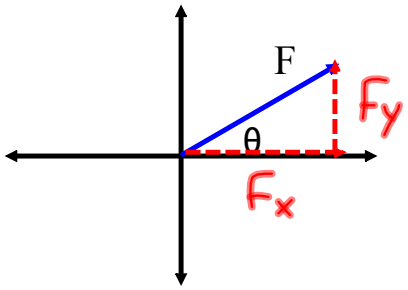
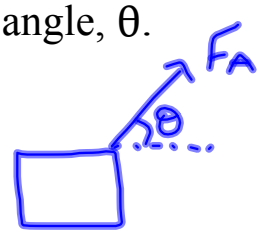
Section 2.4: Component Forces

Forces don't always act in the N, S, E, or W direction. Quite often they act at an angle, which means that part of the force will act in the x-direction and part of the force will act in the y-direction. Using basic trigonometry we can determine how much of a given force acts in the x-direction and how much acts in the y-direction.

Consider the following force which is acting at an angle, θ.





$\frac{\cos\theta}{1} = \frac{F_x}{F}$ $\qquad$ $\frac{\sin\theta}{1} = \frac{F_y}{F}$

$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

To find the horizontal & vertical components of any force that makes an angle θ with the x-axis use

horizontal : $F_x = F\cos\theta$

Vertical : $F_y = F\sin\theta$

Component Method

Steps:

1. Draw each vector separately in the x-y plane.

2. Find the vertical (Fy) and horizontal (Fx) components of all forces. Include direction with each component and compile them in two separate columns.

$F_x = F\cos\theta \qquad F_y = F\sin\theta$

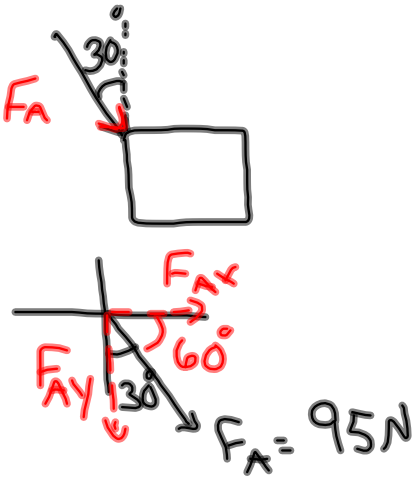
3. Find the sum of the x-components.
   Find the sum of the y-components.

4. Use Pythagorean Theorem and Right Triangle Trigonometry to find the resultant. (F_{net})

Examples

1 A lawnmower is pushed across a lawn by applying a force of 95 N along the handle of the mower. The handle makes an angle of 30º with the vertical.

A) What are the horizontal and vertical components of the force?



$F_{Ax} = F_A \cos\theta$

$F_{Ax} = 95N \cos 60°$

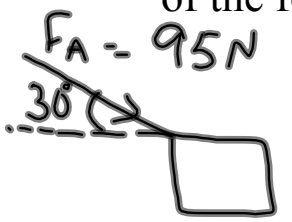
$F_{Ax} = 48N$ (Right)

$F_{Ay} = F_A \sin\theta$

$F_{Ay} = 95N \sin 60°$

$F_{Ay} = 82N$ (down)

B) The handle is lowered so that it makes an angle of 30º with the horizon, what are the horizontal and vertical components of the force?



$F_{Ax} = F_A \cos\theta$

$= 95N \cos 30°$

$= 82N$ (Right)

$F_{Ay} = F_A \sin\theta$

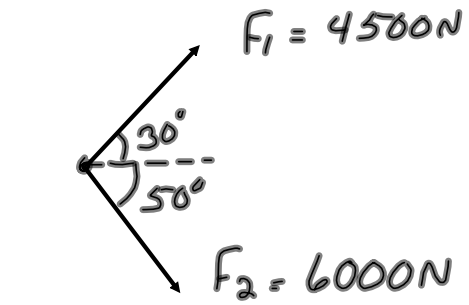
$= 95N \sin 30°$

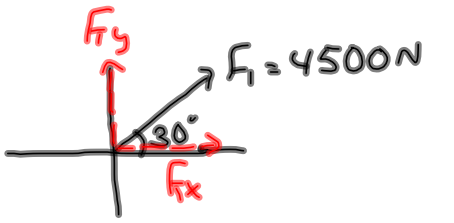
$= 48N$ (Down)

