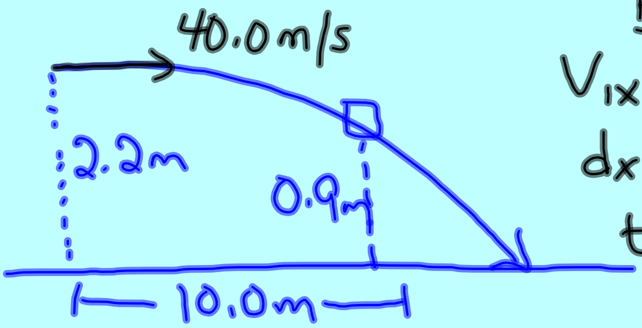
One more question dealing with horizontally launched projectiles.

Ex) Will a tennis ball, served horizontally at 40.0 m/s, from a height of 2.2 m clear a net 0.90 m high and 10.0 m away?



H		V
$V_{1x} = 40.0\,m/s$		$V_{1y} = 0\,m/s$
$d_x = 10.0\,m$		$a = -9.8\,m/s^2$
$t = ?$	⟷ same	$t = 0.25\,s$
		$d_y = ?$

$$t = \frac{d_x}{V_x} = \frac{10\,m}{40\,m/s} = 0.25\,s$$

$$d_y = \cancel{V_1 t} + \frac{1}{2}at^2$$

$$d_y = \frac{1}{2}(-9.8\,m/s^2)(0.25\,s)^2$$

$$d_y = -0.31\,m$$

This means the ball fell 0.31 m from its initial height.

$$d = 2.2\,m - 0.31\,m$$

$$d = 1.89\,m \text{ or } 1.9\,m$$

After 0.25 s the ball is 1.9 m above the ground, so it will clear a net 0.9 m high.

Section 1.4 Launching Projectiles at Any Angle

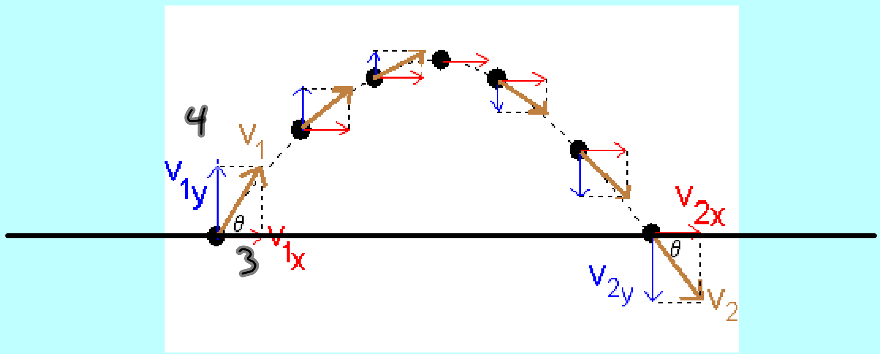
In this lesson

You will calculate:

- the total time of the motion when the initial velocity and initial height are given
- the range and final velocity when the initial velocity and initial height are given
- the velocity and displacement at anytime when the initial velocity and initial height are given
- the height from which the projectile is fired when its final velocity is given
  (e.g., the projectile may be fired at an angle from a roof top or from a cliff.)
- the initial velocity when the final velocity and change in position are given

A projectile is launched at an angle above the horizontal as shown below.

0
3
4
5



- **v_1** is the **initial velocity** of the ball. It is directed at an angle θ above the horizontal. The initial **vertical** component of the velocity is **v_{1y}** and the initial **horizontal** component of the velocity is **v_{1x}**.

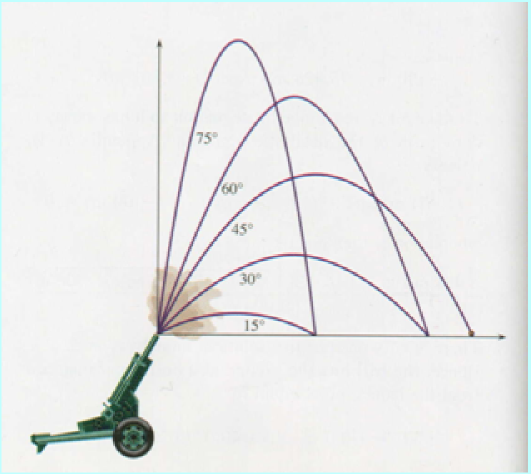
- The vertical velocity decreases until it becomes 0.0 m/s at the very top of the trajectory. Then, it increases in exactly the same increments as the ball falls back to earth. Just as the ball reaches the level at which it started, the **magnitudes** of **v_{1y}** and **v_{2y}** are identical. These two velocities are, of course, in exactly opposite directions.

