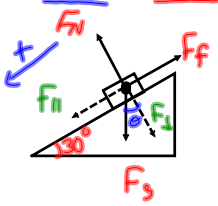


down the incline



Examples

1. A 35.0 kg box sits on a plank while a worker raises one end so that the box slides down the plank to a co-worker at the other end. At the very instant the box is about to slide, the plank makes an angle of 30.0° with the ground. What is the coefficient of static friction? MC



Common mistake on exam
$F_f = \mu mg$

$F_{net} = F_{||} - F_f$

$F_{net} = mg\sin\theta - \mu F_N$

Impt.

$F_N = F_\perp = mg\cos\theta$

$F_{net} = mg\sin\theta - \mu mg\cos\theta$

$0 = (35\,kg)(9.8\,m/s^2)\sin 30° - \mu(35\,kg)(9.8\,m/s^2)\cos 30°$

$\mu = 0.577$

$F_{net} = mg\sin\theta - \mu mg\cos\theta$

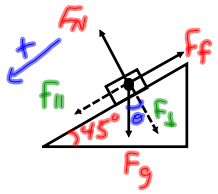
$0 = mg\sin\theta - \mu mg\cos\theta$

$\frac{\mu mg\cos\theta}{mg\cos\theta} = \frac{mg\sin\theta}{mg\cos\theta}$

$\mu = \frac{\sin\theta}{\cos\theta}$

$\mu_s = \tan\theta$

2. A 45.0 kg object is placed on a ramp that makes an angle of 45° with the ground. If the coefficient of kinetic friction is 0.43, find the acceleration of the box.



$F_{net} = F_{||} - F_f$

$ma = mg\sin\theta - \mu mg\cos\theta$

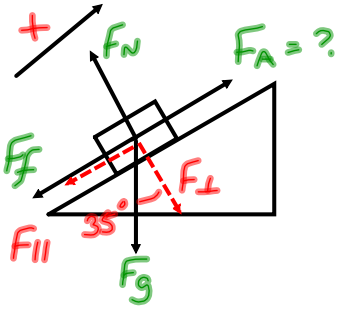
$a = g\sin\theta - \mu g\cos\theta$

$a = 9.8\sin 45° - 0.43(9.8\,m/s^2)\cos 45°$

$a = 3.9\,m/s^2$

$a = g(\sin\theta - \mu\cos\theta)$

3. A rocket has a mass of 1200 kg and is accelerated up a ramp at 5.0g's. The coefficient of friction between the ramp and the rocket is 0.60. What must be the thrust of the rocket if the angle of the ramp is 35°?



* cannot cancel "m".

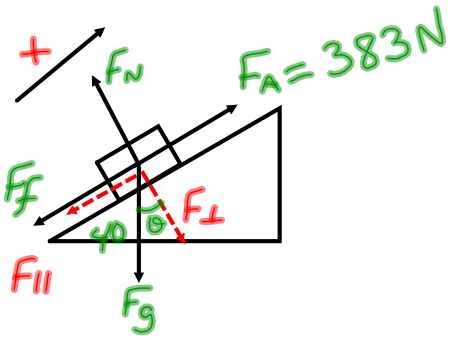
$$F_{net} = F_A - F_{||} - F_f$$

$$ma = F_A - mg\sin\theta - \mu mg\cos\theta$$

$$(1200)(49) = F_A - (1200)(9.8)\sin 35° - (0.6)(1200)(98)\cos 3$$

$$71000\,N = F_A$$

4. A person pushes a 25 kg box up an incline. He applies a force of 383 N parallel to the surface of the incline. The box accelerates up the incline at 0.75 m/s^2. Find the coefficient of kinetic friction between the box and the incline if the angle of the ramp is 40.0°.



$$F_{net} = F_A - F_{||} - F_f$$

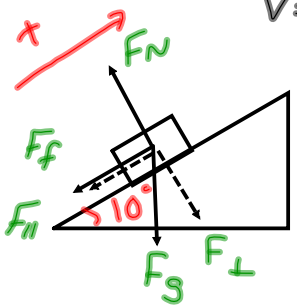
$$ma = F_A - mg\sin\theta - \mu mg\cos\theta$$

$$(25)(0.75) = 383 - (25)(9.8)\sin 40° - \mu(25)(9.8)\cos 40°$$

$$-206.77 = -187.7\mu$$

$$1.1 = \mu$$

5. A skier skiing downhill reaches the bottom of a hollow with a velocity of 20.0 m/s and then coasts up a hill that has a 10.0° slope. If the coefficient of kinetic friction is 0.10, how far up the hill will she travels before she stops?



$0.10 = \mu$

$d = ?$

$$F_{net} = -F_{||} - F_f$$

$$ma = -mg\sin\theta - \mu mg\cos\theta$$

$$a = -9.8\,m/s^2 \sin 10^\circ - (0.1)(9.8)\cos 10^\circ$$

$$a = -2.67\,m/s^2$$

$$d = \frac{V_2^2 - V_1^2}{2a}$$

$$= \frac{0 - (20\,m/s)^2}{2(-2.67\,m/s^2)}$$

$$= 75\,m$$

Textbook: Page 196. Questions 1, 2, 4.
Page 224. Questions 6-10.

Do worksheet on incline planes.