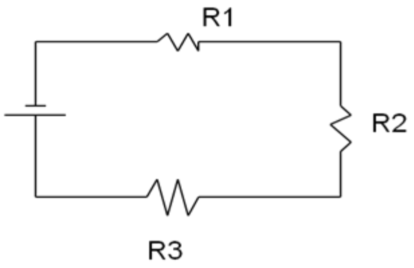
Section 9: Resistors in Series and Parallel

Series Circuit



Recall, In series,

$I_T = I_1 = I_2 = I_3$, and

$V_T = V_1 + V_2 + V_3$

Recall also, Ohm's Law, V = IR

$$V_T = V_1 + V_2 + V_3$$

Substituting V = IR

$$I_T R_T = I_1 R_1 + I_2 R_2 + I_3 R_3$$

And since I is the same throughout it can be factored and cancelled.

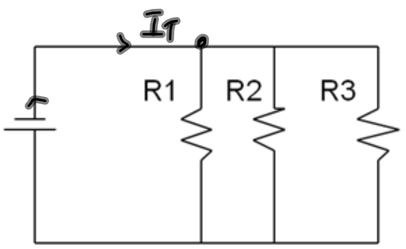
$$IR_T = I(R_1 + R_2 + R_3)$$

$$R_T = R_1 + R_2 + R_3$$

In other words, if resistors are connected in series the total resistance is equal to the sum of the individual resistors.

If all resistors have the same value (i.e. $R_1 = R_2 = R_3 = \ldots R_n$), then the total resistance may be found using $R_T = nR$.

Parallel Circuit



Recall, In Parallel,

$I_T = I_1 + I_2 + I_3$ and

~~$V_T = V_1 + V_2 + V_3$~~

$V_T = V_1 = V_2 = V_3$

Recall also, Ohm's Law: V = IR or I = V/R

$$I_T = I_1 + I_2 + I_3$$

Since I = V/R it can be substituted

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

Since V is the same everywhere, it can be factored and cancelled.

$$\frac{V}{R_T} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

When resistors are connected in parallel, their equivalent resistance is ALWAYS less than the value of the lowest resistance.

If all resistors have the same value (i,e. $R_1 = R_2 = R_3 = ...R_n$), then the equivalent resistance may be found using $R_T = \frac{R}{n}$.

$$\frac{1}{R_T} = \frac{n}{R_1}$$

Examples

1 In order for a certain series circuit to work properly a resistor of 1.2 kΩ is required. In your junk drawer you find two resistors with values of 350 Ω and 450 Ω respectively. What is the size of a third resistor that would provide the required 1.2 kΩ?

$R_T = R_1 + R_2 + R_3$

$1200\ \Omega = 350\ \Omega + 450\ \Omega + R_3$

$400\ \Omega = R_3$

2 a) Calculate the total resistance of three 120 Ω resistors connected in series.

$R_T = nR$

$= 3(120\ \Omega)$

$= 360\ \Omega$

b) Calculate the total resistance of three 120 Ω resistors connected in parallel.

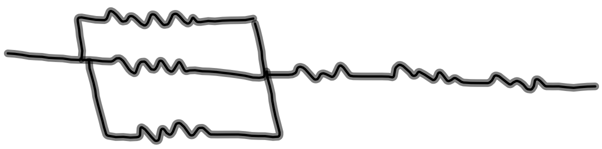
$\frac{1}{R_T} = \frac{n}{R}$

$\frac{1}{R_T} = \frac{3}{120\ \Omega}$

$R_T = \frac{120\ \Omega}{3}$

$= 40\ \Omega$

c) Calculate the total resistance of three 120 Ω resistors connected in parallel which are then connected to three 120 Ω resistors that are connected in series.



from B $R_{\parallel} = 40\ \Omega$

A $R_S = 360\ \Omega$

$R_T = 40\ \Omega + 360\ \Omega$

$R_T = 400\ \Omega$

3 A string of eight lights connected in series has a total resistance of 120 Ω. If the lights are identical, what is the resistance of each light bulb?

$R_T = 120\,\Omega$
$R = ?$
$n = 8$

$R_T = nR$
$120\,\Omega = 8R$
$\boxed{15\,\Omega = R}$

4 The above string of lights is connected in parallel. What is the resistance of each light bulb if the total resistance is still 120 Ω?

