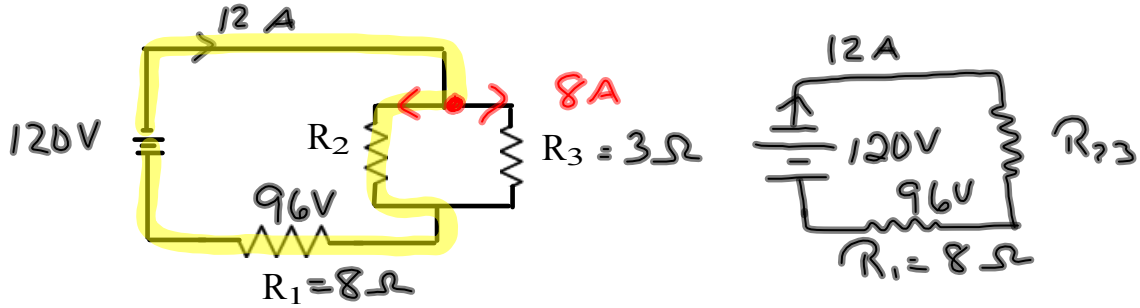
Section 10: Circuit Analysis II - Combination Circuits - Series - Parallel Circuits

1. An 8.0 Ω (R_1) is connected in series with 2 resistors that are connected in parallel. One of the parallel resistors (R_3) has a resistance of 3.0 Ω. The circuit is connected to a 120 V power supply and draws a total current of 12 A. Solve the circuit.



R1	8Ω	I1	12 A	V1	96 V
R2	6Ω	I2	4A	V2	24V
R3	3Ω	I3	8A	V3	24V
R23		I23	12 A	V23	24V
Rt	10Ω ✓	IT	12A ✓	VT	120V ✓

OPT

① $I_T = I_1 = I_{23} = 12\,A$

② $R_T = \frac{V}{I} = \frac{120V}{12\,A} = 10\,\Omega$

③ $V_1 = IR = (12\,A)(8\,\Omega) = 96V$

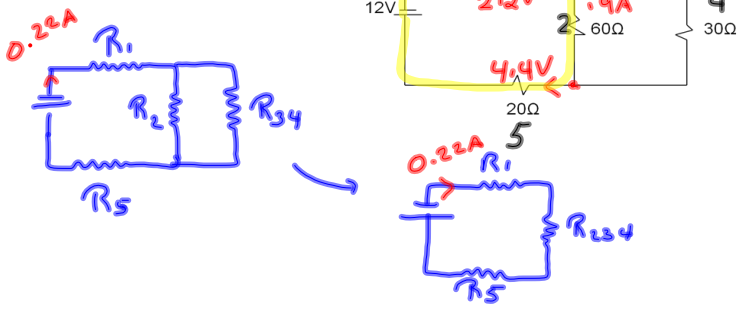
④ $V_{23} = V_T - V_1 = 120V - 96V = 24V = V_2 = V_3$

⑤ $I_3 = \frac{V}{R} = \frac{24V}{3\Omega} = 8.0\,A$

⑥ $I_2 = I_T - I_3 = 12\,A - 8\,A = 4A$

⑦ $R_2 = \frac{V}{I} = \frac{24V}{4A} = 6.0\,\Omega$

2. Solve the circuit.



R1	10Ω	I1	0.22 A	V1	2.2V
R2	60Ω	I2	0.09A	V2	5.4V
R3	10Ω	I3	0.13A	V3	1.3V
R4	30Ω	I4	0.13A	V4	3.9V
R5	20Ω	I5	0.22A	V5	4.4V
R34	40Ω	I34		V34	
R234	24Ω	I234		V234	
~~RN~~	~~~~	~~I1~~	~~~~	~~VII~~	~~~~
Rt	54Ω	It	0.22A	V_T	12V

① $R_{34} = 10\Omega + 30\Omega = 40\Omega$

② $\frac{1}{R_{234}} = \frac{1}{60\Omega} + \frac{1}{40\Omega}$

$R_{234} = 24\Omega$

③ $R_T = 10\Omega + 24\Omega + 20\Omega = 54\Omega$

④ $I_T = \frac{V_T}{R_T} = \frac{12V}{54\Omega} = 0.22A$

⑤ $I_T = I_1 = I_5 = 0.22A$ $(I_{234} = 0.22A)$

⑥ $V_1 = I_1 R_1 = (0.22A)(10\Omega) = 2.2V$

⑦ $V_5 = IR = (0.22A)(20\Omega) = 4.4V$

$V_{234} = IR = (0.22A)(24\Omega) = 5.3V$

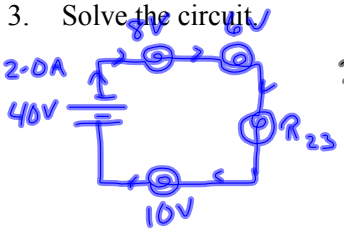
⑧ KVL $V_2 = V_T - V_1 - V_5 = 12V - 4.4V - 2.2V = 5.4V$

