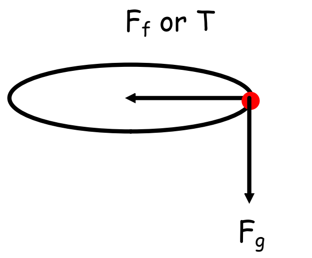
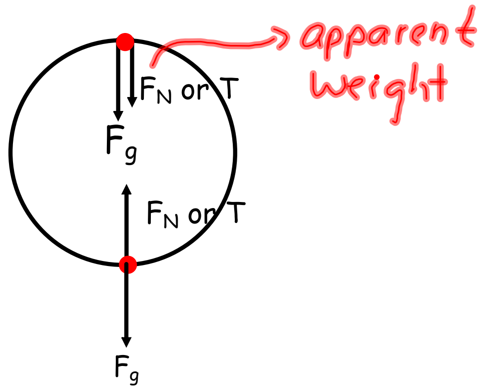
Section 5: Solving Circular Motion Problems the Vertical Circle

In the last section, we worked with objects moving in a circle in a horizontal plane. In these situations, the weight of the moving object was often ignored because the weight vector was "perpendicular" to the force providing the centripetal force (ie. tension, friction, etc). When dealing with circular motion in a vertical plane, weight (or the force of gravity) cannot be ignored. The pictures below show why this is so.

Horizontal Plane



Vertical Plane



In the first picture the weight is perpendicular to the force of friction or tension (which provides the centripetal force) and does not interact with it. In the second picture the weight and normal force (or tension) are in the same direction at the top of the path, but in opposite directions at the bottom of the path. Therefore, the weight of the object (or force of gravity) now plays a very important role. (Don't forget that centripetal force is always towards the center and centripetal force is a net force.)

$$F_c = F_{net} = ma_c = mv^2/r \text{ .}$$

So, for uniform circular motion there are two terms for F_{net}:

$F_c = mv^2/r$ and
$F_c = \Sigma F$, which means that
$mv^2/r = \Sigma F$.

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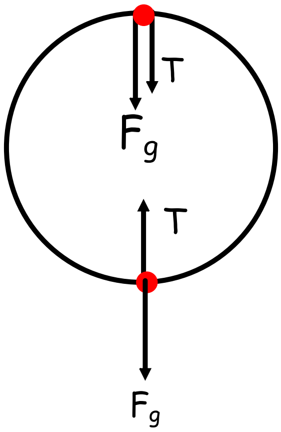
Consider a ball swung in a vertical circle.



Where is the tension in the string the greatest? the least?

Bottom Top

For uniform circular motion in the vertical plane, we will take toward the center to be positive.

At the top:

$$F_{Net} = F_C = T + F_g$$
$$T = F_C - F_g$$
$$T = \frac{mv^2}{r} - mg$$

At the bottom:

$$F_{Net} = F_C = T - F_g$$
$$T = F_C + F_g$$
$$T = \frac{mv^2}{r} + mg$$

At the top $T = F_C - F_g$ while at the bottom $T = F_C + F_g$. From these two equations, it is clear that the tension in the string at the top of the circle is less than the tension in the string at the bottom of the circle.

Note the same would be true for F_N. If a you were riding on a roller coaster (upside down at the top), you would experience a smaller normal force at the top than at the bottom. In other words, you would weigh less at the top and more at the bottom.

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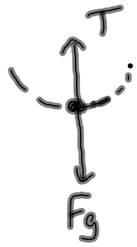
Examples

1. A 0.250 kg ball is being swung on a 1.3 m string in a vertical circle. If its tangential speed at the bottom of the path is 4.2 m/s, what is the tension in the string for that position?

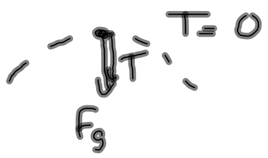


* toward center of circle is +

$$F_c = T - F_g$$
$$T = F_c + F_g$$
$$T = \frac{mv^2}{r} + mg$$
$$T = \frac{(0.250\,kg)(4.2\,m/s)^2}{(1.3\,m)} + (0.25\,kg)(9.8\,m/s^2)$$
$$T = 5.8\,N$$

2. For the ball in the previous example, what is the minimum speed that the ball can have at the top and still move in a circle?

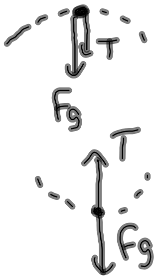
Key: The minimum speed is that speed which cause the tension in the string to go to 0 N. At even smaller speeds, the string will go limp and the ball will fall out of its circular path.



$$F_c = F_g + \cancel{T} \quad T = 0$$
$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$
$$v = \sqrt{gr}$$
$$v = \sqrt{(9.8\,m/s^2)(1.3\,m)}$$
$$v = 3.6\,m/s$$

The minimum speed is 3.6 m/s. At speeds less than 3.6 m/s, the ball will not revolve.

