

Section 1.6: Graphing Accelerated Motion

In this lesson you will

- Construct $d-t$ and $v-t$ graphs for an object undergoing uniformly accelerated motion.
- Use the $d-t$ graphs to determine the instantaneous and average velocity.
- Use the $v-t$ graphs to determine the displacement of the object during any specific time interval.
- Use the $v-t$ graphs to determine the acceleration of the object.
- Describe the motion of an object given its displacement-time graph or velocity-time graph.
- Compare and contrast displacement vs. time and velocity vs. time graphs.

Recall:

Uniform Motion

1. Position-time graph (Displacement-time graph)

- The **position vs. time** graph for uniform motion is linear. (a diagonal line or a horizontal line)
- The **slope** of the line represents the velocity.
 - If the slope is positive: moving to the right at a constant speed
 - If the slope is negative: moving to the left at a constant speed.
 - If the slope is 0: the object is at rest
- The **y-intercept** of the graph represents the initial position.

2. Velocity-time graph

- The **velocity vs. time** graph (in future referred to as a $v-t$ graph) for uniform motion is a horizontal line.
- The **y-intercept** of the graph presents the average velocity.
- If the graph is in the first quadrant, the object is moving to the right.
- If the graph is in the fourth quadrant, the object is moving to the left.
- The **area "under"** any part of a $v-t$ graph (between the x-axis and the time axis) represents the displacement for that time interval.
 - If the area is above the time axis: positive displacement
 - If the area is below the time axis: negative displacement
- Average velocity = area \div time

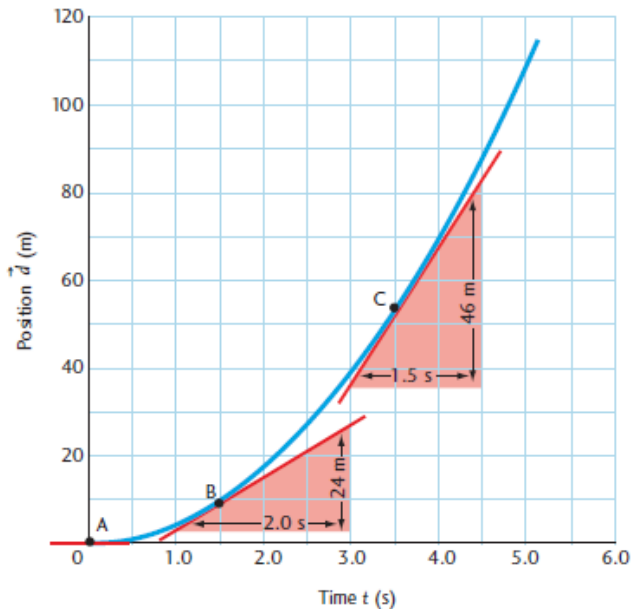
Accelerated Motion

1. Position-time graph (Displacement-time graph)

- The position-time graph for uniform acceleration is parabolic. (curved line)
- The **slope** of the line tangent to the curve represents the instantaneous velocity.

To construct a tangent to a curve:

- 1 Choose a point on the curve.
- 2 Draw a straight line parallel to the direction of the curve at that point. The line touches the curve at that point only: and the angles between the curve and the line on either side of the point are equal.
- 3 Then draw a rise-run triangle and calculate the slope.



- The **slope** of the line that joins two points on the curve represents the average velocity.
- The **y-intercept** of the graph represents the initial position.

2. Velocity-Time Graphs

- The **velocity vs. time** graph for uniform acceleration is a diagonal line.
- The slope of the line represents the acceleration of the object.
- The **y-intercept** represents the initial velocity.
- If the graph is in the first quadrant, the object is moving to the right.
- If the graph is in the fourth quadrant, the object is moving to the left.
- The **area** "under" any part of the graph (between the x-axis and the time axis) represents the displacement for that time interval.
 - If the area is above the time axis: positive displacement
 - If the area is below the time axis: negative displacement
- Average velocity = area \div time
- The **velocity vs. time** graph for non-uniform acceleration is a parabolic curve.
- The **slope** of the line or the slope of the line tangent to the curve gives the instantaneous acceleration of the object.

Note: Important

If velocity and acceleration are in the same direction (same signs) the object is speeding up.

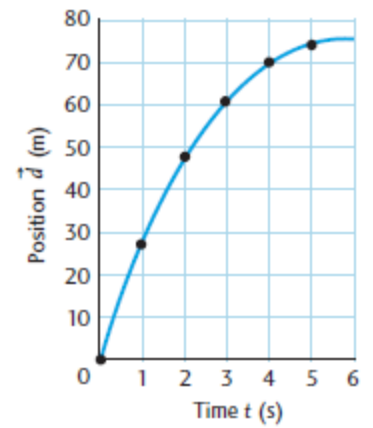
If velocity and acceleration are in opposite directions (opposite signs), the object is slowing down.

