

Section 3.1: Relative Displacements

Have you ever had an experience something like this? You are sitting in a vehicle (a bus or a train is good) and looking out the window at another vehicle very close to yours. Suddenly you feel that your vehicle is moving forward. However, after a second or two, you realize that your vehicle is not moving at all. Instead it is the other vehicle that is backing up! If one of your friends is standing on the road looking at both vehicles, there is no way that she would have been fooled. Right away she would be able to tell which vehicle was moving.

Why is that?

The answer has to do with your **frame of reference** and with your friend's frame of reference.

What is a frame of reference?

A frame of reference is a place from which motion is observed. The description of any motion depends on the frame of reference of the observer.

Your **frame of reference** was the **vehicle you were sitting in**. You assumed, at least momentarily, that your vehicle was at rest, and you viewed all motion from your **reference point**. On the other hand, your friend's frame of reference is the surface of the earth where she is standing. From her reference point she says that your vehicle stayed at rest while the other moved backwards.

Are you tempted to say that your friend's frame of reference is "better" or more valid than yours? DON'T say that. They are both equally valid, especially as far as physics is concerned. Let us have a look at another example.

You are on a train watching a ping pong ball bounce on a table.
Does it go straight up and down in a vertical line?



YES, otherwise it would miss the table on the return trip.

Next ask the driver to drive at a fixed speed down a straight track (there should be no sudden turns or braking) while you bounce the ball up and down. Because the train is moving when you toss it up, does the ball go flying back over your shoulder?



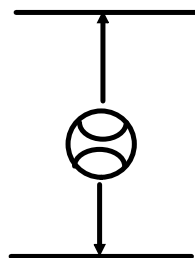
Of course NOT. It would be really amusing to see such a thing; it would also be annoying to someone sitting behind you if that happened.

OK, then, what does the ball do?



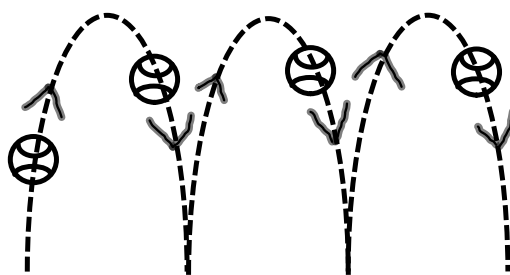
It goes up, up, up past your nose and down, down, down onto the table in exactly the same manner as before.

From other experiences you know that you would have no trouble bouncing the ball in this manner even though the train is moving. From the **frame of reference of the** train you see the ball going straight up and down exactly as it did before the car started.



Next we put your friend outside the train, some distance into the field on one side of the road. We will even give your friend binoculars so that she can watch you bounce the ball as you ride by. Suppose that on the other side of the road there is a fence with posts driven into the ground, and your friend can see the posts through the car as you drive by. She sees you bounce the ball into the air just as you pass post 1. While the ball is in the air, you, the train, and the ball race towards post 2. Just as you pass post 2, the ball arrives back on the table. It bounces into the air and back again as the car passes post 3. What does your friend see?

From your friend's **frame of reference** the ball is **not** going straight up and down. Instead it is moving in an arc or trajectory. The motion as seen from each frame of reference is illustrated below.



http://www.cdli.ca/courses/phys2204/unit01_org02_ilo02/b_activity.html



Examples:

1. An ant travels 10 cm to the right on a plate **relative to the plate** as a disgusted picnicker pushes the plate a distance of 40 cm to the right **relative to the earth**. What is the **displacement** of the ant with **respect to the earth**?

Solution:**Method 1 - In your head**

You can reason like this: if the ant crawls 10 cm to the right on the plate, and then if the plate is pushed an additional 40 cm to the right, the ant will altogether move 50 cm to the right. (relative to the earth).