$n = 8$
$R_T = 120\,\Omega$
R

$$\frac{1}{R_T} = \frac{n}{R}$$
$$\frac{1}{120\,\Omega} = \frac{8}{R}$$
$$R = 960\,\Omega$$

5 Find the total resistance of three resistors having values 5.0 Ω, 10 Ω, and 30 Ω when they are connected in series and when they are connected in parallel.

Series: $R_T = R_1 + R_2 + R_3 = 5\,\Omega + 10\,\Omega + 30\,\Omega = 45\,\Omega$

Parallel:
$$\frac{1}{R_T} = \frac{1}{5\,\Omega} + \frac{1}{10\,\Omega} + \frac{1}{30\,\Omega}$$
$$\frac{1}{R_T} = \frac{6}{30\,\Omega} + \frac{3}{30\,\Omega} + \frac{1}{30\,\Omega}$$
$$\frac{1}{R_T} = \frac{10}{30\,\Omega}$$
$$R_T = 3\,\Omega$$

$$\frac{1}{R_T} = \frac{1}{5\,\Omega} + \frac{1}{10\,\Omega} + \frac{1}{30\,\Omega}$$
$$\frac{1}{R_T} = 0.\overline{3}$$
$$R_T = 3\,\Omega$$

$5^{-1} + 10^{-1} + 30^{-1}$
$\frac{1}{R_T} = 0.\overline{33}$
$R_T = .33^{-1}$

6 What resistance would have to be added in parallel with a 40 Ω hair dryer to reduce the equivalent resistance to 8 Ω?

$R_1 = 40\,\Omega$
$R_T = 8\,\Omega$
$R_2 = ?$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{1}{8\,\Omega} = \frac{1}{40\,\Omega} + \frac{1}{R_2}$$
$$\frac{1}{8\,\Omega} - \frac{1}{40\,\Omega} = \frac{1}{R_2}$$
$$\frac{5}{40\,\Omega} - \frac{1}{40\,\Omega} = \frac{1}{R_2}$$
$$\frac{4}{40\,\Omega} = \frac{1}{R_2}$$
$$R_2 = 10\,\Omega$$

$$8^{-1} - 40^{-1} = \frac{1}{R_2}$$
$$0.1 = \frac{1}{R_2}$$
$$10\,\Omega = R_2$$

7 A toaster draw a current of 6.0 A from a 120 V source. What resistance would have to be added in series with the same toaster to reduce its current to 4.0 A?

$V = I R$ (V constant; I ↓, R ↑)

to increase R must add resistor in series.

$$R_{toaster} = \frac{V}{I} = \frac{120V}{6A} = 20\,\Omega$$

$$R_{Total} = \frac{V}{I} = \frac{120V}{4A} = 30\,\Omega$$

∴ You need a 10 Ω resistor added in series ($R_2 = 30\,\Omega - 20\,\Omega = 10\,\Omega$)

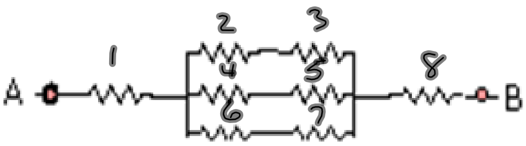
8 How many 160 Ω resistors must be connected in parallel to draw a current of 6.0 A from a 120 V source?

$$R_T = \frac{V_T}{I_T} = \frac{120V}{6A} = 20\,\Omega$$

$$\frac{1}{R_T} = \frac{n}{R}$$
$$\frac{1}{160\,\Omega} = \frac{n}{20\,\Omega}$$

n = 8 resistors

9 Each resistor in the picture to the right has a resistance of 15 Ω. What is the total resistance between A and B.



$R_{23} = R_{45} = R_{67} = 30\,\Omega$

$\frac{1}{R_{\parallel}} = \frac{1}{R_{23}} + \frac{1}{R_{45}} + \frac{1}{R_{67}}$

$\frac{1}{R_{\parallel}} = \frac{1}{30\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{30\,\Omega}$

$R_{\parallel} = 10\,\Omega$

$R_T = \underset{(1)}{15\,\Omega} + \underset{(R_{\parallel})}{10\,\Omega} + \underset{(8)}{15\,\Omega} = 40\,\Omega$

R_T

10 A technician requires a 3Ω resistor for an electronic project. However, all that is available in the supply room is a box of 12 Ω resistors. Are these any good to him? Explain.