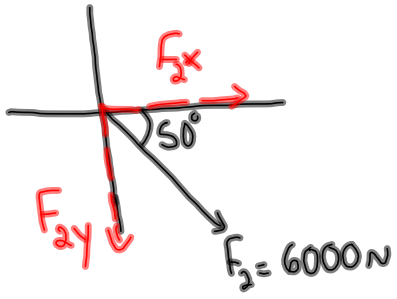
midterm Test

2. Two forces, ***F***1 and ***F***2 act on the same object. Find the resultant if ***F***1 = 4500 N [30° NE] and ***F***2 = 6000 N [50° SE].

(Assume 2sfs)







$F_{1x} = F_1 \cos\theta$

$F_{1x} = 4500N \cos 30°$

$= 3897.1N[E]$

$F_{1y} = F_1 \sin\theta$

$= 4500N \sin 30°$

$= 2250N[N]$

$F_{2x} = 6000N \cos 50°$

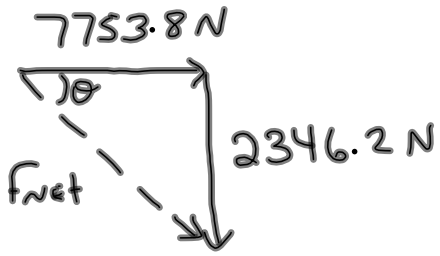
$= 3856.7N[E]$

$F_{2y} = 6000N \sin 50°$

$= 4596.2N[S]$

$\Sigma x = 7753.8N[E]$

$\Sigma y = 2346.2N[S]$



$F_{net}^2 = (7753.8N)^2 + (2346.2N)^2$

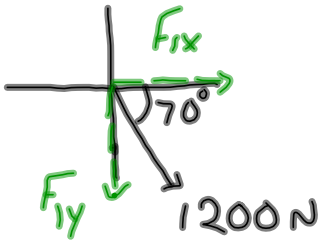
$F_{net} = 8100N[E\ 17°S]$

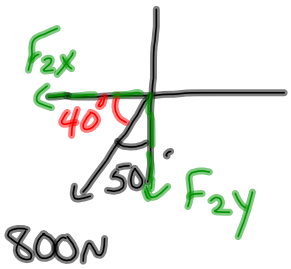
$\tan\theta = \frac{2346.2N}{7753.8N}$

$\theta = 17°$

3. Two forces ***F***1 and ***F***2 act on the same object. Find the resultant if ***F***1 = 1200 N [E 70° S] and ***F***2 = 800 N [S 50° W].

(Assume 3 sfs)





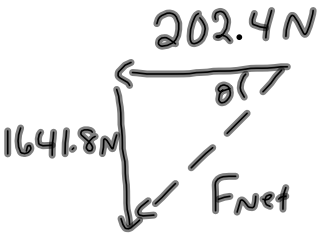
$F_{1x} = F_1 \cos\theta$
$= 1200N \cos 70°$
$= 410.4N\ [E]$

$F_{1y} = F_1 \sin\theta$
$= 1200N \sin 70°$
$= 1127.6N\ [S]$

$F_{2x} = 800N \cos 40°$
$= 612.8N\ [W]$

$F_{2y} = 800N \sin 40°$
$= 514.2N\ [S]$

$\Sigma x = 202.4N\ [W]$
$\Sigma y = 1641.8N\ [S]$

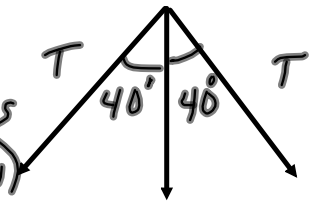


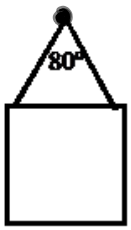
$F_{Net}^2 = (202.4N)^2 + (1641.8N)^2$
$F_{Net} = 1650N\ [W\ 83°\ S]$

