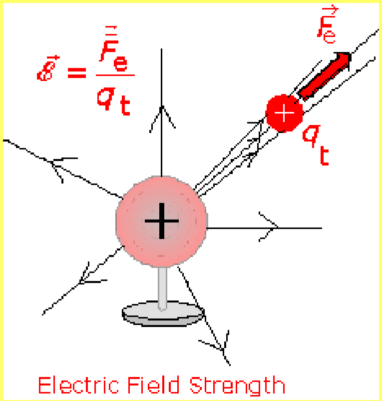
Section 15: Electric Field Strength

In this lesson you will

- define electric field strength and use the definition to express the concept mathematically
- use Coulomb's Law to write a second expression for electric field strength
- compare electric field strength with gravitational field strength
- determine the electric field strength at a particular point due to the presence of one or more charged objects.

You can exert a force on things because you have strength. Since an electric field exerts a force on a charge that is in the field, the field must also have strength.

A charged spherical metal ball is placed on a glass insulating stand. All around it exists an electric field. When a small positive test charge, q, is brought into the vicinity of the charged sphere, the electric field, E, will exert a force, F, on the test charge.



The force exerted on the test charge will depend on the strength or intensity of the field at the point where the test charge is placed and on the size of the test charge.

$$F \propto E \quad \text{and} \quad F \propto q$$

Definition: The **Electric field strength** at a particular point in the field is defined as the force per unit charge on a positive test charge placed at that point, or in mathematical terms:

$$E = \frac{F_e}{q}$$

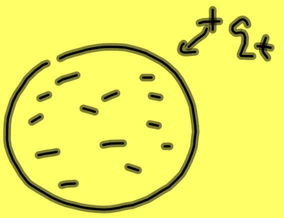
Not given

where: F_E is the electrical force in Newtons (N)
q is the positive test charge in Coulombs (C)
E is the electric field strength in Newtons/Coulomb(N/C)

The units of electric field strength are N/C. Unless it becomes critical that the direction be maintained in the computations, we will compute the magnitude *E* from $E = \frac{F_e}{q}$, and add the appropriate direction to the final answer if required. In many textbooks the term **electric field strength** is shortened to **electric field**.

Examples

1. A charged rubber balloon has an electric field around it. At a point in the field, a test charge of 1.5 μC experiences a force of 3.0×10^{-4} N. What is the electric field strength?



Balloon

$$\vec{\mathcal{E}} = \frac{F}{q_t} = \frac{3.0\times10^{-4}\,N}{1.5\times10^{-6}\,C}$$

$$\vec{\mathcal{E}} = 2\times10^{2}\,N/C$$

(toward the main charge)

2. A test charge of 1.3×10^{-7} C is placed in an electric field of 3.9×10^{3} N/C. What is the electric force exerted on the test charge?

q_t $\qquad \mathcal{E} = \frac{F}{q_t}$

(q_m)

$$F = \mathcal{E} q_t$$

$$= (3.9\times10^{3}\,N/C)(1.3\times10^{-7}\,C)$$

$$= 5.1\times10^{-4}\,N$$

Direction: toward q_m if q_m is −

: away from q_m if q_m is +

3. What is the value of a test charge that experiences an electric force of 7.6×10^{-2} N in an electric field of 9.5×10^{3} N/C?

$$\mathcal{E} = \frac{F}{q_t}$$

$$q_t = \frac{F}{\mathcal{E}} = \frac{7.6 \times 10^{-2} N}{9.5 \times 10^{3} N/C} = 8.0 \times 10^{-6} C$$

Note: the main charge (the one causing the field) did not enter into any calculations. This means that we don't have to worry about the main charge being a point charge, as was required by Coulomb's Law.

In other words, it is permissible to use $E = F_e / q_t$ when the main charge is distributed over different shape surfaces--it doesn't have to be on a nice, neat, tiny sphere.

Another way to calculate Electric Field Strength

However, we are willing to restrict the main charge, qm, to a point source then we can derive another expression for ***E***.