Therefore:

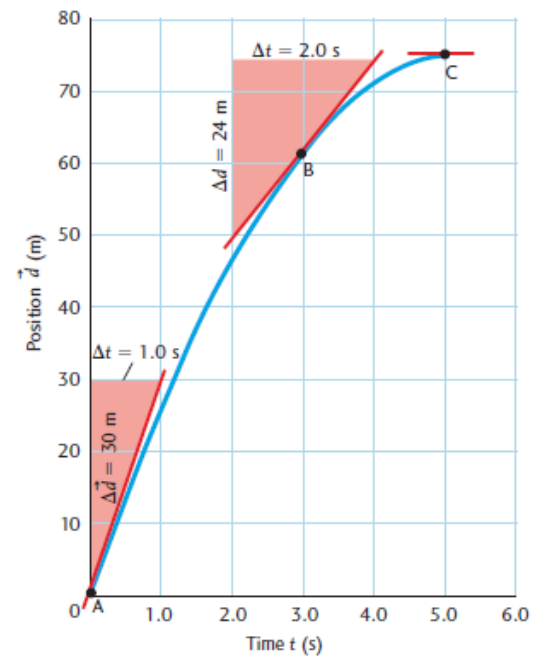
- 1 If v is + and a is +: object is moving to the right and speeding up.
- 2 If v is - and a is -: object is moving to the left and speeding up.
- 3 If v is + and a is -: object is moving to the right and slowing down.
- 4 If v is - and a is +: object is moving to the left and slowing down.

Examples

1. To the right is a graph of a section of a roller coaster ride.
 - a) Find the instantaneous velocity at 0 s, 3.0 s and 5.0 s.
 - b) Find the average velocity of the roller coaster between 0 - 3.0 s, and 3.0 s - 5.0 s.
 - c) Describe the motion of the roller coaster.
 - d) Sketch a graph of the corresponding v-t graph.

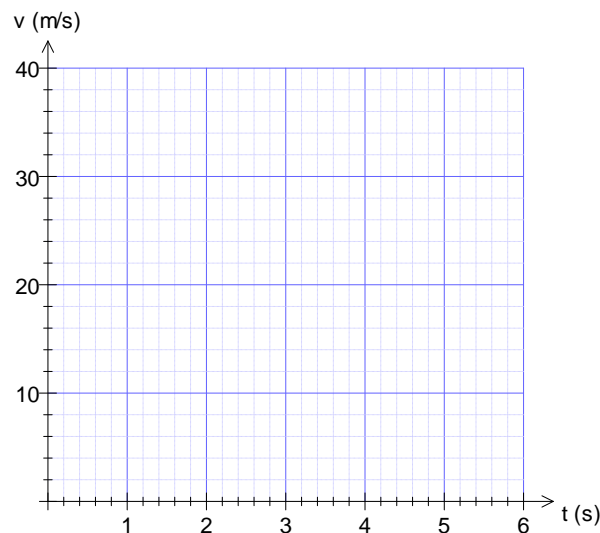


- a) To find the instantaneous velocity we need to draw tangents to the graph at 0 s, 3.0s, and 5.0 s and then find the slope of each.



- b) Average velocity = slope of the line joining two points.

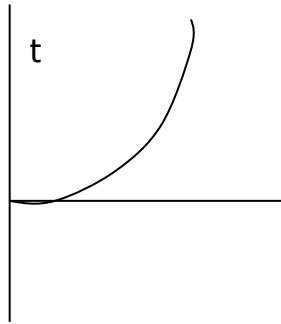
c)



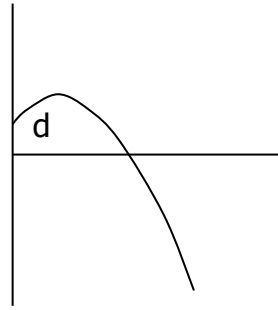
d)

2. A) Describe the motion of the following objects.

i)



ii)

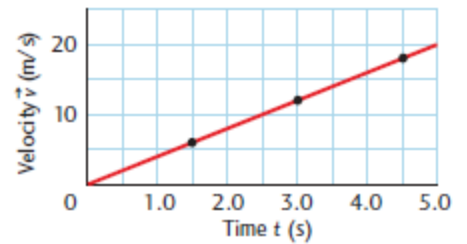


B) Sketch the corresponding velocity-time graphs.

3. Draw a position-time graph to represent a ball falling from the top of a building. Draw the corresponding velocity-time graph.

