

Section 2.8: Newton's Second Law of Motion

Recall:

The first law of motion states that as long as the forces are balanced

- an object at rest stays at rest
- an object in motion stays in motion at a constant speed in a straight line.

What if an unbalanced force acts on object? What happens?

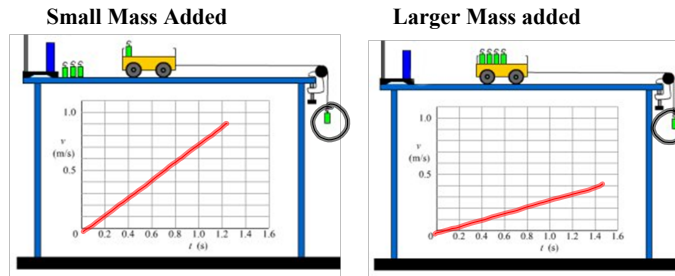
The object will accelerate!!

Keep the Unbalanced Force constant and Vary Mass

Below is a similar setup as before:

- Now, we will keep the suspended mass (and thus the unbalanced force) equal.
- We will vary the mass, by adding extra masses to cart.

Compare how the motion of each cart below is different?



If the unbalanced force is constant and the mass of an object increases, the acceleration decreases; if the mass of an object decreases, its acceleration increases.

THE ACCELERATION OF AN OBJECT IS INVERSELY PROPORTIONAL TO ITS MASS.

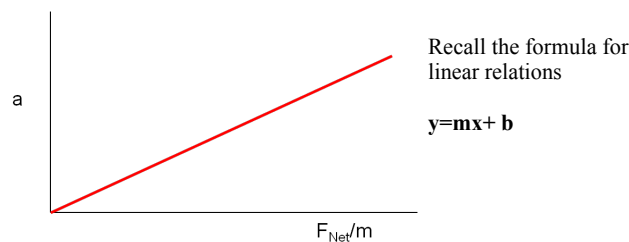
In short, $a \propto 1/m$

$$a \propto \frac{1}{m}$$

Those two parts make up Newton's Second Law! We can combine the two

$$a \propto \frac{F_{Net}}{m}$$

This suggests that there is a linear relation between a and $\frac{F_{Net}}{m}$



This means that $a = k \left(\frac{F_{Net}}{m} \right)$ where k is a constant – the slope of the graph of the two quantities.

We can rewrite this as $k F_{Net} = ma$

Now, if we define the unit of Force as ONE NEWTON which is the AMOUNT OF FORCE THAT WILL GIVE A MASS OF ONE KILOGRAM AN ACCELERATION OF 1 m/s^2 , k becomes 1 and the equation becomes

$$F_{Net} = ma$$

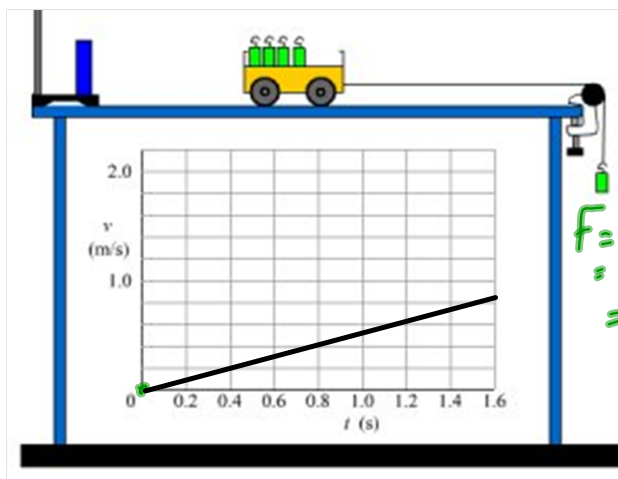
where: m is the mass of the object in kg
 a is the acceleration of the object in m/s^2
 F is the net force on the object in kgm/s^2 or Newtons (N)

Vary the Unbalanced Force and Observe the Effect on Motion (Constant Mass)

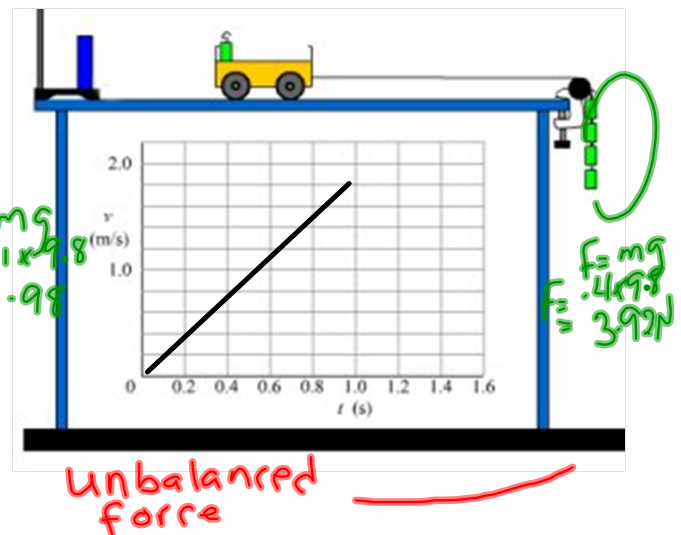
Consider setup below:

- Cart and mass is attached to a suspended mass. If there is no friction, what happens as suspended mass is released?
- How does cart's motion compare in each case below?