$\tan\theta = \frac{1641.8N}{202.4N}$
$\theta = 83°$

4. The picture shown below weighs 60.0 N. The angle made by the string at the nail is 80.0º. Determine the tension in the strings.

* Draw an FBD at the nail head
* Each string supports half the weight (30N)

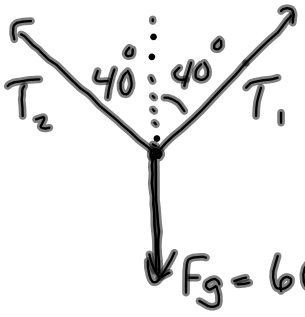




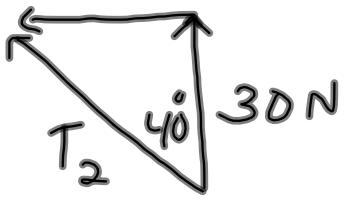
$$\cos 40^\circ = \frac{30\,N}{T}$$

$$T = \frac{30\,N}{\cos 40^\circ} = 39.2\,N$$

5. Find the tension in the string if the picture is hung as shown below. The weight is still 60.0 N and the angle between the string is 80.0º.



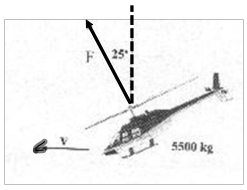
Each string supports $\frac{1}{2}$ the weight

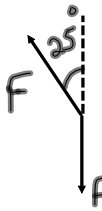


$$\cos 40^\circ = \frac{30\,N}{T_2}$$

$$T_2 = \frac{30\,N}{\cos 40^\circ} = 39.2\,N$$

6. What is the force, F, provided by the rotor, if the helicopter shown is travelling in level flight?

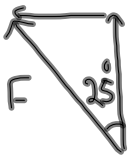




$F_g = mg$

$= (5500\,kg)(9.8\,m/s^2)$

$= 53\,900\,N$

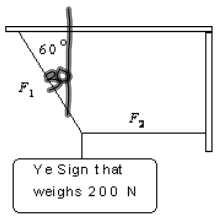


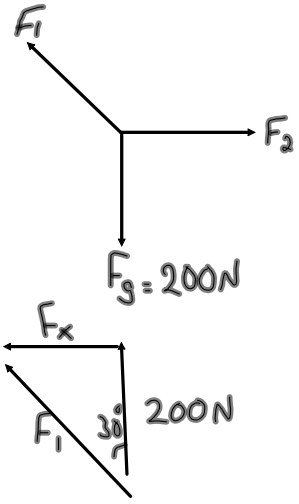
$\cos 25^\circ = \frac{53900\,N}{F}$

$F = \frac{53\,900\,N}{\cos 25^\circ}$

$F = 59\,500\,N$

7. A sign weighing 200.0 N is held in place by two wires as shown in the picture. One wire is horizontal and the other makes an angle of 60.0° with the top support. Find the tensional forces, F_1 and F_2 in the wires.





F_1 is the force responsible for supporting the weight of the sign. F_2 just holds it in position.