4. A Honda Civic produced the following v-t graph.

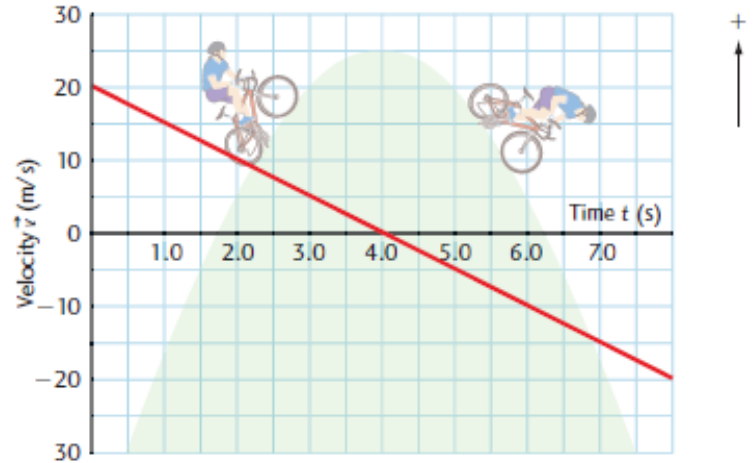
a) Find the acceleration of the car.



b) Find the displacement of the car in 5.0 s.

c) Describe the motion of the car.

5. The $v-t$ graph at the right represents a cyclist's motion.
- a) Find the cyclist's acceleration for each section.



b) Find the cyclist's total displacement.

c) Describe the cyclist's motion.

d) What is the cyclist's velocity at 4.0 s? What is his acceleration at 4.0 s?

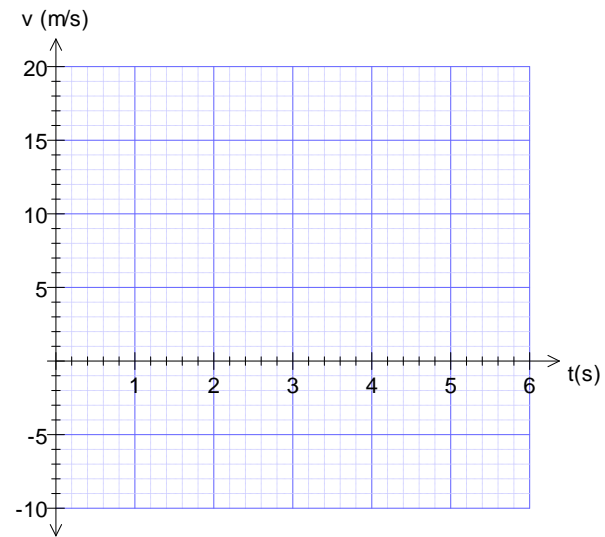
6. Using the velocity -time graph below, answer the following questions.

a) What is the object's initial velocity?

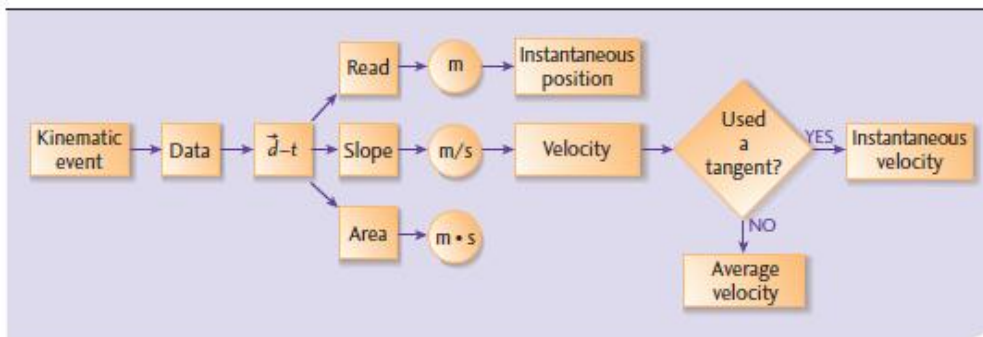
b) What is the object's acceleration?

c) What is the acceleration of the object at 2.4 s?

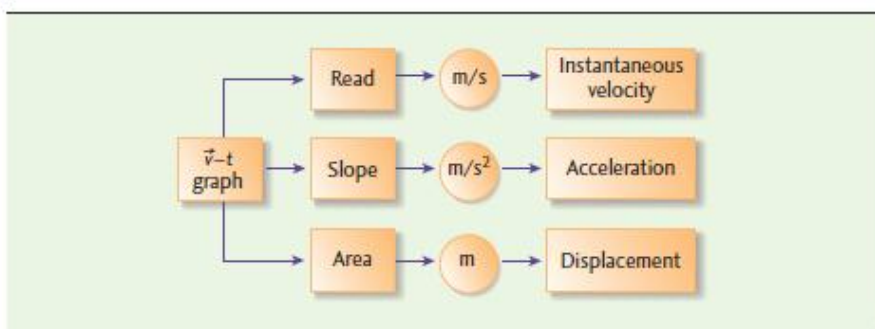
d) Describe the motion of the object.



