Section 1.3: Solving Projectile Motion Problems
Horizontal Launching

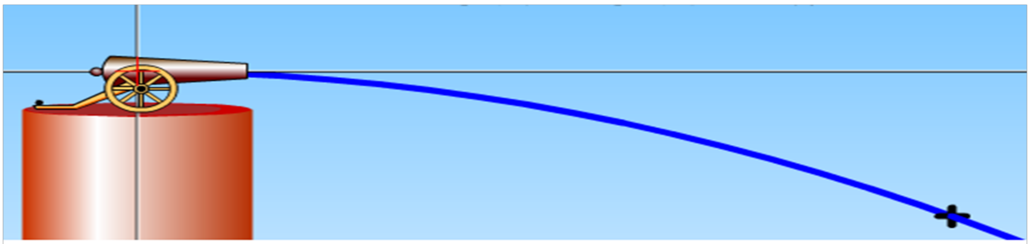
This lesson deals with the case when the projectile is propelled in a *horizontal direction* from a high point like a roof or a cliff.

You will calculate:

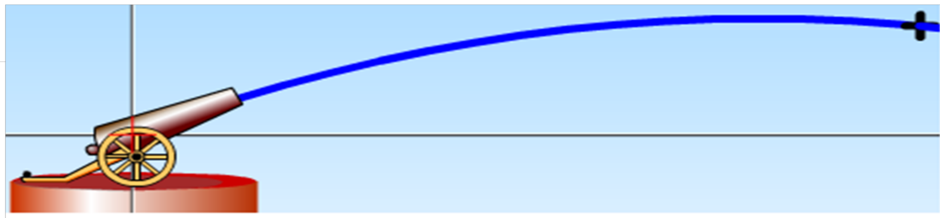
- total time of motion when the initial height is given
- range and final velocity when the initial height and velocity are given
- velocity and displacement at any time when the initial velocity is given
- initial velocity when the launch height and range are known
- height from which the projectile was fired when its final velocity is given
- initial velocity when the final velocity is given

Problems in this unit of work will include:

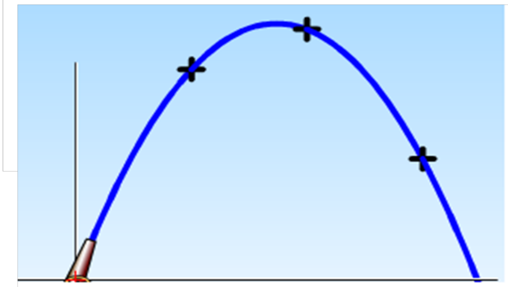
-- a projectile that is launched horizontally and lands BELOW the point of projection



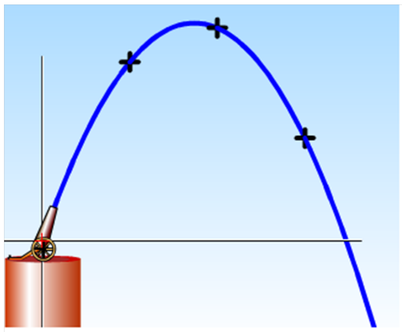
….a projectile that is launched AT AN ANGLE and lands ABOVE the point of projection



…. a projectile is launched AT AN ANGLE and lands AT the same level as the point of projection

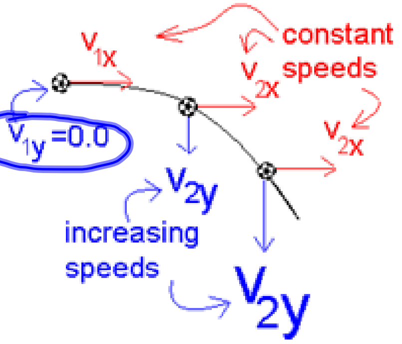


….the projectile is launched AT AN ANGLE and lands BELOW the point of projection



Horizontally Launched Projectiles

always true



$V_{1x} = V_{2x} = V_{3x} = \ldots$

Horizontal Motion
(Uniform Motion)

$v_{1x} = v_{2x} = v_{3x} = \ldots$
$d_x = v_x t$

d_x is the range

Vertical Motion
(Uniform Acceleration)

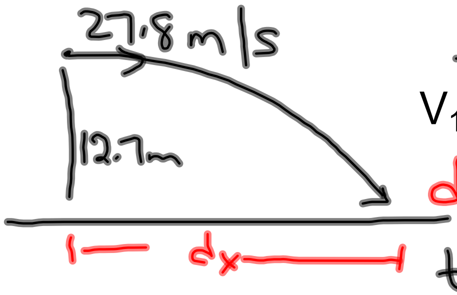
$v_{1y} = 0$
$a = -9.8\ \text{m/s}^2$

use kinematics equations to find the unknowns in the vertical direction.