R_1

$\frac{1}{R_T} = \frac{n}{R}$

$\frac{1}{3\,\Omega} = \frac{n}{12\,\Omega}$

$n = 4$

Yes, he can connect 4 - 12 Ω resistors in parallel.

11 Three resistors of 12 ohms each are connected in series, and three resistors of 108 ohms each are connected in parallel. If these arrangements are connected across identical 1.5 V cells, which cell will be drained first?

$R_S = 3 \times 12\,\Omega = 36\,\Omega$

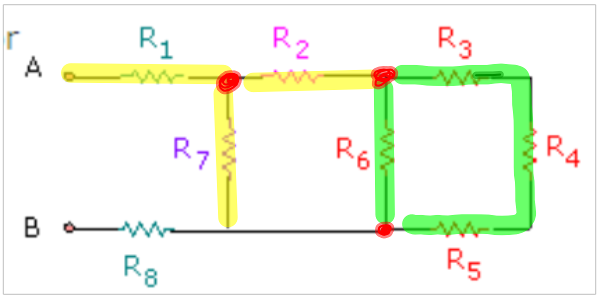
$\frac{1}{R_{\parallel}} = \frac{n}{R}$

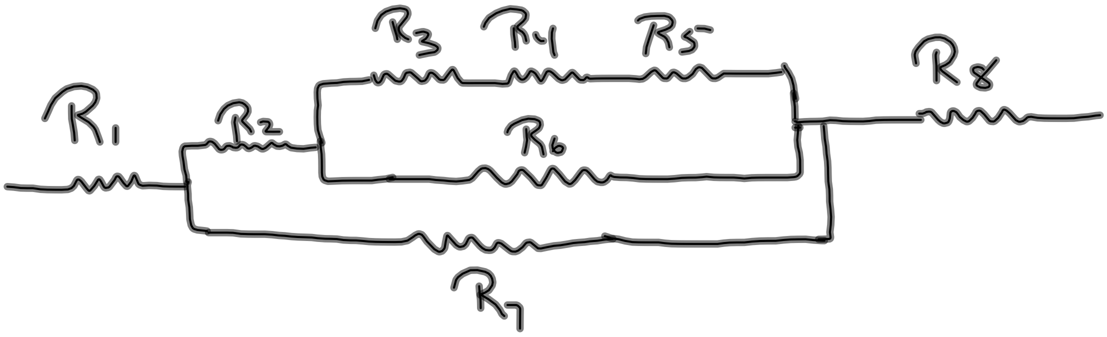
$\frac{1}{R_{\parallel}} = \frac{3}{108\,\Omega}$

$R_{\parallel} = 36\,\Omega$

Since both have the same resistance, they will drain the battery at the same rate.

12 Find the total resistance between points A and B in the drawing to the right. Each resistor has a value of 10 Ω.





$R_{345} = 3 \times 10\,\Omega = 30\,\Omega$

$\frac{1}{R_{3456}} = \frac{1}{30\,\Omega} + \frac{1}{10\,\Omega}$

$R_{3456} = 7.5\,\Omega$

$R_{23456} = 10\,\Omega + 7.5\,\Omega = 17.5\,\Omega$

$\frac{1}{R_{11}} = \frac{1}{17.5\,\Omega} + \frac{1}{10\,\Omega}$

$R_{11} = 6.4\,\Omega$

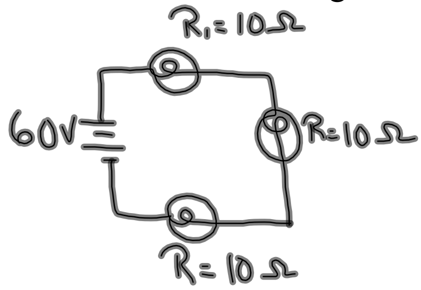
$R_T = 2 \times 10\,\Omega + 6.4\,\Omega = 26.4\,\Omega$

For Resistors in Series: Consider 3 resistors in series

- If one of the resistors is shut off so that there is a break in the circuit, there will be no current passing through the remaining resistors. Hence, for light bulbs, if one of the lights go out, they all go out.

