Section 3.3: Velocity Vectors in Two Dimensions

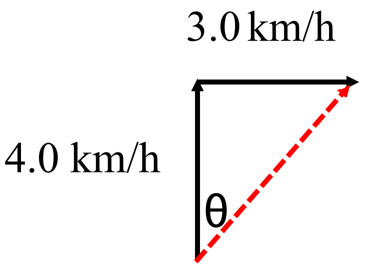
Type 1

1 A swimmer jumps into a river and swims for the opposite shore. Her velocity in still water is 4.0 km/h [N]. The current in the river is 3.0 km/h [E]. Find the swimmer's velocity relative to the shore.

${}_sv_w$ = 4.0 km/h [N] (still water - swimmer velocity in the water)
${}_wv_e$ = 3.0 km/h [E] (current)
${}_sv_e$ = ? (Swimmer wrt the earth - Resultant Velocity)

$${}_sv_w + {}_wv_e = {}_sv_e$$
4.0 km/h [N] + 3.0 km/h [E] = ${}_sv_e$

To add these vectors we need to use a vector diagram.



$v^2 = (4\text{ km/h})^2 + (3\text{km/h})^2$
v = 5.0 km/h [N 37° E]

$$tan\theta = \frac{3km/h}{4km/h}$$
$$\theta = 37^\circ$$

2 The velocity of the current in a river is 10.0 m/s [E] and a boat on the river has a velocity of 7.0 m/s [N]. On the boat there is a cart that has a velocity of 4.0 m/s [W]. A turtle is moving on the cart at 3.0 m/s [E] and an ant is moving at 1.0 m/s [S] on the turtle's back. Find the velocity of the ant with respect to the:

A) boat B) earth

A)

${}_w v_e$ = 10 m/s [E]

${}_b v_w$ = 7. 0 m/s [N]

${}_c v_b$ = 4.0 m/s [W]

${}_t v_c$ = 3.0 m/s [E]

${}_a v_t$ = 1.0 m/s [S]

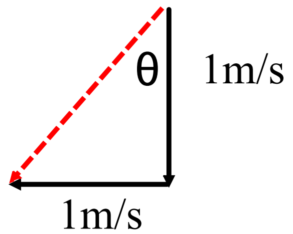
${}_a v_b$ = ?

${}_a v_b = {}_a v_t + {}_t v_c + {}_c v_b$

${}_a v_b$ = 1 m/s [S] + 3 m/s [E] + 4 m/s [W]

= 1m/s [S] + 1 m/s [W]

To add these vectors we need to use a vector diagram.



$v^2 = (1\ \text{km/h})^2 + (1\ \text{km/h})^2$

v = 1.4 km/h [S 45° W]

$$tan\theta = \frac{1 km/h}{1 km/h}$$

$\theta = 45°$

B)

${}_w v_e$ = 10 m/s [E]

${}_b v_w$ = 7. 0 m/s [N]

${}_c v_b$ = 4.0 m/s [W]

${}_t v_c$ = 3.0 m/s [E]

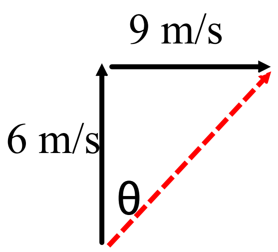
${}_a v_t$ = 1.0 m/s [S]

${}_a v_e$ = ?

${}_a v_e = {}_a v_t + {}_t v_c + {}_c v_b + {}_b v_w + {}_w v_e$

${}_a v_b$ = 1 m/s [S] + 3 m/s [E] + 4 m/s [W] + 7 m/s [N] + 10 m/s [E]

= 6 m/s [N] + 9 m/s [E]



$v^2 = (6\ \text{km/h})^2 + (9\ \text{km/h})^2$

v = 11 km/h [S 56° W]

$$tan\theta = \frac{9 km/h}{6 km/h}$$

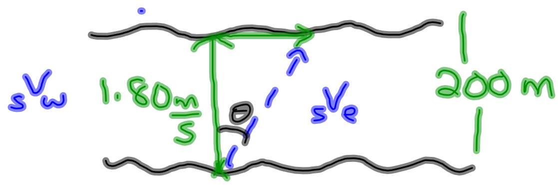
$\theta = 56°$

Doe

3 A swimmer can swim at a speed of 1.80 m/s in still water. If the current in a river 200.0 m wide is 1.00 m/s [E] and the swimmer starts on the south bank and swims ~~so that she is always headed directly across~~ the river, determine:

A) the swimmer's resultant velocity.

$_sV_w = 1.80\,m/s\,[N]$
$_wV_e = 1.0\,m/s\,[E]$ +
$_sV_e = ?$



$$V^2 = (1.80\,m/s)^2 + (1.00\,m/s)^2$$
$$V = 2.06\,m/s\ [N\,29.1°E]$$

$$\tan\theta = \frac{1.00\,m/s}{1.80\,m/s}$$
$$\theta = 29.1°$$

B) how long she will take to reach the far shore.

*

Pull

The time to cross the river is the same in still water as it is in water with a current because the water's velocity and the swimmer's velocity are at right angles to each other and are therefore independent of each other. Therefore the current neither aids nor hinders the swimmer. What it does is cause her to move downstream.

$$t = \frac{d}{_sV_w} = \frac{200\,m}{1.80\,m/s} = 111\,s$$

C) how far downstream she will land.

