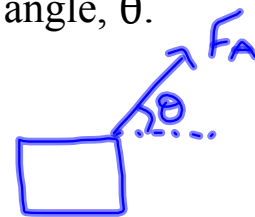
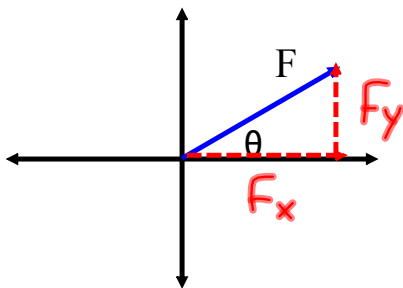


## Section 2.4: Component Forces

Forces don't always act in the N, S, E, or W direction. Quite often they act at an angle, which means that part of the force will act in the x-direction and part of the force will act in the y-direction. Using basic trigonometry we can determine how much of a given force acts in the x-direction and how much acts in the y-direction.

Consider the following force which is acting at an angle,  $\theta$ .



$$\frac{\cos\theta}{1} = \frac{F_x}{F}$$

$$\frac{\sin\theta}{1} = \frac{F_y}{F}$$

$$F_x = F \cos\theta \quad F_y = F \sin\theta$$

To find the horizontal & vertical components of any force that makes an angle  $\theta$  with the x-axis, use

horizontal :  $F_x = F \cos\theta$

vertical :  $F_y = F \sin\theta$

## Component Method

Steps:

1. Draw each vector separately in the x-y plane.
2. Find the vertical ( $F_y$ ) and horizontal ( $F_x$ ) components of all forces. Include direction with each component and compile them in two separate columns.

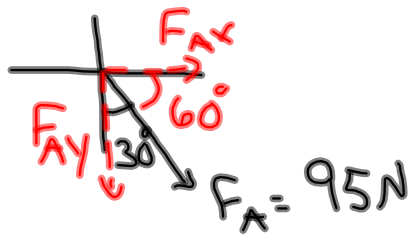
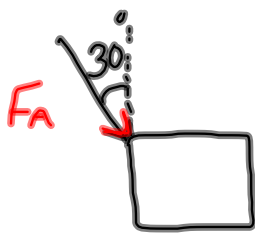
$$F_x = F \cos \theta \quad F_y = F \sin \theta$$

3. Find the sum of the x-components.  
Find the sum of the y-components.
4. Use Pythagorean Theorem and Right Triangle Trigonometry to find the resultant. ( $F_{net}$ )

## Examples

- 1 A lawnmower is pushed across a lawn by applying a force of 95 N along the handle of the mower. The handle makes an angle of  $30^\circ$  with the vertical.

A) What are the horizontal and vertical components of the force?



$$F_{Ax} = F_A \cos \theta$$

$$F_{Ax} = 95\text{ N} \cos 60^\circ$$

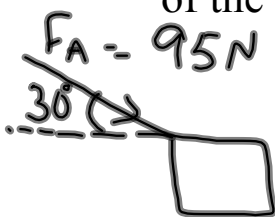
$$F_{Ax} = 48\text{ N (Right)}$$

$$F_{Ay} = F_A \sin \theta$$

$$F_{Ay} = 95\text{ N} \sin 60^\circ$$

$$F_{Ay} = 82\text{ N (down)}$$

B) The handle is lowered so that it makes an angle of  $30^\circ$  with the horizon, what are the horizontal and vertical components of the force?



$$F_{Ax} = F_A \cos \theta$$

$$= 95\text{ N} \cos 30^\circ$$

$$= 82\text{ N (Right)}$$

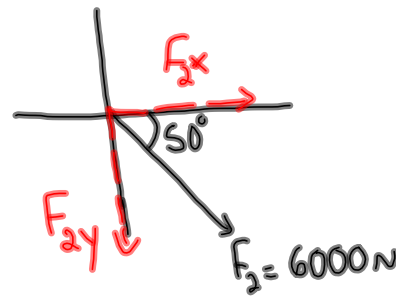
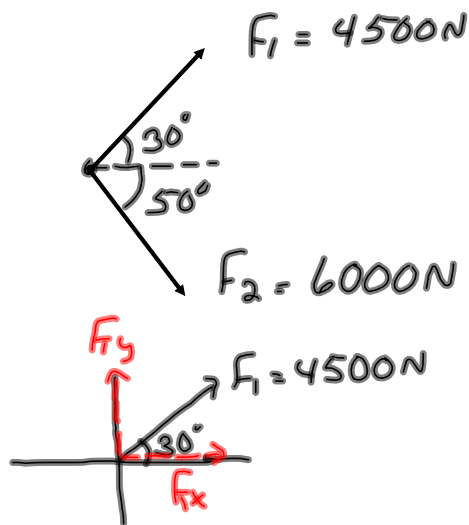
$$F_{Ay} = F_A \sin \theta$$

$$= 95\text{ N} \sin 30^\circ$$

$$= 48\text{ N (Down)}$$

## midterm test

2. Two forces,  $F_1$  and  $F_2$  act on the same object. Find the resultant if  $F_1 = 4500 \text{ N}$  [ $30^\circ \text{ NE}$ ] and  $F_2 = 6000 \text{ N}$  [ $50^\circ \text{ SE}$ ].  
(Assume 255)



