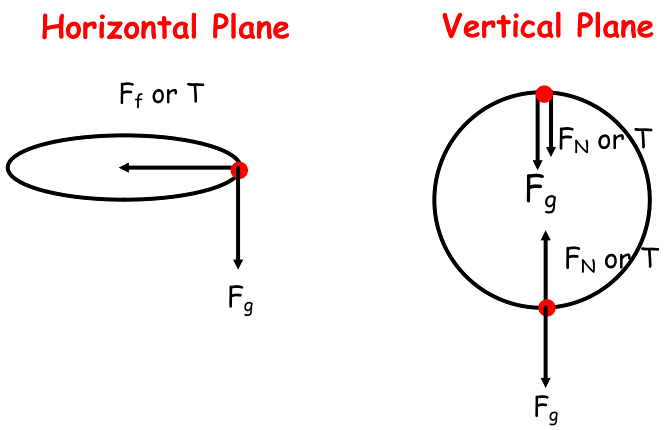
Section 5: Solving Circular Motion Problems the Vertical Circle

In the last section, we worked with objects moving in a circle in a horizontal plane. In these situations, the weight of the moving object was often ignored because the weight vector was "perpendicular" to the force providing the centripetal force (ie. tension, friction, etc). When dealing with circular motion in a vertical plane, weight (or the force of gravity) cannot be ignored. The pictures below show why this is so.



In the first picture the weight is perpendicular to the force of friction or tension (which provides the centripetal force) and does not interact with it. In the second picture the weight and normal force (or tension) are in the same direction at the top of the path, but in opposite directions at the bottom of the path. Therefore, the weight of the object (or force of gravity) now plays a very important role. (Don't forget that centripetal force is always towards the center and centripetal force is a net force.)

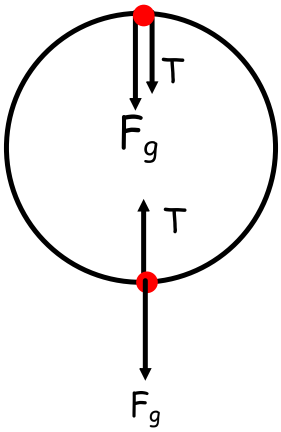
$$F_c = F_{net} = ma_c = mv^2/r .$$

So, for uniform circular motion there are two terms for F_{net}:

$F_c = mv^2/r$ and
$F_c = \Sigma F$, which means that
$mv^2/r = \Sigma F$.

Consider a ball swung in a vertical circle.

Where is the tension in the string the greatest? the least?



For uniform circular motion in the vertical plane, we will take toward the center to be positive.

At the top:

$$F_{Net} = F_C = T + F_g$$
$$T = F_C - F_g$$
$$T = \frac{mv^2}{r} - mg$$

At the bottom:

$$F_{Net} = F_C = T - F_g$$
$$T = F_C + F_g$$
$$T = \frac{mv^2}{r} + mg$$

At the top $T = F_C - F_g$ while at the bottom $T = F_C + F_g$. From these two equations, it is clear that the tension in the string at the top of the circle is less than the tension in the string at the bottom of the circle.

Note the same would be true for F_N. If a you were riding on a roller coaster (upside down at the top), you would experience a smaller normal force at the top than at the bottom. In other words, you would weigh less at the top and more at the bottom.