Recall Coulomb's Law: $\boldsymbol{F = \frac{kq_1q_2}{r^2}}$

If the source charge, q_m, is a point charge, then the force experience by a test charge qt placed near the source charge, q_m, is given by: $\boldsymbol{F = \frac{kq_mq_t}{r^2}}$

Substitute this expression into $\boldsymbol{E = \frac{F}{q_t}}$ and simplify.

$$\mathcal{E} = \frac{\frac{kq_mq_t}{r^2}}{q_t}$$

$$\mathcal{E} = \frac{kq_m\cancel{q_t}}{r^2} \times \frac{1}{\cancel{q_t}}$$

$$\mathcal{E} = \frac{kq_m}{r^2}$$

TWO EXPRESSIONS FOR ELECTRIC FIELD STRENGTH

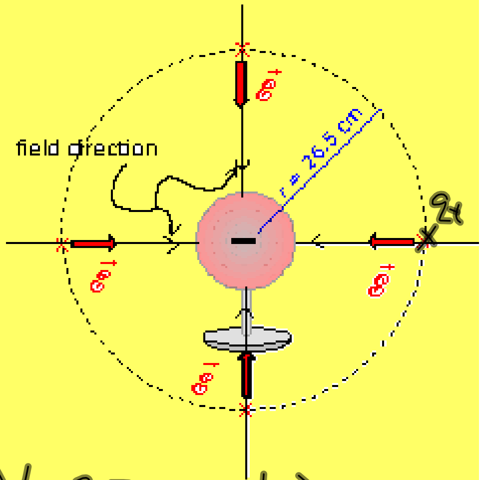
Use $E = \frac{F_e}{q}$ when the main charge is unknown or not a point source.

Use $E = \frac{kq_m}{r^2}$, when there is no test charge given and when both the main charge and the distance from the main charge are known. This reinforces the idea that the electric field has strength even if there are no test charges around.

Examples

1. Calculate the field strength 26.5 cm from a charge of -9.7 μC.

The information does not specify the exactly location, only that the point in question lies somewhere on a circle of radius 26.5 cm. The magnitude of the field strength will be the same everywhere on the circle. Only the direction changes.



$$E = \frac{kq_m}{r^2} = \frac{(9.0\times10^9\ N m^2/C^2)(-9.7\times10^{-6}\ C)}{(0.265\ m)^2}$$

$$E = -1.2\times10^6\ N/C$$

$$E = 1.2\times10^6\ N/C\ \text{(toward main charge)}$$

The magnitude of the electric field strength anywhere on 26.5 cm spherical surface is 1.2×10^6 N/C. It is obvious from the picture that the negative sign in the answer does not mean to the left or West. Rather it indicates that the direction of the field strength is towards the main charge. Note that the field strength is in the same direction as the field lines, which is the direction that a positive test would move if place at point x in the field. A negative charge would move a direction opposite the field lines.

2. The electric field strength is 2.6×10^3 N/C at a point which is 68.4 cm from a charged object. What is the charge on the object?

$\mathcal{E}$ r

$$\mathcal{E} = \frac{kq_m}{r^2}$$

$$q_m = \frac{\mathcal{E} r^2}{k}$$

$$= \frac{(2.6 \times 10^3 \text{ N/C})(0.684 \text{ m})^2}{(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2)}$$

$$= 1.4 \times 10^{-7} \text{ C}$$

3. At what distance from a charge of 6.2 μC is the electric field 4.2×10^2 N/C?

r q_m $\mathcal{E}$

$$\mathcal{E} = \frac{kq}{r^2}$$

$$r = \sqrt{\frac{kq}{\mathcal{E}}}$$

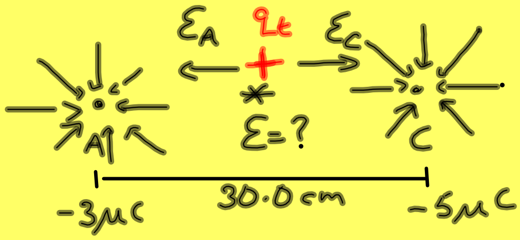
$$r = \sqrt{\frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(6.2 \times 10^{-6} \text{ C})}{4.2 \times 10^2 \text{ N/C}}}$$

$$r = 11.5 \text{ m}$$

$$r = 12 \text{ m}$$

Test

4. A) Charges A, and C are in a straight line a distance 30.0 cm apart. Find the net electric field strength at the point midway between the charges. The charges are as follows: A................ ⁻3.0 μC
C............... ⁻5.0 μC



$$\mathcal{E}_A = \frac{kq_A}{r^2} = \frac{(9\times10^9\,Nm^2/C^2)(-3\times10^{-6}\,C)}{(0.15\,m)^2}$$
$$= -1.2\times10^6\,N/C$$
$$= 1.2\times10^6\,N/C \text{ left (toward A)}$$

$$\mathcal{E}_C = \frac{kq_C}{r^2} = \frac{(9\times10^9\,Nm^2/C^2)(-5\times10^{-6}\,C)}{(0.15\,m)^2}$$
$$= -2.0\times10^6\,N/C$$
$$= 2.0\times10^6\,N/C \text{ Right (toward C)}$$

$$\mathcal{E}_{net} = 1.2\times10^6\,N/C \text{ left} + 2.0\times10^6\,N/C \text{ Right}$$
$$= 8.0\times10^5\,N/C \text{ Right.}$$

B) If a +4μC is placed midway between the two charges, what electric force will it experience?