- Assuming no air friction (even though that's unreal!) there is no force either aiding or opposing the horizontal vectors. Therefore the horizontal components, **v_x**, remain constant.

- Assuming the ball returns to the level at which it started, the magnitudes of **v_1** and **v_2** are the same. The difference is that **v_1** makes an angle of θ above the horizontal, while **v_2** makes an angle θ below the horizontal.

Launch Angles

Suppose a cannon ball is launched at a speed of 20.0 m/s at 15°, 30°, 45°, 60°, 75°.



15° $v_x = 20\frac{m}{s}\cos 15^\circ$ $v_y = 20\,m/s\,\sin 15^\circ$
$= 19.3\,m/s$ $= 5.2\,m/s$

30° $v_x = 20\,m/s\,\cos 30^\circ$ $v_y = 20\,m/s\,\sin 30^\circ$
$= 17.3\,m/s$ $= 10\,m/s$

45° $v_x = 14.1\,m/s$ $v_y = 14.1\,m/s$

60° $v_x = 10\,m/s$ $v_y = 17.3\,m/s$

75° $v_x = 5.2\,m/s$ $v_y = 19.3\,m/s$

1 **Which ball is in the air for the shortest period of time?**

The one launched at 15° will hit the ground first. The hang time of a projectile (i.e., the length of time the ball is in the air) is determined by the initial value of the vertical velocity component (v_{1y}). The smaller v_{1y}, the shorter the hang time.

2 **Which ball has the highest peak?**

The one launched at 75°. The peak height is determined by v_{1y}. The greater v_{1y}, the higher it will rise.

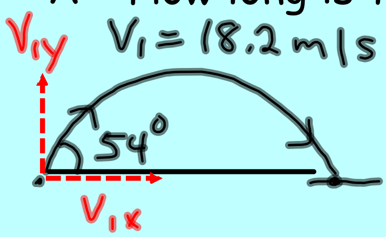
3 **Which ball has the greatest range?**

The one launched at 45°. Range is determined by v_{1x} and the hang time is determined by v_{1y}.

Examples

1 A soccer ball is kicked so that its initial velocity is 18.2 m/s at an angle of 54° with the field.

A How long is the ball in the air?



H

$V_{1x} = 18.2 m/s \cos 54^\circ$
$= 10.7 m/s$

$t = \frac{V_{2y} - V_{1y}}{a}$

$t = \frac{-14.7 m/s - 14.7 m/s}{-9.8 m/s^2}$

$t = 3.0 s$

V

$V_{1y} = 18.2 m/s \sin 54^\circ$
$= 14.7 m/s$
$a = -9.8 m/s^2$
$V_{2y} = -14.7 m/s$
(when it hits ground)

B Determine the range of the ball.

$d_x = V_x t$
$= (10.7 m/s)(3.0 s)$
$= 32 m$

C Determine the speed and direction with which the ball hits the ground.

$V_2 = 18\,m/s$ at 54° below the horizontal.

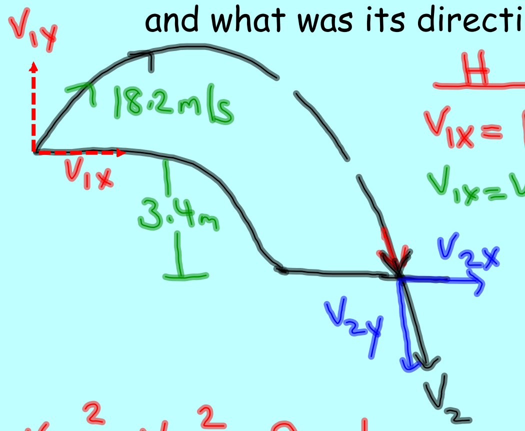
V_1 & V_2 are at ground level so V_1 & V_2 are equal in magnitude but opposite in direction.

D How high did the ball rise above the field? * use 1.5s Not 3.0s

$$d = V_1 t + \frac{1}{2} a t^2$$
$$= (14.7\,m/s)(1.5\,s) + \frac{1}{2}(-9.8\,m/s^2)(1.5\,s)^2$$
$$= 11\,m$$

2 A ball is kicked so that its initial velocity is 18.2 m/s at an angle of 54° with the playing field. This time the ball cleared an embankment and landed on a sandy beach 3.4 m below the level of the playing field.

A How fast was the ball traveling when it hit the beach and what was its direction?