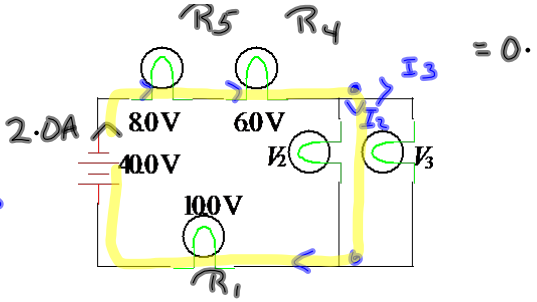
⑨ $I_2 = \frac{V}{R} = \frac{5.4V}{60\Omega} = 0.09A$

⑩ KCL: $I_{34} = I_T - I_2 = 0.22A - 0.09A$
$= 0.13A = I_3 = I_4$

⑪ $V_3 = IR = 1.3V$
$V_4 = IR = 3.9V$

3. Solve the circuit.





R_1	5Ω	I_1	2.0A	V_1	10.0V
R_2	10.7Ω	I_2	1.5A	V_2	16V
R_3	32Ω	I_3	0.5A	V_3	16V
R_4	3Ω	I_4	2.0A	V_4	6V
R_5	4Ω	I_5	2.0A	V_5	8V
R_{23}	8Ω	I_{23}	2.0A	V_{23}	16V
R_T	20Ω	I_T	2.0A	V_T	40V

① $I_T = I_1 = I_4 = I_5 = (I_{23})$

② $V_2 = V_T - V_1 - V_4 - V_5 = 40V - 8V - 6V - 10V = 16V = V_3$

③ $R_4 = \frac{V}{I} = \frac{6V}{2A} = 3\Omega$

④ $R_5 = \frac{8V}{2A} = 4\Omega$

⑤ $R_T = \frac{40V}{2A} = 20\Omega$

⑥ $R_1 = \frac{10V}{2A} = 5\Omega$

⑦ $R_{23} = \frac{16V}{2A} = 8\Omega$

⑧ $R_3 = \frac{V}{I} = \frac{16V}{0.5A} = 32\Omega$

⑨ $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}$

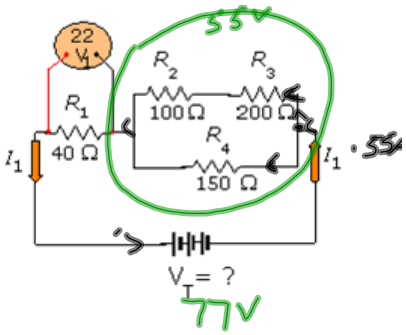
$\frac{1}{8\Omega} = \frac{1}{R_2} + \frac{1}{32\Omega}$

$\frac{1}{8\Omega} - \frac{1}{32\Omega} = \frac{1}{R_2}$

$10.7\Omega = R_2$

⑩ $I_2 = \frac{V}{R} = \frac{16V}{}$ 1.5A

4. The voltmeter in the circuit to the right measures a potential difference of 22 V across the 40 Ω resistor. Solve the circuit.



	R_1	40Ω	I_1	0.55A	V_1	22V
	R_2	100Ω	I_2	0.18A	V_2	18V
	R_3	200Ω	I_3	0.18A	V_3	36V
	R_4	150Ω	I_4	0.37A	V_4	55V
x	R_{23}	300Ω	I_{23}	0.18A	V_{23}	55V
x	R_{234}	100Ω	I_{234}	0.55A	V_{234}	55V
	R_T	140Ω	I_T	0.55A	V_T	77V

