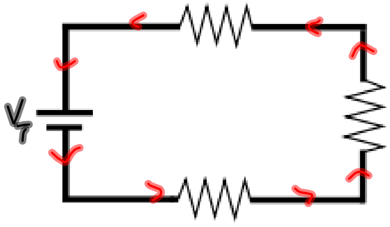
Section 8: Kirchoff's Laws (Text 14.6, p. 605 - 607)

A Series Circuit

- Consists of a source, one or more energy users/resistors and a single conducting path.

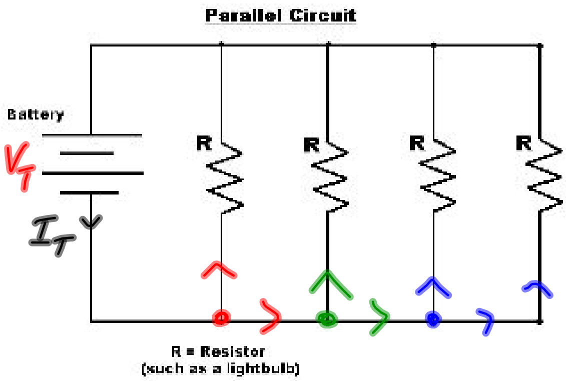


$I_T = I_1 = I_2 = I_4 = \cdots$

$V_T = V_1 + V_2 + V_3 + \cdots$

A Parallel Circuit

- Consists of a source, one or more resistors on each of at least two separate conducting paths.
- Every electron that goes through the parallel circuit goes through **only one** of the circuit loads before returning to the source.

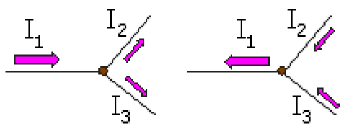


$I_T = I_1 + I_2 + I_3 + \cdots$

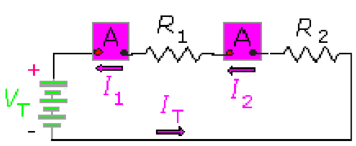
$V_T = V_1 = V_2 = V_3 = \cdots$

Kirchhoff's Current Law: The total amount of current into a junction point of a circuit equals the total amount of current that flows out of that same junction.

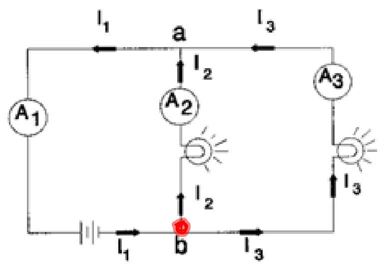
$I_1 = I_2 + I_3$



Kirchoff's current rule



Series (a single path)



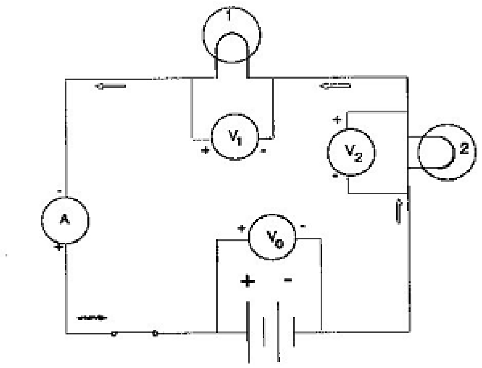
In series

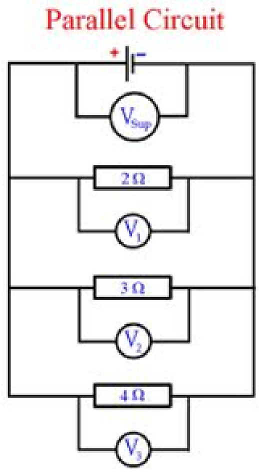
$I_T = I_1 = I_2$

In Parallel

$I_1 = I_2 + I_3$

Kirchhoff's Voltage Law: Around any complete path through an electric circuit the sum of the increases in electric potential is equal to the sum of decreases in electric potential. In other words, in any closed loop, the sum of the voltage rises is equal to the sum of the voltage drops.





In Series

$V_o = V_1 + V_2$

In Parallel

$V_T = V_1 = V_2 = V_3$

Properties of Series Circuits

1 **Current**
Since there is only one pathway for the current to pass through, all parts of the series circuit must have the same current.