H	V
$V_{1x} = 10.7\,m/s$	$V_{1y} = 14.7\,m/s$
$V_{1x} = V_{2x} = V_{3x} = \dots$	$a = -9.8\,m/s^2$
	$d_y = -3.4\,m$
	$V_{2y} = ?$

$$V_{2y}^2 = V_{1y}^2 + 2ad$$
$$= (14.7\,m/s)^2 + 2(-9.8\,m/s^2)(-3.4\,m)$$
$$V_{2y} = -16.8\,m/s$$

$$V_2^2 = V_{2x}^2 + V_{2y}^2$$
$$= (10.7\,m/s)^2 + (16.8\,m/s)^2$$
$V_2 = 19.9\,m/s$ at 58° below horizontal.

$$\tan\theta = \frac{V_y}{V_x} = \frac{16.8\,m/s}{10.7\,m/s}$$
$$\theta = 58°$$

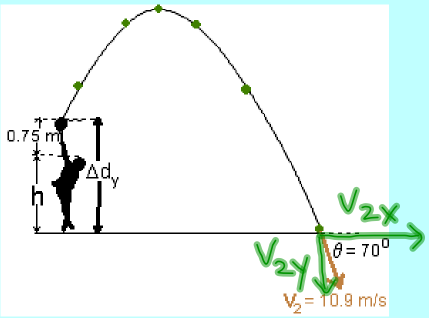
B How much longer was the ball in the air (compared to the ball in question 1)?

$$t = \frac{V_{2y} - V_{1y}}{a}$$
$$= \frac{-16.8\,m/s - 14.7\,m/s}{-9.8\,m/s^2}$$
$$= 3.2\,s$$

The ball was in the air an extra 0.20 s. (3.2 s − 3.0 s = 0.2 s)

3 A novice tennis player lobs the ball in a high arc as shown. A radar device clocks the ball at 10.9 m/s as it hits the court 1.83 s after being struck by the racket. The player has a reach of 0.75 m.

A How tall is the player?



H

$V_{2x} = 10.9\,m/s \cos 70^\circ$
$= 3.73\,m/s$
$t = 1.83\,s$

V

$V_{2y} = 10.9\,m/s \sin 70^\circ$
$= 10.2\,m/s$
use $V_{2y} = -10.2\,m/s$
$a = -9.8\,m/s^2$
$t = 1.83\,s$

$d = V_2 t - \frac{1}{2}at^2$
$d = (-10.2\frac{m}{s})(1.83s) - \frac{1}{2}(-9.8\frac{m}{s^2})(1.83s)^2$
$d = -2.26\,m$

∴ height ball was hit is 2.26m
height of player = 2.26m − 0.75m
= 1.51m

B How fast was the tennis ball traveling when first struck by the racket?

($V_1 = ?$) Find V_{1x} & V_{1y}

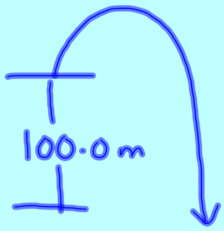
$V_{1x} = 3.73\,m/s$
$V_{1y} = V_{2y} - at$
$= -10.2\,m/s - (-9.8\,m/s^2)(1.83\,s)$
$= 7.73\,m/s$

$V_1^2 = (3.73\,m/s)^2 + (7.73\,m/s)^2$
$V_1 = 8.6\,m/s$ at 64° above the horizontal

$\tan\theta = \frac{7.73\,m/s}{3.73\,m/s}$

4 A ball is shot out of a cannon with a horizontal velocity component of 40.0 m/s and a vertical velocity component of 20.0 m/s. The cannon is sitting at the top of a 100.0 m high cliff.

A How far will it travel? (range)



H

$v_{1x} = 40 m/s$

$d_x = ?$

V

$v_{1y} = 20 m/s$

$a = -9.8 m/s^2$

$d = -100 m$

$v_{2y} = ?$

$$v_{2y}^2 = v_{1y}^2 + 2ad$$

$$= (20 m/s)^2 + 2(-9.8 m/s^2)(-100 m)$$

$$v_{2y} = \pm 48.6 m/s$$

$$v_{2y} = -48.6 m/s$$

$$t = \frac{v_{2y} - v_{1y}}{a} = \frac{-48.6 m/s - 20 m/s}{-9.8 m/s^2}$$

$$t = 7.0 s$$

$$d_x = v_x t = (40 m/s)(7.0 s)$$

$$= 280 m$$

B What will be the cannon ball's maximum height?

$v_{1y} = 20 m/s$

$a = -9.8 m/s^2$

$v_{2y} = 0$

$d = ?$

$$d_y = \frac{v_{2y}^2 - v_{1y}^2}{2a}$$

$$d_y = \frac{0 - (20 m/s)^2}{2(-9.8 m/s^2)}$$

$$d_y = 20.4 m$$

wrt cliff: max $h = 20.4 m$

wrt ground max $h = 120.4 m$ or $1.20 \times 10^2 m$

Homework: 19, 29 – 31, 33 page 115 – 116