$V = IR$

- If a fourth resistor is added in series, the total resistance of the circuit increases ($R_T = R_1 + R_2 + R_3 + R_4$) which means that the total current in the circuit will decrease. (From Ohm's Law, current is inversely proportional to the resistance when the voltage remains constant.) As a result, the voltage drop across each resistor will also be less. This means that the lights will become dimmer as more and more lights are added in series. (NOTE: it it the voltage drop across the bulb that determines the brightness; the greater the voltage drop across the bulb the bright the bulb.)

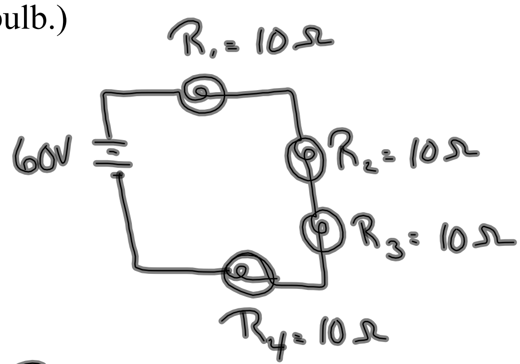


$R_T = 30\,\Omega$

$I_T = \frac{V}{R} = \frac{60V}{30\Omega} = 2A$

$V_1 = IR = (2A)(10\Omega) = 20V$

Each Bulb has a V drop of 20V



$R_T = 40\,\Omega$

$I_T = \frac{V}{R} = \frac{60V}{40\Omega} = 1.5A$

$V_1 = I_1R_1 = (1.5A)(10\Omega) = 15V$

Each bulb has a voltage Drop of 15V

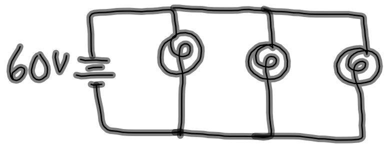
For Resistors in Parallel: Consider 3 resistors in parallel

- If one of the resistors is shut off so that there is a break in the circuit, the remaining branches will not be affected. For lights bulbs, this means that if one of the lights go out, the others will be unaffected. The brightness of the remaining bulbs does not change because the voltage across each bulb stays the same, even though one of the bulbs burnt out.

- If a fourth resistor (or bulb) is added in parallel, the resistors (or bulbs) will still have the same voltage drop across it. For light bulbs this means that the bulbs maintain a constant brightness no matter how many resistors (or bulbs) are added. However, adding too many resistors (or bulbs) in parallel can be dangerous.

- As more and more resistors are added in parallel, the total resistance of the circuit decreases, which means the total current increases. (I = V/R; Current is inversely proportional to the resistance when the voltage remains constant.) This is called overloading a circuit. If the current becomes too great, it will trip a breaker or blow a fuse and the circuit will cease to operate. However, if there is no fuse or circuit breaker to protect the circuit, the increase current would damage the appliances or resistors. (Too much current can cause overheating and may result in a fire.)

Each bulb has a resistance of 10Ω.

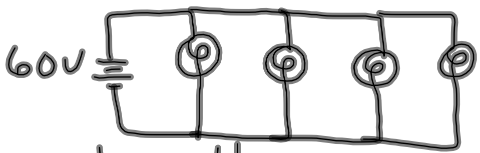


$\frac{1}{R_T} = \frac{3}{10\Omega}$

$R_T = 3.3\Omega$

$I_T = \frac{V_T}{R_T} = \frac{60V}{3.3\Omega} = 18.2A$

$V_1 = V_2 = V_3 = V_T = 60V$



$\frac{1}{R_T} = \frac{4}{10\Omega}$

$R_T = 2.5\Omega$

$I_T = \frac{V}{R} = \frac{60V}{2.5\Omega} = 24A$

$V_1 = V_2 = V_3 = V_T = 60V$

Do questions 1 – 4, p. 609 ,
3 - 6 and 29 – 34, p. 620