Small Suspended Mass – Small Force



Larger Suspended Mass – Larger Force



IF THE MASS STAYS CONSTANT AND THE UNBALANCED FORCE BECOMES LARGER, THE ACCELERATION BECOMES LARGER.

*** THE ACCELERATION OF AN OBJECT IS DIRECTLY PROPORTIONAL TO THE NET (OR UNBALANCED) FORCE ACTING ON IT.**

In short, we write

$$a \propto F_{\text{NET}}$$

Newton's Second Law of Motion or Newton's Law of Acceleration states:

If an unbalanced force acts on an object, the object **accelerates in the direction of the force**. The acceleration varies directly with the unbalanced force and inversely with the mass of the object.

$$a \propto \frac{1}{m} \quad \text{if the force remains constant}$$

$$a \propto F_{\text{Net}} \quad \text{if the mass remains constant.}$$

$$\mathbf{F_{Net} = ma}$$

Question: A rocket fired from its launching pad not only picks up speed, but its acceleration increases significantly as firing continues. Why is this so?

Examples

1. What acceleration results when a force of 22.0 N is applied to an object that has a mass of 2.2 kg?

$$\frac{F_{net}}{m} = \frac{ma}{m} \rightarrow a = 1.0 \times 10^1 \text{ m/s}^2$$

$$\frac{22 \text{ N}}{2.2 \text{ kg}} = a$$

2. A rock is resting on an ice surface. What is its mass if an applied force of 3.2 N gives it an acceleration of 8.0 m/s²?

$$\frac{F_{net}}{a} = \frac{ma}{a} \rightarrow 0.40 \text{ kg} = m$$

$$\frac{3.2 \text{ N}}{8.0 \text{ m/s}^2} = m$$

3. A car accelerates at 0.80 m/s². What accelerating force is required if the car has a mass of 1200 kg?

$$F_{net} = ma$$

$$= (1200 \text{ kg})(0.80 \text{ m/s}^2)$$

$$= 960 \text{ N}$$

4. A bicycle goes from rest to 8.0 m/s in a time of 2.0 s. What must have been the accelerating force if the bike and rider have a combined mass of 132 kg?

$$V_1 = 0$$

$$V_2 = 8.0 \text{ m/s}$$

$$t = 2.0 \text{ s}$$

$$m = 132 \text{ kg}$$

$$F_{net} = ? \text{ (ma)}$$

$$a = ?$$

$$a = \frac{V_2 - V_1}{t} = \frac{8.0 \text{ m/s}}{2.0 \text{ s}} = 4.0 \text{ m/s}^2$$

$$F_{net} = ma$$

$$= (132 \text{ kg})(4.0 \text{ m/s}^2)$$

$$= 528 \text{ N}$$

$$= 530 \text{ N}$$

5. A truck with a mass of 4000 kg is initially traveling at 10.0 m/s when the driver presses down the accelerator. The truck's engine applies an accelerating force of 1000.0 N. What will be the truck's velocity after 5.0 s?

$$m = 4000 \text{ kg}$$

$$V_1 = 10.0 \text{ m/s}$$

$$F_{net} = 1000 \text{ N}$$

$$t = 5.0 \text{ s}$$

$$V_2 = ?$$

$$* \text{ need to find } a?$$

$$\frac{F_{net}}{m} = \frac{ma}{m}$$

$$\frac{1000 \text{ N}}{4000 \text{ kg}} = a$$

$$0.25 \text{ m/s}^2 = a$$

$$V_2 = V_1 + at$$

$$V_2 = 10.0 \text{ m/s} + (0.25 \text{ m/s}^2)(5 \text{ s})$$

$$V_2 = 11 \text{ m/s}$$

6. A driver in a car notices that, at full power, with a net force of 900 N, it accelerates at 1.0 m/s^2 . Suppose that car takes on passengers with a combined extra mass of 250 kg. What will be the acceleration at full power? HINT: first find the mass of the van + driver. This helps to give the total mass which is needed for the last step.

<u>Before</u>	<u>After</u>
$F_{\text{net}} = 900 \text{ N}$	$m_{\text{extra}} = 250 \text{ kg}$
$a = 1.0 \text{ m/s}^2$	$a = ?$
$m = ?$ (driver + car)	$F_{\text{net}} = 900 \text{ N}$

$$\textcircled{1} m = \frac{F_{\text{net}}}{a} = \frac{900 \text{ N}}{1.0 \text{ m/s}^2} = 900 \text{ kg}$$

$$\textcircled{2} M_{\text{Total}} = 900 \text{ kg} + 250 \text{ kg} = 1150 \text{ kg}$$

$$\textcircled{3} a = \frac{F_{\text{net}}}{m} = \frac{900 \text{ N}}{1150 \text{ kg}} = 0.8 \text{ m/s}^2$$

7. A minivan and its driver have a combined mass of 1220 kg and can accelerate from rest to 28 m/s (about 100 km/hr) in a time of 13.0 s. Suppose that the van stops to let some "hockey playing buddies" along with their gear, to get on. The hockey players and the gear have a combined mass of 450 kg. How long will it now take the van to accelerate to 28 m/s? HINT: find the accelerating force first.