Method 2: Vector Algebra Method

We will introduce some neat shorthand:

- ${}_a\vec{d}_p$ the displacement of the ant with respect to the plate
- ${}_p\vec{d}_e$ the displacement of the plate with respect to the earth
- ${}_a\vec{d}_e$ the displacement of the ant with respect to the earth.

The **displacement** of the ant with **respect to the earth**
EQUALS the **displacement** of the ant with **respect to the plate**
PLUS the **displacement** of the plate with **respect to the earth**.

In short-hand, this becomes ${}_a\vec{d}_e = {}_a\vec{d}_p + {}_p\vec{d}_e$

Trick to remind yourself which subscript goes where.

- Notice that the subscripts on the **LEFT HAND SIDE** of the equation (a and e) appear on the extreme left and extreme right on **THE RIGHT HAND SIDE** of the equation.
- Notice that the "subscripts in the middle" of the right hand side are both the same (they are p's).

Before we practice this trick by writing another equation, let us find the displacement of the ant by Method 2:

$$\begin{aligned}
 {}_a\vec{d}_e &= {}_a\vec{d}_p + {}_p\vec{d}_e \\
 {}_a\vec{d}_e &= 10\text{ cm} + 40\text{ cm} \\
 {}_a\vec{d}_e &= 50\text{ cm}
 \end{aligned}$$

2. Write the following three terms in an equation:

$$\textcircled{hP_w} \quad {}_sP_w \quad \textcircled{sP_h}$$

$${}_sP_h + {}_hP_w = {}_sP_w$$

3. An ant travels 10 cm to the right on a plate **relative to the plate** as a disgusted picnicker pushes the plate a distance of 40 cm to the left **relative to the earth**. What is the **displacement** of the ant with **respect to the earth**?

$$\begin{aligned} a_{d\boxed{p}} &= 10 \text{ cm} & a_{d_p} + {}_p d_e &= a_{d_e} \\ \boxed{p} d_e &= -40 \text{ cm} & 10 \text{ cm} + -40 \text{ cm} &= a_{d_e} \\ a_{d_e} &= ? & -30 \text{ cm} &= a_{d_e} \\ & & 30 \text{ cm [L]} &= a_{d_e} \end{aligned}$$

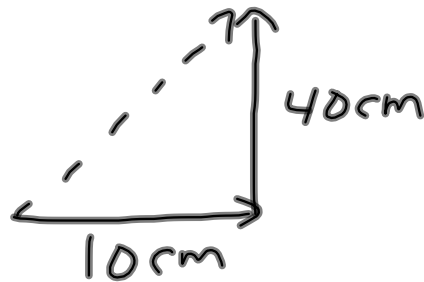
4. As a disgusted picnicker pushes a plate at 30 cm to the left **relative to the earth**, it is noticed that the ant's displacement **relative to the earth** was only 20 cm to the left. This could only occur if the ant was walking on the plate as the plate was being pushed. Determine the ant's **displacement relative to the plate**.

$$\begin{aligned} \boxed{p} d_e &= -30 \text{ cm} \\ a_{d_e} &= -20 \text{ cm} \\ a_{d\boxed{p}} &= ? \end{aligned}$$

$$\begin{aligned} a_{d_p} + {}_p d_e &= a_{d_e} \\ a_{d_p} - 30 \text{ cm} &= -20 \text{ cm} \\ a_{d_p} &= 10 \text{ cm} \\ a_{d_p} &= 10 \text{ cm [Right]} \end{aligned}$$

5. An ant travels 10.0 cm [E] on a plate as a disgusted picnicker pushes the plate a distance of 40.0 cm [N]. What is the **displacement** of the ant with **respect to the earth**?

$$\begin{aligned} d_{P} &= 10 \text{ cm [E]} \\ d_{e} &= 40 \text{ cm [N]} \\ d_{e} &= \end{aligned}$$



6. An ant travels 10.0 cm [E] on a plate as a disgusted picnicker pushes the plate a distance of 40.0 cm [E 60.0°EN]. What is the **displacement** of the ant with **respect to the earth**?