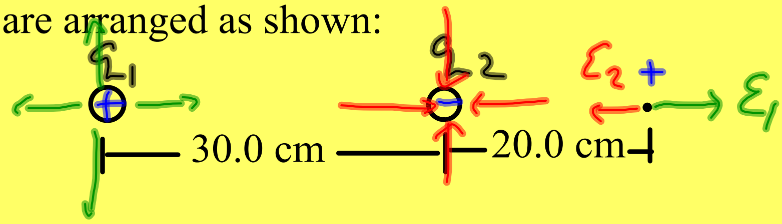
$\mathcal{E} = 8.0\times10^5\,N/C$ (Right)

$q_t = 4\times10^{-6}\,C$

$$\mathcal{E} = \frac{F}{q_t}$$

$$F = \mathcal{E}q_t$$
$$= (8.0\times10^5\,N/C \text{ (Right)})(4\times10^{-6}\,C)$$
$$= 3.2\,N \text{ [Right]}$$

5. Two point charges $q_1 = 3.6 \times 10^{-6}$ C and $q_2 = -2.7 \times 10^{-6}$ C are arranged as shown:



Find the net electric field strength at point A due to the combined electric fields of both charges.

$$\varepsilon_1 = \frac{kq_1}{r^2} = \frac{(9 \times 10^9 \, Nm^2/C^2)(3.6 \times 10^{-6} C)}{(0.50 m)^2}$$

$= 1.3 \times 10^5 N/C$ [outward]

$= 1.3 \times 10^5 N/C$ [Right]

$$\varepsilon_2 = \frac{kq_2}{r^2} = \frac{(9 \times 10^9 \, Nm^2/C^2)(-2.7 \times 10^{-6} C)}{(0.20 m)^2}$$

$= -6.1 \times 10^5 N/C$

$= 6.1 \times 10^5 N/C$ (Inward)

$= 6.1 \times 10^5 N/C$ [Left]

$\vec{\varepsilon}_{net} = 1.3 \times 10^5 N/C [R] + 6.1 \times 10^5 N/C [L]$

$= 4.8 \times 10^5 N/C [L]$

Homework 557, 1, 3, 4
579-580, 7, 17, 19
584 59-65

Electric Field Strength

Test yourself

1. A force of 4×10^{-2} N acts on a charge of 2.0 μC when it is placed in a uniform electric field. What is the magnitude of the electric field?

- ● a) 20,000 N/C
- ○ b) 2×10^{-2} N/C
- ○ c) 8×10^{-2} N/C
- ○ d) 5.0×10^{-5} N/C

$$\mathcal{E} = \frac{F}{q}$$

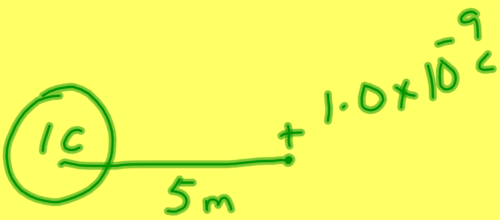
2. At 3 times the distance from a point source, what is the strength of the electric field?

- ○ a) it is 9 times its original value
- ○ b) it is 3 times it original value
- ○ c) it is one-third its original value
- ● d) it is one-ninth its original value

$$\mathcal{E} \propto \frac{1}{r^2} \propto \frac{1}{(3)^2} = \frac{1}{9}$$

3. A 1.0 C charge is 5.0 m away from a 1.0 nC test charge. What is the electric field at the 1.0 nC charge?
[nC = nano coulomb = 10^{-9} C.]

- a) 1.8×10^{9} N/C
- b) 3.6 nN/C
- c) 3.6×10^{8} N/C
- d) 1.8 N/nC



$\mathcal{E} = \frac{F}{q_t}$

$F = \frac{k q_1 q_2}{r^2}$

$\mathcal{E} = \frac{k q_m}{r^2}$

4. A 7.0 μC charge is place at the 0 cm mark of a meter stick and a -5.0 μC charge is placed at the 60 cm mark. What is the electric field at the 40 cm mark?

- a) 3.0×10^{5} N/C
- b) 1.5×10^{6} N/C
- c) 7.3×10^{5} N/CC
- d) -7.3×10^{5} N/C

5. A particle with a charge of 4.0 μC has a mass of 5.0×10^{-3} kg. What electric field directed upward will exactly balance the the weight of the particle?

- a) 1.2×10^4 N/C
- b) 4.1×10^4 N/C
- c) 8.2×10^4 N/C
- d) 5.1×10^4 N/C