The **time** of flight is the **same** for the x- and y-directions.

Examples

1 A) A golf ball is hit horizontally from a knoll that is 12.7 m above the level course. It leaves the tee traveling at 27.8 m/s. When it lands, how far will it be from the base of the knoll?



H	V
V_{1x}=27.8 m/s	d = -12.7m
$d_x =$	$a = -9.8\,m/s^2$
$t = ?$	$v_{1y} = 0$
	$t = ?$

Same

Need t use vertical givens.

$$d = \cancel{v_1 t} + \frac{1}{2}at^2$$

$$2d = at^2$$

$$\sqrt{\frac{2d}{a}} = t$$

$$\sqrt{\frac{2(-12.7m)}{-9.8m/s^2}} = t$$

$$1.61s = t$$

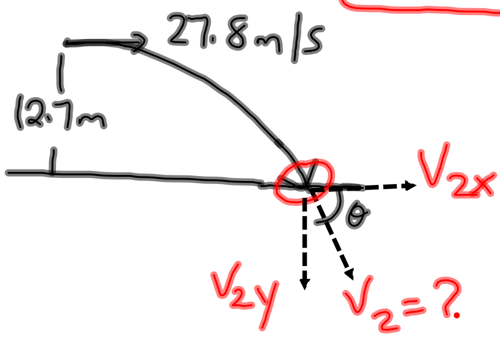
Range

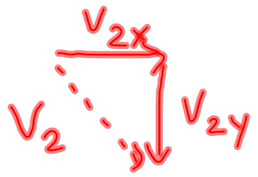
$$d_x = vt$$

$$= (27.8m/s)(1.61s)$$

$$= 44.8m$$

B) Find the velocity with which the ball hits the ground. → does not mean find V_{2y}





H	V
$V_{1x} = 27.8\frac{m}{s}$	$V_{1y} = 0$
$V_{2x} = ?$	$a = -9.8m/s^2$
	$d = -12.7m$
	$V_{2y} = ?$

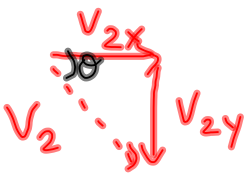
$V_{2x} = 27.8m/s$

$V_{2y}^2 = \cancel{V_{1y}^2} + 2ad$

$V_{2y}^2 = 2(-9.8m/s^2)(-12.7m)$

$V_{2y} = \pm 15.8m/s$

$V_{2y} = -15.8m/s$



$V_2^2 = V_{2x}^2 + V_{2y}^2$

$= (27.8m/s)^2 + (15.8m/s)^2$

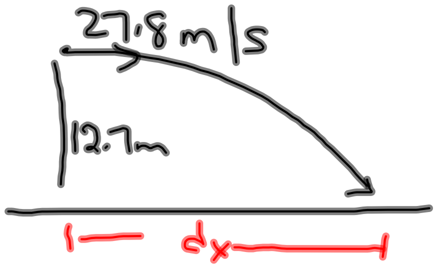
$V_2 = 32.0m/s$

$\tan\theta = \frac{15.8m/s}{27.8m/s}$

$\theta = 29.6°$

Velocity is 32.0m/s at 29.6° below the horizontal.

2. A golf ball hit horizontally from the knoll is 12.7 m above the level course. It leaves the tee traveling at 27.8 m/s. Find where the ball is at 1.2 s and how fast it is going.