① $R_{23} = 100\Omega + 200\Omega = 300\Omega$

② $\frac{1}{R_{234}} = \frac{1}{300\Omega} + \frac{1}{150\Omega}$

$R_{234} = 100\Omega$

③ $R_T = 100\Omega + 40\Omega = 140\Omega$

④ $I_1 = \frac{V}{R} = \frac{22V}{40\Omega} = 0.55A$

⑤ $I_T = I_1 = I_{234} = 0.55A$

⑥ $V_T = IR = (0.55A)(140\Omega) = 77V$

$V_{234} = (0.55A)(100\Omega) = 55V$

$V_{234} = 55V = V_4 = V_{23}$

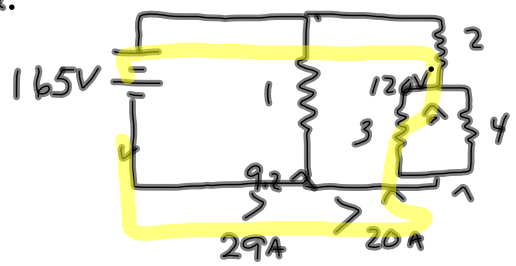
⑦ $I_4 = \frac{V}{R} = \frac{55V}{150\Omega} = 0.37A$

⑧ $I_{23} = \frac{V}{R} = \frac{55V}{300\Omega} = 0.18A = I_2 = I_3$

⑨ $V_2 = IR = (0.18A)(100\Omega) = 18V$

⑩ $V_3 = IR = (0.18A)(200V) = 36V$

Ex.



$R_1 = 18\Omega$

$R_2 = 6.0\Omega$

$R_3 = 4.0\Omega$

$R_4 = 5.0\Omega$

R_1	18Ω	I_1	9.2A	V_1	165V
R_2	6.0Ω	I_2	20A	V_2	120V
R_3	4.0Ω	I_3		V_3	45V
R_4	5.0Ω	I_4	11A	V_4	45V
R_{34}	2.2Ω	I_{23}	9A	V_{23}	45V
R_{234}	8.2Ω	I_{234}		V_{234}	165V
R_T	5.6Ω	I_T	20A 29A	V_T	165V

① $\frac{1}{R_{34}} = \frac{1}{4\Omega} + \frac{1}{5\Omega}$

$R_{34} = 2.2\Omega$

② $R_{234} = R_2 + R_{34}$
$= 2.2\Omega + 6\Omega$
$= 8.2\Omega$

③ $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_{234}}$

$\frac{1}{R_T} = \frac{1}{18\Omega} + \frac{1}{8.2\Omega}$

$R_T = 5.6\Omega$

④ $I_T = \frac{V_T}{R_T} = \frac{165V}{5.6\Omega} = 29A$

⑤ KVL

$V_T = V_1 = 165V$

$V_T = V_{234} = 165V$

⑥ $I_1 = \frac{V}{R} = \frac{165V}{18\Omega} = 9.2A$

⑦ $I_{234} = \frac{V}{R} = \frac{165V}{8.2\Omega} = 20A$

⑧ $I_2 = 20A$

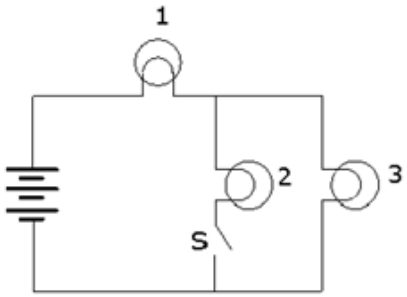
⑨ $V_2 = IR = (20A)(6\Omega)$
$= 120V$

⑩ $V_{34} = V_3 = V_4 = V_T - V_2 = 165V - 120V = 45V$

⑪ $I_3 = \frac{V}{R} = \frac{45V}{4\Omega} = 11A$

$I_4 = \frac{45V}{5\Omega} = 9A$

5. The circuit below contains three identical light bulbs. Compare the brightness of bulb 1and bulb 3 when switch, S, (i) is closed (ii) is opened.