<u>Before</u>	<u>After</u>
$m = 1220 \text{ kg}$	$m = 1220 \text{ kg} + 450 \text{ kg}$
$V_1 = 0 \text{ m/s}$	$= 1670 \text{ kg}$
$V_2 = 28 \text{ m/s}$	$V_1 = 0 \text{ m/s}$
$t = 13.0 \text{ s}$	$V_2 = 28 \text{ m/s}$
$\textcircled{1} a = ?$	$\textcircled{1} t = ?$
$\textcircled{2} F_{\text{net}} = ?$	$\textcircled{2} a = ?$

Same $\Rightarrow F_{\text{net}} (a = \frac{F_{\text{net}}}{m})$

$$\textcircled{1} a = \frac{V_2 - V_1}{t} = \frac{28 \text{ m/s} - 0 \text{ m/s}}{13 \text{ s}} = 2.15 \text{ m/s}^2$$

$$\textcircled{2} F_{\text{net}} = ma = (1220 \text{ kg})(2.15 \text{ m/s}^2) = 2623 \text{ N}$$

$$\textcircled{3} a = \frac{F_{\text{net}}}{m} = \frac{2623 \text{ N}}{1670 \text{ kg}} = 1.57 \text{ m/s}^2$$

$$\textcircled{4} t = \frac{V_2 - V_1}{a} = \frac{28 \text{ m/s} - 0 \text{ m/s}}{1.57 \text{ m/s}^2} = 18 \text{ s}$$

8. A car having a mass of 1000 kg comes to a stop in 40 m. If the initial speed was 20 m/s, what average stopping force was supplied by the road acting on the car? HINT: first determine the acceleration.

$m = 1000 \text{ kg}$	$\frac{2ad}{2d} = \frac{V_2^2 - V_1^2}{2d}$
$d = 40 \text{ m}$	
$V_1 = 20 \text{ m/s}$	
$V_2 = 0$	$a = \frac{0 - (20 \text{ m/s})^2}{2(40 \text{ m})}$
$F_{\text{net}} = ?$	$a = -5.0 \text{ m/s}^2$
$a = ?$	$F_{\text{net}} = ma = (1000 \text{ kg})(-5.0 \text{ m/s}^2)$
	$= -5000 \text{ N}$

Proportionality Exercises

$$a \propto \frac{F_{\text{net}}}{m}$$

1. What happens to the acceleration of a truck if its mass is tripled?

$$a \propto \frac{F_{\text{net}}}{m}$$

$$a \propto \frac{1}{3}$$

Acceleration dec. by a factor of 3.

2. What happens to the acceleration of a truck if the net force is doubled and the mass is decreased by a factor of 3?

$$a \propto \frac{F_{\text{net}}}{m}$$

$$a \propto \frac{2}{1/3}$$

$$a \propto 2 \times 3$$

$$a \propto 6$$

The acceleration is 6 times greater.

3. An object accelerates at 4.5 m/s^2 . Suppose the net force is halved and the mass is increased by a factor of 3. What is its new acceleration?

$$a \propto \frac{F_{\text{net}}}{m}$$

$$\propto \frac{1/2}{3}$$

$$a \propto \frac{1}{2} \times \frac{1}{3}$$

$$a \propto \frac{1}{6}$$

$$\left((1 \div 2) \div 3 \right)$$

$$0.1666 \dots$$

$$\text{Math} \rightarrow \text{frac}$$

$$\frac{1}{6}$$

$$a = \frac{1}{6} \times 4.5 \text{ m/s}^2$$

$$a = 0.75 \text{ m/s}^2$$

Newton's 2nd Law of Motion

Test yourself

1. Suppose that a constant NET force (or UNBALANCED force) acts on an object. Which choice correctly describes the motion of the object?

- (a) Constant velocity
- (b) Constant acceleration
- (c) Increasing acceleration
- (d) Decreasing acceleration

2. Which statement describes the relation between net force and acceleration. The quantities are...

- (a) not related
- (b) equal
- (c) in inverse proportion
- (d) in direct proportion

3. Which statement describes the relation between mass and acceleration. The quantities are...

- (a) not related
- (b) equal
- (c) in inverse proportion
- (d) in direct proportion

4. What will happen to the acceleration of an object if the net force is doubled and the mass is held constant? It will...

- (a) increase by a factor of 2
- (b) increase by a factor of 4
- (c) decrease by a factor of 2
- (d) decrease by a factor of 4

5. What will happen to the acceleration of an object if the mass is doubled and the net force is held constant? It will...

- (a) increase by a factor of 2
- (b) increase by a factor of 4
- (c) decrease by a factor of 2
- (d) decrease by a factor of 4

Answers

- 1. b
- 2. d
- 3. c
- 4. a
- 5. c