Pull

Again, the swimmer's downstream velocity is independent of her swimming velocity. As a result she will move downstream at the same rate whether she is swimming or floating.

$$d = {_wV_e}\,t$$
$$= (1.00\,m/s)(111\,s)$$
$$= 111\,m$$

Type 2

4 A swimmer on the south shore of a river wishes to swim to a dock due north of his starting point. His speed in still water is 4.0 km/h and there is a current in the river flowing at 2.5 km/h to the West.

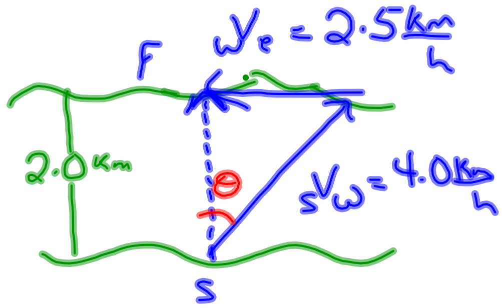
A) In what direction must he swim to get directly across the river?

③ ${}_sV_w$ = 4.0 km/h [?] (a)

② ${}_wV_e$ = 2.5 km/h [W]

① ${}_sV_e$ = (?) [N]
(b)

connect head to head



$$\sin\theta = \frac{2.5\ km/h}{4.0\ km/h}$$

$\theta = 39°$ [N39°E]

B) If the river is 2.0 km wide, how long does it take him to make the crossing?

so we need to find ${}_sV_e$.

$${}_sV_e^2 = (4.0\ km/h)^2 - (2.5\ km/h)^2$$

$${}_sV_e = 3.1\ km/h\ [N]$$

$$t = \frac{d}{V} = \frac{2.0\ km}{3.1\ km/h} = 0.65\ h$$

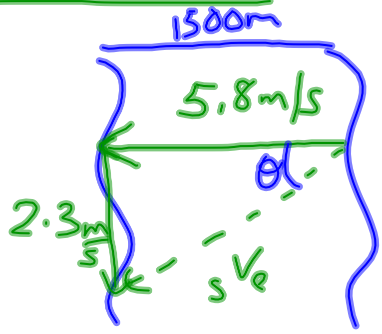
5. A man attempts to swim at 5.8 m/s due west across a river which flows south with a 2.3 m/s current.

A. i. What will be his resultant velocity?

$_sV_w = 5.8 m/s$ [W]

$_wV_e = 2.3 m/s$ [S]

$_sV_e = ?$



$$_sV_e^2 = (5.8 m/s)^2 + (2.3 m/s)^2$$

$$_sV_e = 6.2 m/s \text{ [W22°S]}$$

$$\tan\theta = \frac{2.3 m/s}{5.8 m/s} \quad \theta = 22°$$

ii. How long does it take him to cross the river, if the river is 1500 m wide?

$$t = \frac{d}{v} = \frac{1500 m}{5.8 m/s} = 260 s$$

iii) How far downstream does he land?

$$d = {}_wV_e t$$

$$d = (2.3 m/s)(260 s)$$

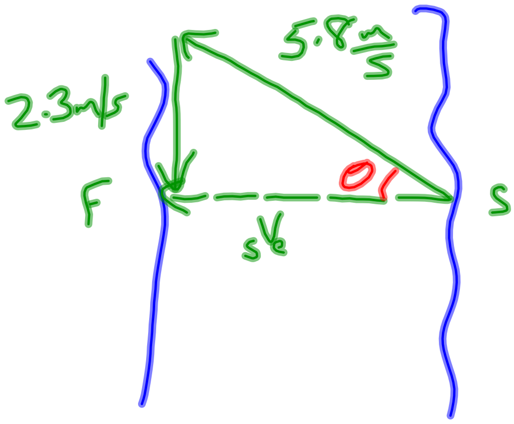
$$d = 6.0 \times 10^2 m$$

B. i. In what direction must he swim at his 5.8 m/s speed to get directly across the river?

$_sV_w$ = 5.8 m/s [?]

② $_wV_e$ = 2.3 m/s [S]) head to head

① $_sV_e$ = ? [W]



$$\sin\theta = \frac{2.3\,m/s}{5.8\,m/s}$$

$$\theta = 23^\circ$$

[W 23° N]

ii. If he does get directly across the river, what was his resultant velocity in crossing the river?

$$_sV_e^2 = (5.8\,m/s)^2 - (2.3\,m/s)^2$$

$$_sV_e = 5.3\,m/s\ [W]$$

iii. If the river is 1500 m wide, how long will it take him to cr the river?

$$t = \frac{d}{V} = \frac{1500\,m}{5.3\,m/s} = 280\,s$$

6. A canoeist paddles north across a river at 3.0 m/s. The river is flowing east at 4.0 m/s and is 150 m wide.

A) What is the velocity of the canoe relative to the river?

B) Calculate the time required to cross the river.

C) How far downstream does she land?

7. A girls can swim at 3.0 m/s in still water. She jumps into a river that is 1.0 km wide and has a current of 2.0 m/s [E].

   A) If she wants to end up directly north of her starting point, in which direction should she head?

   B) What is her velocity relative to the earth?

   C) How long does it take her to cross the river?