$I_T = I_1 = I_2 = I_3 = \ldots I_n$ (Conservation of Electric Charge)

2 **Voltage**
As electrons move through an electric circuit, they gain energy in the sources (voltage rise or potential rise) and lose energy in the loads (voltage drop or potential drop), but the total energy gained is equal to the total energy lost. (Conservation of Energy)

Total Voltage Rises = Total Voltage Drops
$(V_T = V_1 + V_2 + \ldots V_n)$

Properties of Parallel Circuits

1 **Current**

The total amount of current entering the circuit must be the same as the total amount of current exiting the circuit. The current may be reduced as it travels along different pathways; however the total electric current into a junction (node) is equal to the total electric out of the junction (node).

$$I_T = I_1 + I_2 + \ldots I_n$$

2 **Voltage**

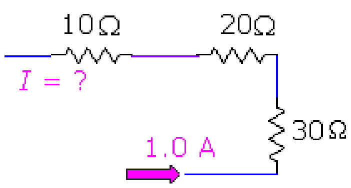
The potential difference across each of the loads or 'loss' of energy is the same in all **parallel** circuits.

$$V_T = V_1 = V_2 = \ldots V_n$$

Examples

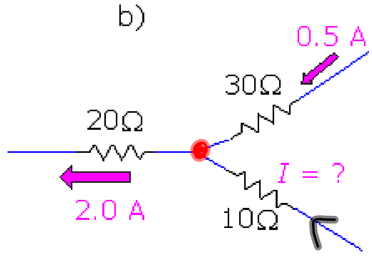
1 For the circuit branches shown, what is the current through the 10Ω resistor?

a)



Since there is only one path, the current through 10Ω resistor is 1A

b)



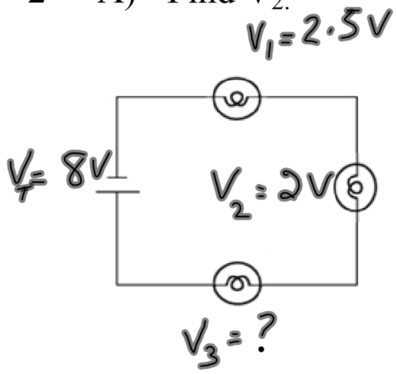
KCL

$I_{in} = I_{out}$

$0.5A + I = 2A$

$I = 1.5A$

2 A) Find V_2.



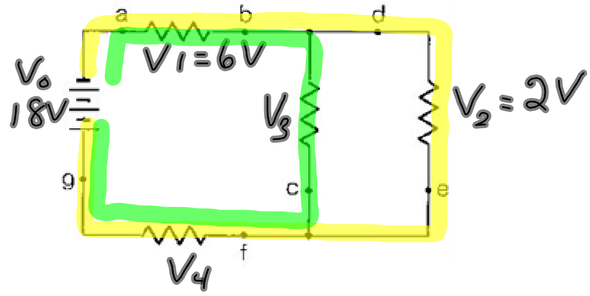
$V_{Rise} = V_{Drops}$

$V_T = V_1 + V_2 + V_3$

$8V = 2.5V + 2V + V_3$

$3.5V = V_3$

B) Find V_3 and V_4.



KVL: $V_{Rises} = V_{Drops}$

$V_0 = V_1 + V_2 + V_4$

$18V = 6V + 2V + V_4$

$10V = V_4$

KVL: $V_{Rise} = V_{Drops}$

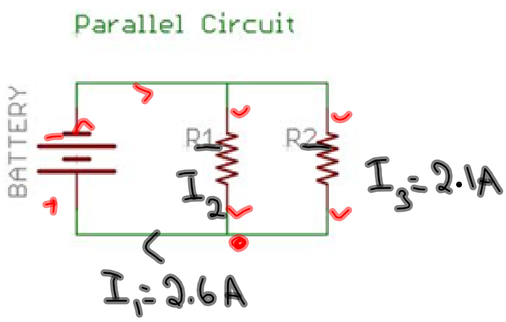
$V_0 = V_1 + V_3 + V_4$

$18V = 6V + V_3 + 10V$

$2V = V_3$

* Note $V_2 = V_3$ b/c they are in parallel with each other.

3 A) Find I_2.



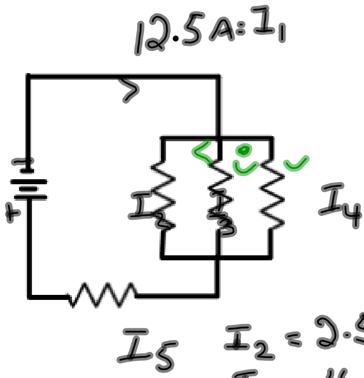
KCL: $I_{in} = I_{out}$

$I_2 + I_3 = I_1$

$I_2 + 2.1A = 2.6A$

$I_2 = 0.5A$

B) Find I_4 and I_5.