$\cos 30^\circ = \frac{200\,N}{F_1}$

$F_1 = \frac{200\,N}{\cos 30^\circ}$

$F_1 = 231\,N$

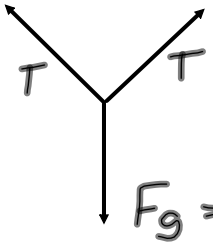
* Now find F_x. F_2 is equal but in the opposite direction to F_x

$\tan 30^\circ = \frac{F_x}{200\,N}$

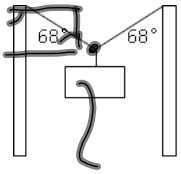
$F_x = 200\,N \tan 30^\circ$ so $F_2 = 115\,N$

$= 115\,N$

8 A sign is held by 2 wires as shown. What is the tension in each wire?

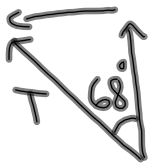


*Each string supports $\frac{1}{2} F_g$ (weight)



$m = 75\,kg$

$F_g = mg$
$= (75\,kg)(9.8\,m/s^2)$
$= 735\,N$

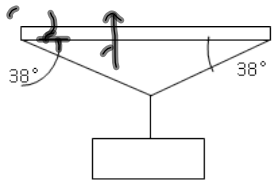


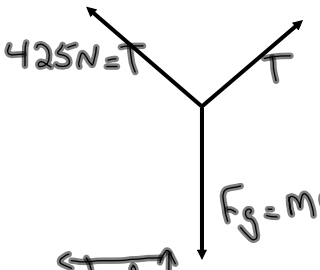
$\cos 68° = \frac{367.5\,N}{T}$

$T = \frac{367.5\,N}{\cos 68°}$

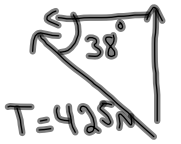
$T = 980\,N$

9 A sign hangs from 2 cables as shown. If the tension in each wire is 425 N, what is the mass of the sign?





Each string supports $\frac{1}{2} F_g$.



$(x = \frac{1}{2} F_g)$

$\sin 38° = \frac{x}{425\,N}$

$x = 261.7\,N$

$F_g = 2(261.7\,N)$

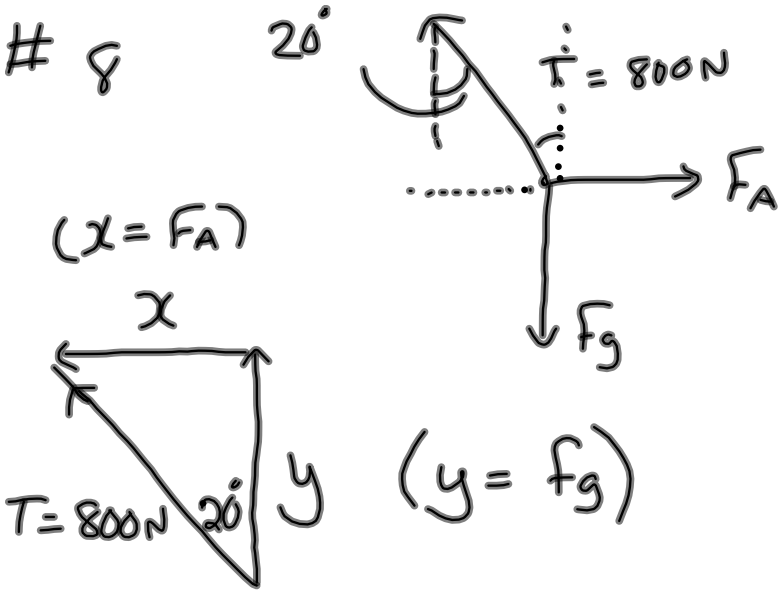
$F_g = 523.4\,N$

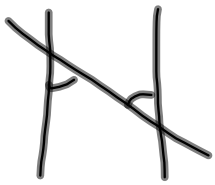
$\frac{F_g}{g} = \frac{mg}{g}$

$\frac{523.4\,N}{9.8\,m/s^2} = m$

$53.4\,kg$

8





$\cos 20^\circ = \frac{y}{800N}$

$y = 751.8N$

$\therefore f_g = 751.8N$

$\sin 20^\circ = \frac{x}{800N}$

$x = 273.6N$

$F_A = 273.6N$