H	V
$V_{1x} = 27.8\,m/s$	$V_{1y} = 0$
$t = 1.2\,s$	$a = -9.8\,m/s^2$
d_x	$t = 1.2\,s$
V_{2x}	d_y
	$V_{2y} = ?$

$V_2 = ?$

A)

$d_x = v_x t$
$= (27.8\,m/s)(1.2\,s)$
$= 33.4\,m$

$d_y = \cancel{v_1 t} + \frac{1}{2}at^2$

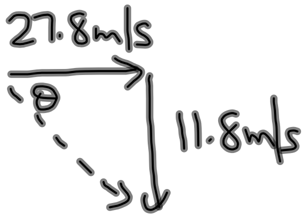
$d_y = \frac{1}{2}(-9.8\,m/s^2)(1.2\,s)^2$

$d_y = -7.1\,m$

The ball is 33.4 m to the right of the hill & 7.1 m below the top of the hill.

B) $V_{2x} = 27.8\,m/s$

$V_{2y} = V_{1y} + at$
$= (-9.8\,m/s^2)(1.2\,s)$
$= -11.8\,m/s$



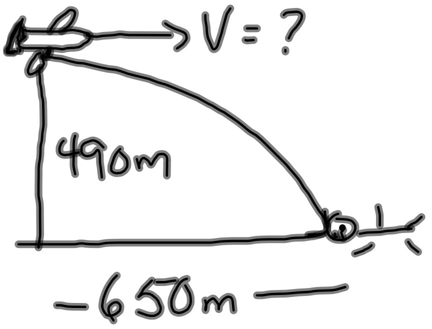
$V_2^2 = (27.8\,m/s)^2 + (11.8\,m/s)^2$

$V_2 = 30.2\,m/s$ at 23.0° below the horizontal

$\tan\theta = \dfrac{11.8\,m/s}{27.8\,m/s}$

$\theta = 23.0^\circ$

3. A swimmer is in a bit of trouble and an inflatable raft is dropped to him from a height of 490 m. The raft travels a horizontal distance of 650 m. What is the speed of the plane?



H	V
$d_x = 650\ m$	$V_{1y} = 0$
$V_x = ?$	$a = -9.8\ m/s^2$
$t = ?$	$d = -490\ m$

use Vertical givens to get t.

$$d = \cancel{V_1 t} + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2d}{a}}$$

$$t = \sqrt{\frac{2(-490\ m)}{-9.8\ m/s^2}}$$

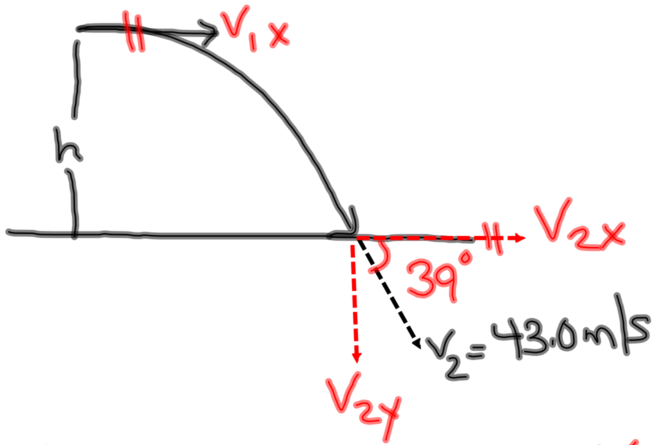
$$t = 10.0\ s$$

$$V_x = \frac{d}{t}$$

$$= \frac{650\ m}{10\ s}$$

$$= 65\ m/s$$

4. A) A ski jumper approaches the motion sensor at the landing site at a speed v_s which is determined to be 43.0 m/s. The pitch of the hill at the landing site is 39°. How high is the launch site above the bottom of the hill?



H

$V_{2x} = 43.0\text{ m/s} \cos 39^\circ$
$= 33.4$ m/s

V

$V_{2y} = 43\text{ m/s} \sin 39^\circ$
$= 27.1$ m/s

use
$V_{2y} = -27.1$ m/s
b/c V_{2y} is downward
$a = -9.8\text{ m/s}^2$
$V_{1y} = 0$
$\vec{d} = ?$

Soln

$$d = \frac{V_{2y}^2 - V_{1y}^2}{2a}$$

$$d = \frac{(-27.1\text{ m/s})^2}{2(-9.8\text{ m/s}^2)}$$

$d = -37.5$ m

∴ height of hill = 38m

B) What was the skier's speed when he left the jump?

His speed was 33 m/s.

Do questions 1-3, 5, 10, 17-28, p. 113