$I_2 = 2.5A$

$I_3 = 4.0A$

KCL $I_{in} = I_{out}$

$I_1 = I_2 + I_3 + I_4$

$12.5A = 2.5A + 4A + I_4$

$6A = I_4$

$I_1 = I_5$

$12.5A = I_5$

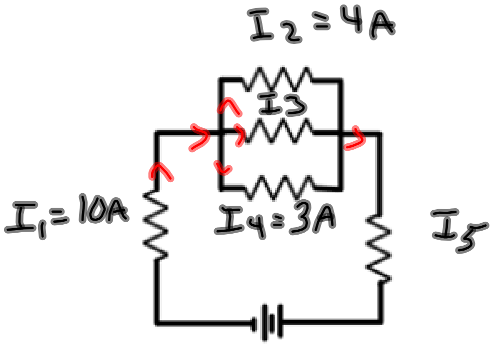
4 Find I_3 and I_5.

$I_1 = I_2 + I_3 + I_4$

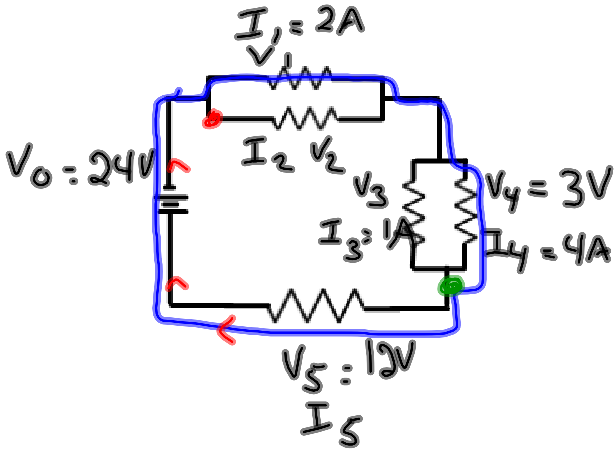
$10A = 4A + I_3 + 3A$

$3A = I_3$

$I_1 = I_5 = 10A$



5 Find V_1, V_2, V_3, I_2, and I_5.



KVL: ① $V_0 = V_1 + V_4 + V_5$

$24V = V_1 + 3V + 12V$

$9V = V_1$

② $V_1 = V_2 = 9V$

③ $V_3 = V_4 = 3V$

KCL: $I_{in} = I_{out}$

$I_5 = I_3 + I_4$

$I_5 = 1A + 4A$

$I_5 = 5A$

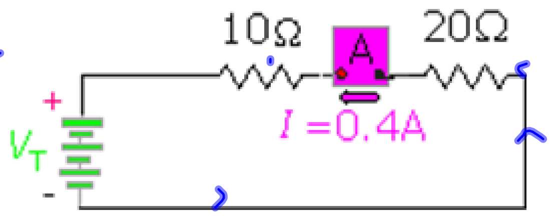
$I_{in} = I_{out}$

$I_5 = I_1 + I_2$

$5A = 2A + I_2$

$3A = I_2$

6 An ammeter connected between the two resistors in the diagram below reads 0.4 A. What is the voltage of the battery?



R_1: 20Ω I_1: 0.4A V_1:
R_2: 10Ω I_2: 0.4A V_2:
R_T: I_T: 0.4A V_T:

B/c there is only one path, the current through each resistor is 0.4A. ($I_1 = I_2 = I_T = 0.4A$)

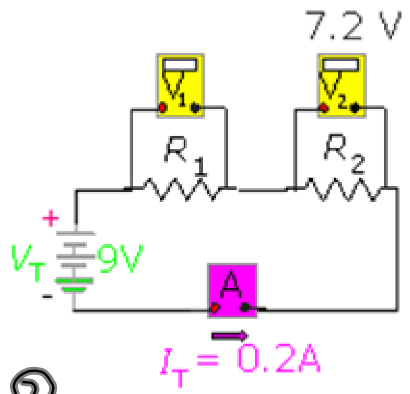
Ohms Law:

$V_1 = I_1 R_1 = (0.4A)(20\Omega)$
$= 8V$

$V_2 = (0.4A)(10\Omega) = 4V$

KVL: $V_T = V_1 + V_2$
$V_T = 8V + 4V$
$V_T = 12V$

7 According to the parameters in the circuit to the right, what is the value of R_1?



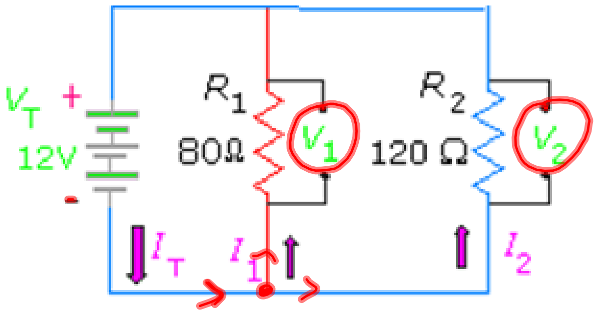
* Series: $I_T = I_1 = I_2 = 0.2A$

KVL: $V_T = V_1 + V_2$
$9V = V_1 + 7.2V$
$1.8V = V_1$

Ohm's law: $R_1 = \frac{V_1}{I_1} = \frac{1.8V}{0.2A} = 9\Omega$

R_1 ② $I_1 = 0.2A$ ______ ①
R_2 $I_2 = 0.2A$ $V_2 = 7.2V$
R_T $I_T = 0.2A$ $V_T = 9V$

8 Use Kirchoff's rules to solve for all the unknown parameters in the circuit to the right.



① KVL (loops)

$V_T = V_1 = V_2 = 12V$

② $I_1 = \frac{V_1}{R_1} = \frac{12V}{80\Omega} = 0.15A$

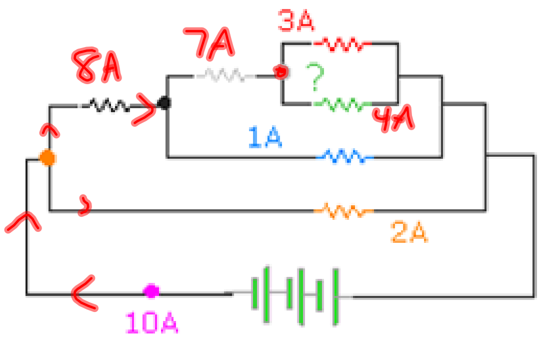
$I_2 = \frac{V_2}{R_2} = \frac{12V}{120\Omega} = 0.1A$

③ $I_T = I_1 + I_2$

$= 0.15A + 0.1A$

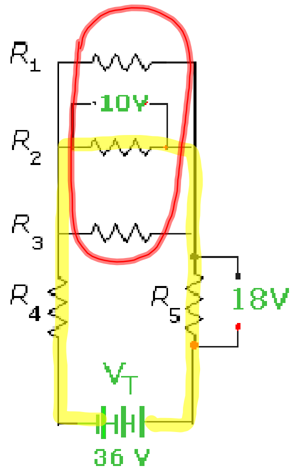
$= 0.25A$

9 In the circuit to the right the current at the pink dot is 10A. The currents through the red, blue and orange resistors are 3A, 1A and 2A, respectively. What current is passing through the green resistor?



4A

10 In the circuit to the right the voltage of the source is 36V. The voltage drops across R_2 and R_5 are 10V and 18V, respectively. What are the voltage drops across the other resistors?



<u>2nd Loop</u>

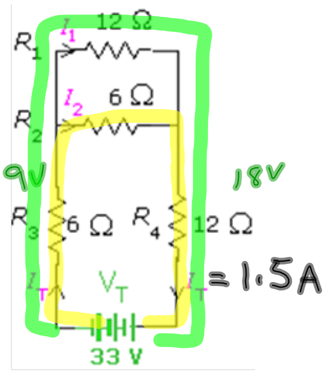
$V_4 = V_T - V_2 - V_5$

$= 36V - 10V - 18V$

$V_4 = 8V$

$V_1 = V_2 = V_3 = 10V$

11 Two resistors of 6Ω and 12Ω are connected in parallel and then the parallel combination is connected in series with a 6Ω and a 12Ω resistor, plus a 33 V power supply. Use Kirchoff's rules to find the current through and the voltage across each resistor.



<u>KCL</u>: $I_T = I_3 = I_4 = 1.5A$

<u>Ohm's Law</u> $V_3 = I_3 R_3 = (1.5A)(6\Omega) = 9V$

$V_4 = (1.5A)(12\Omega) = 18V$

<u>KVL</u>: $V_T = V_3 + V_1 + V_4$

$33V = 9V + V_1 + 18V$

$6V = V_1$

$V_1 = V_2 = 6V$

<u>Ohm's Law</u> $I_1 = \frac{V_1}{R_1} = \frac{6V}{12\Omega} = 0.50A$

$I_2 = \frac{6V}{6\Omega} = 1.0A$