3. A 0.450 kg stone is swinging in a vertical circle of radius 75.0 cm. The constant speed of the stone is 6.21 m/s. Determine the tension in the string at the bottom of the circle and at the top of the circle.



Top: $F_c = T + F_g$

$T = F_c - F_g$

$T = \frac{mv^2}{r} - mg$

$T = \frac{(0.45\,kg)(6.21\,m/s)^2}{(0.75\,m)} - (0.45\,kg)(9.8\,\frac{m}{s^2})$

$T = 18.7\,N$

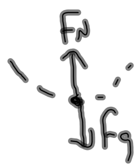
Bottom: $T = F_c + F_g$

$T = \frac{(0.45\,kg)(6.21\,m/s)^2}{(0.75\,m)} + (0.45\,kg)(9.8\,\frac{m}{s^2})$

$T = 27.5\,N$

4. As a 58.0 kg pilot comes out of a dive in a circular arc at a speed of 350 km/h, he experiences an acceleration of 9.0 g's. What force is applied upward by F_N his seat and what must be the radius of the path?

$* \; F_c = \frac{mv^2}{r}$



$m = 58\,kg$

$V = 350\,km/h = 97.2\,m/s$

$F_c = ma_c$

$a_c = 9 \times 9.8\,m/s^2 = 88.2\,m/s^2$

$r = ?$

$F_N = ?$

$F_c = F_N - F_g$

$F_N = F_c + F_g$

$F_N = ma_c + mg$

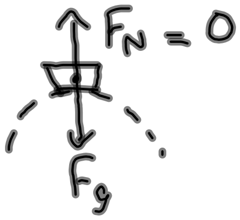
$F_N = (58\,kg)(88.2\,m/s^2) + (58\,kg)(9.8\,m/s^2)$

$F_N = 5700\,N$

$r = \frac{v^2}{a} = \frac{(97.2\,m/s)^2}{88.2\,m/s^2} = 110\,m$

5. One of the vertical circular rides in Fisics Fantasy Land has a radius of 35.0 m. You are sitting in a car that is just cresting the top of the ride. How fast must the car be moving in order that you momentarily lift off your seat and feel weightless?

Key: F_N is the force of the seat pushing up on you. Since you feel weightless, you are momentarily free of your seat. Since it is not pushing on you, F_N is equal to 0 N.



$$F_{Net} = F_c = F_g + \cancel{F_N}^{=0}$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

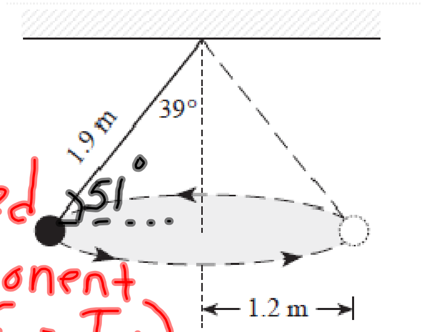
$$v = \sqrt{gr}$$

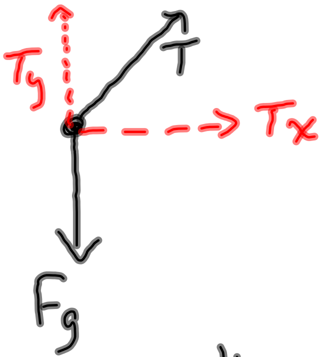
$$v = \sqrt{(9.8\,m/s^2)(35.0\,m)}$$

$$v = 18.5\,m/s$$

6. The diagram shows an object of mass 3.0 kg travelling in a circular path of radius 1.2 m while suspended by a piece of string of length 1.9 m.

A. What is the centripetal force on the mass?





* F_c is provided by the x-component of Tension. ($F_c = T_x$)
To find T_x we need to find T_y.

y-direction

$T_y = F_g$

$T\sin\theta = mg$

$T\sin 51^\circ = (3\text{kg})(9.8\text{ m/s}^2)$

$T = 37.8\text{ N}$

x-direction

$F_c = T_x$

$F_c = T\cos\theta$

$F_c = 37.8\text{N}\cos 51^\circ$

$F_c = 24\text{N}$

B. What is the velocity?

$$F_c = \frac{mv^2}{r}$$

$$V = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(24\text{N})(1.2\text{m})}{3\text{kg}}} = 3.1\text{ m/s}$$

C. What is its period?

$$V = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{V}$$

$$T = \frac{2\pi(1.2\text{m})}{3.1\text{ m/s}}$$

$$T = 2.4\text{s}$$