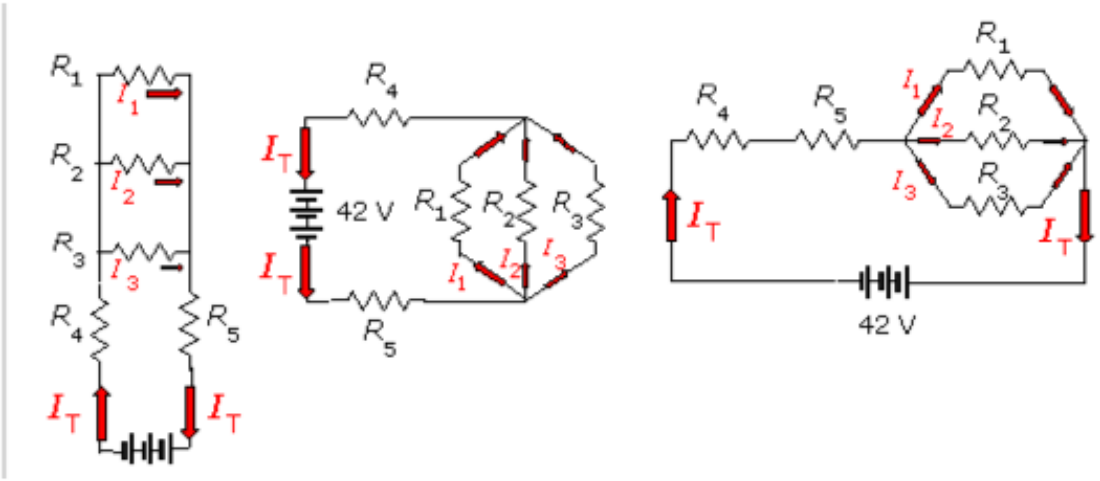


When the switch is open, bulb 2 will not light. Bulb 1 and 3 will be in series and each will have the same brightness b/c each bulb will have the same voltage drop across it.

When the switch is close, all three bulbs will light. Bulb 2 and 3 are in parallel and their equivalent resistance is less than the individual resistance. This means that the total resistance of the circuit is decreased and therefor the total current of the circuit will increase. Hence the voltage drop across bulb 1 will be greater than before (bulb is brighter). As a result the current through Bulb 2 and 3 will be less. Therefore, bulb 3 will be less bright b/c of the smaller voltage drop across it.

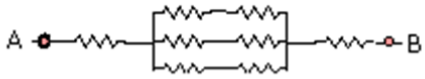
6. R_1, R_2, and R_3 are connected in parallel and the combination is connected in series with R_4 and R_5. Find the current through and the voltage drop across each of the resistors if the circuit has a 42 V source, and the values of the resistors are as follows:

$$R_1 = 75\ \Omega\ R_2 = 150\ \Omega\ R_3 = 120\ \Omega\ R_4 = 48\ \Omega\ R_5 = 36\ \Omega$$



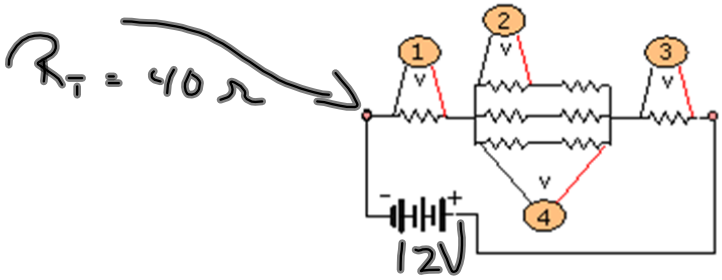
Test yourself

1. Each resistor in the picture to the right has a resistance of 15 Ω. What is the total resistance between points A and B?



- a) 30 Ω
- b) 40 Ω
- c) 45 Ω
- d) 60 Ω

2. Each resistance in the circuit to the right has a value of 15 Ω. Which voltmeter will have the smallest reading?



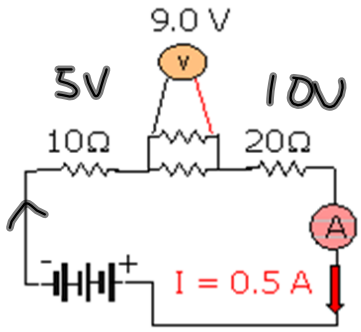
- a) 1
- b) 2
- c) 3
- d) 4

3. If the battery in the diagram of #2 is rated at 12.0 V, and if each resistor is 15 Ω, what current is the battery providing?

- a) 0.4 A
- b) 0.3 A
- c) 0.26 A
- d) 0.2 A

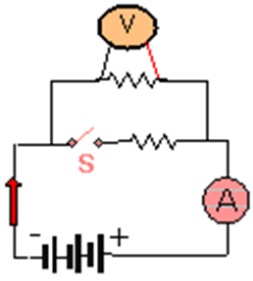
4. From the information in the circuit diagram to the right, what is the voltage of the battery?

- a) 9.0 V
- b) 33 V
- c) 19.5 V
- d) 24 V



5. An ammeter and a voltmeter are connected to two parallel branches as shown to the right. There is an opened switch in one of the branches. How will the readings on the ammeter and voltmeter be affected if the switch is closed?

- a) both V and A readings will increase
- b) the V reading will increase; the A reading will not change
- c) the A reading will increase; the V reading will not change
- d) both V and A readings will decrease



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$V = IR$

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So ↑