

## Section 21: Conservation of Momentum

**Isolated System** - No net force acts from outside.

### Law of Conservation of Momentum

In any isolated system, the total momentum of the parts before an interaction (collision, explosion, etc.) is equal to the total momentum of the parts after the interaction.

There are two types of interactions:

- A) Elastic
- B) Inelastic

- A) Elastic Interactions** - objects bounce apart  
- Both momentum and kinetic energy are conserved.

#### Momentum

$$p = p'$$

$$m_A v_A + m_B v_B + \dots = m_A v_A' + m_B v_B' + \dots$$

#### Kinetic Energy

$$E_k = E_k'$$

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \dots = \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2 + \dots$$

- B) Inelastic Interactions** - objects stick together  
- Only momentum is conserved.

$$p = p'$$


$$m_A v_A + m_B v_B + \dots = (m_A + m_B + \dots)v'$$

$$(m_A + m_B + \dots)v = m_A v_A' + m_B v_B' + \dots$$

## Examples


1. Cart A has a mass of 2.0 kg and is travelling at 10.0 m/s to the right. Cart B has a mass of 3.0 kg and is moving to the left at 6.0 m/s. Cart B collides with cart A which causes cart A to rebound at 4.0 m/s. What is the velocity of cart B after the collision?

Before



$m_A = 2.0 \text{ kg}$     $m_B = 3 \text{ kg}$   
 $v_A = 10 \text{ m/s}$     $v_B = -6.0 \text{ m/s}$

After



$v'_A = -4.0 \text{ m/s}$     $v'_B = ?$   
 $m_A = 2.0 \text{ kg}$     $m_B = 3 \text{ kg}$

$$p = p'$$

$$p_A + p_B = p'_A + p'_B$$


$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$(2 \text{ kg})(10 \text{ m/s}) + (3 \text{ kg})(-6 \text{ m/s}) = (2 \text{ kg})(-4.0 \text{ m/s}) + (3 \text{ kg})v'_B$$

$$3.3 \text{ m/s} = v'_B$$


2. A ball of putty with a mass of 0.10 kg collides with and sticks to a dynamics cart with a mass of 1.20 kg. If the ball of putty was moving to the RIGHT at 32 m/s and the dynamics cart was moving to the LEFT at 2.5 m/s, what will be the velocity of the combined object after the collision?

→   ←



$m_p = 0.10 \text{ kg}$     $m_c = 1.20 \text{ kg}$   
 $v_p = 32 \text{ m/s}$     $v_c = -2.5 \text{ m/s}$

→



$m_p + m_c = 0.1 \text{ kg} + 1.2 \text{ kg}$   
 $= 1.3 \text{ kg}$   
 $v' = ?$


$$p = p'$$

$$m_p v_p + m_c v_c = (m_p + m_c) v'$$

$$(0.1 \text{ kg})(32 \text{ m/s}) + (1.2 \text{ kg})(-2.5 \text{ m/s}) = (1.3 \text{ kg})v'$$

$$0.15 \text{ m/s} = v'$$

3. A 50.0 g bullet is shot from a 4.0 kg rifle. The bullet leaves the rifle at 400 m/s. What is the recoil velocity of the gun?



$$\begin{aligned}
 m_B &= 0.050 \text{ kg} \\
 m_R &= 4.0 \text{ kg} \\
 m_T &= 4.05 \text{ kg} \\
 V &= 0
 \end{aligned}$$

$$\begin{aligned}
 V_R &=? \\
 V_B &= 400 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 p &= p' \\
 0 &= m_B v_B' + m_R v_R' \\
 0 &= (0.050 \text{ kg})(400 \text{ m/s}) + (4 \text{ kg}) v_R' \\
 -5.0 \text{ m/s} &= v_R'
 \end{aligned}$$

4. A 0.50 kg ball traveling at 6.0 m/s collides head-on with a 1.50 kg ball moving in the opposite direction at 12.0 m/s. After the collision the 0.50 kg ball rebounds at 14 m/s. Find the velocity of the 1.50 kg ball after the collision.

$$\begin{aligned}
 m_A &= 0.50 \text{ kg} & m_B &= 1.5 \text{ kg} \\
 V_A &= 6.0 \text{ m/s} & V_B &= -12 \text{ m/s} \\
 V_A' &= -14 \text{ m/s} & V_B' &=?
 \end{aligned}$$

$$\begin{aligned}
 p &= p' \\
 p_A + p_B &= p_A' + p_B' \\
 m_A V_A + m_B V_B &= m_A V_A' + m_B V_B' \\
 (0.5 \text{ kg})(6 \text{ m/s}) + (1.5 \text{ kg})(-12 \text{ m/s}) &= (0.5 \text{ kg})(-14 \text{ m/s}) + (1.5 \text{ kg}) V_B' \\
 5.3 \text{ m/s} &= V_B'
 \end{aligned}$$

5. A 1.6 g air rifle pellet is fired into a small 20.0 g target that is free to move on a frictionless surface. After the pellet becomes imbedded in the target, the target and pellet move off at 8.0 m/s. What was the speed of the pellet just before it hit the target?

<p><u>before</u></p> <p>○</p> <p><math>m_p = 0.0016 \text{ kg}</math></p> <p><math>v_p = ?</math></p>	<p>□</p> <p><math>m_T = 0.020 \text{ kg}</math></p> <p><math>v_T = 0</math></p>	}	<p><u>after</u></p> <p>□</p> <p><math>m_p + m_T = 0.0216 \text{ kg}</math></p> <p><math>v' = 8.0 \text{ m/s}</math></p>
---	---	---	---

$$p = p'$$

$$m_p v_p + m_T v_T = (m_p + m_T) v'$$

$$(0.0016 \text{ kg}) v_p = (0.0216 \text{ kg})(8 \text{ m/s})$$

$$v_p = 110 \text{ m/s}$$

6. Johnny, with a mass of 80 kg, is chatting with his girlfriend, Sally, while standing on the ice at the skating rink. Sally playfully reaches out and pushes Johnny. He moves away with a speed of 3.5 m/s. If Sally has a mass of 60 kg, what will be her velocity?

$$m_J = 80 \text{ kg} \quad m_S = 60 \text{ kg}$$

$$v_J = 0 \quad v_S = 0$$

$$v_J' = 3.5 \text{ m/s} \quad v_S' = ?$$

$$p = p'$$

$$0 = m_J v_J' + m_S v_S'$$

$$0 = (80 \text{ kg})(3.5 \text{ m/s}) + (60 \text{ kg}) v_S'$$

$$-5 \text{ m/s} = v_S'$$