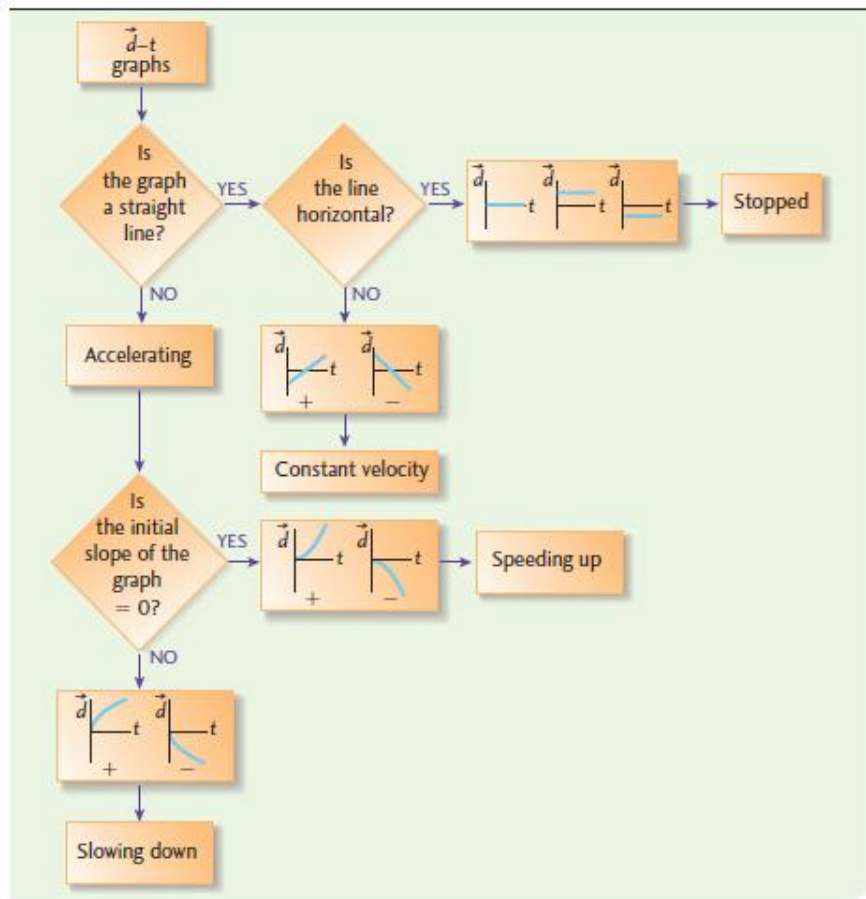
Overview of d-t Graph Manipulations



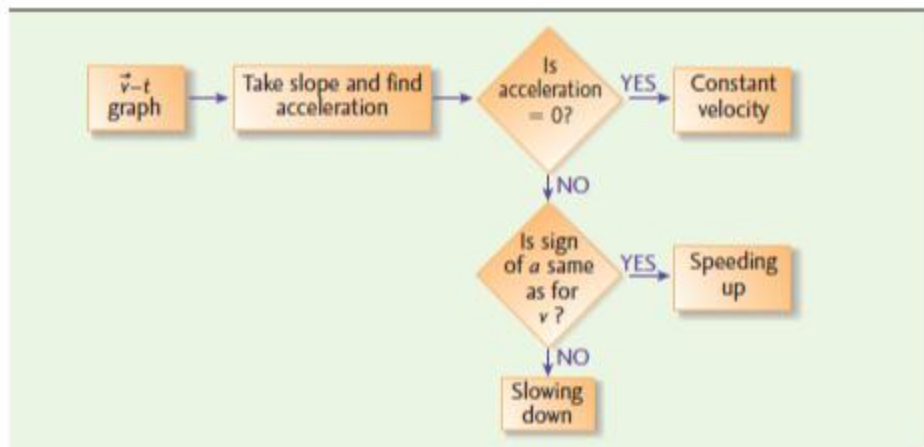
Graph Analysis for a v-t graph



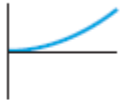
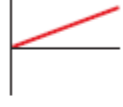


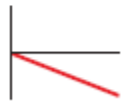

Summary of d-t Graph Analysis






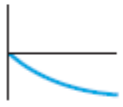


Determining the Action of an Object



Speeding up

d - t	v -t	velocity	acceleration	
		$\vec{v} > 0$	$\vec{a} > 0$	
		$\vec{v} < 0$	$\vec{a} < 0$	

Slowing down

d - t	v -t	velocity	acceleration	
		$\vec{v} > 0$	$\vec{a} < 0$	
		$\vec{v} < 0$	$\vec{a} > 0$	

Accelerated Motion

Recall, when motion is accelerated the d-t graph looks parabolic and the v-t graph is a sloping straight line. The slope of the v-t graph gives the magnitude of acceleration.