$$F_{1x} = F_1 \cos \theta$$

$$F_{1x} = 4500 \text{ N} \cos 30^\circ$$

$$= \underline{3897.1 \text{ N [E]}}$$

$$F_{1y} = F_1 \sin \theta$$

$$= 4500 \text{ N} \sin 30^\circ$$

$$= \underline{2250 \text{ N [N]}}$$

$$\Sigma x = \underline{7753.8 \text{ N [E]}}$$

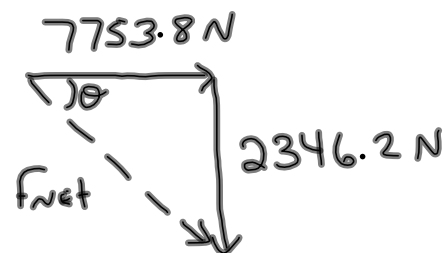
$$\Sigma y = \underline{2346.2 \text{ N [S]}}$$

$$F_{2x} = 6000 \text{ N} \cos 50^\circ$$

$$= \underline{3856.7 \text{ N [E]}}$$

$$F_{2y} = 6000 \text{ N} \sin 50^\circ$$

$$= \underline{4596.2 \text{ N [S]}}$$



$$F_{\text{net}}^2 = (7753.8 \text{ N})^2 + (2346.2 \text{ N})^2$$

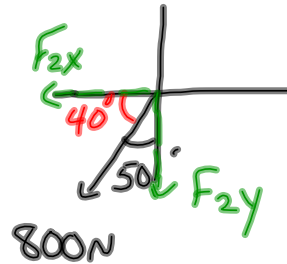
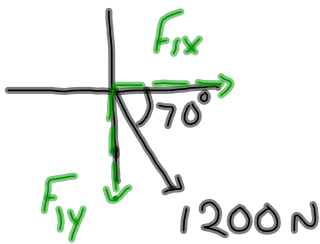
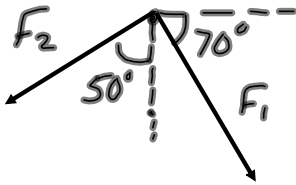
$$F_{\text{net}} = 8100 \text{ N [E } 17^\circ \text{ S]}$$

$$\tan \theta = \frac{2346.2 \text{ N}}{7753.8 \text{ N}}$$

$$\theta = 17^\circ$$

3. Two forces  $F_1$  and  $F_2$  act on the same object. Find the resultant if  $F_1 = 1200 \text{ N [E } 70^\circ \text{ S]}$  and  $F_2 = 800 \text{ N [S } 50^\circ \text{ W]}$ .

(Assume 3 sfs)



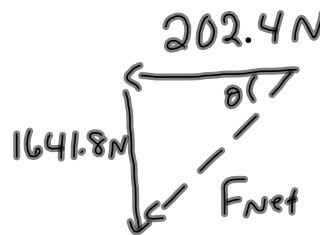
$$\begin{aligned} F_{1x} &= F_1 \cos \theta \\ &= 1200 \text{ N} \cos 70^\circ \\ &= 410.4 \text{ N [E]} \end{aligned}$$

$$\begin{aligned} F_{1y} &= F_1 \sin \theta \\ &= 1200 \text{ N} \sin 70^\circ \\ &= 1127.6 \text{ N [S]} \end{aligned}$$

$$\begin{aligned} F_{2x} &= 800 \text{ N} \cos 40^\circ \\ &= 612.8 \text{ N [W]} \end{aligned}$$

$$\begin{aligned} F_{2y} &= 800 \text{ N} \sin 40^\circ \\ &= 514.2 \text{ N [S]} \end{aligned}$$

$$\begin{aligned} \Sigma x &= 202.4 \text{ N [W]} \\ \Sigma y &= 1641.8 \text{ N [S]} \end{aligned}$$



$$\begin{aligned} F_{\text{net}}^2 &= (202.4 \text{ N})^2 + (1641.8 \text{ N})^2 \\ F_{\text{net}} &= 1650 \text{ N [W } 83^\circ \text{ S]} \end{aligned}$$

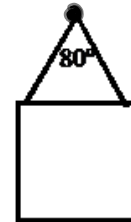
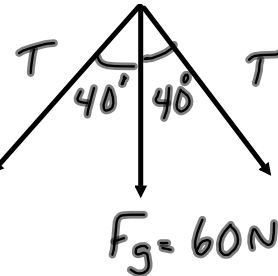
$$\tan \theta = \frac{1641.8 \text{ N}}{202.4 \text{ N}}$$

$$\theta = 83^\circ$$

4. The picture shown below weighs 60.0 N. The angle made by the string at the nail is  $80.0^\circ$ . Determine the tension in the strings.

\* Draw an FBD at the nail head

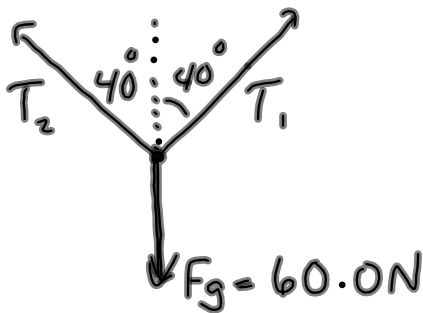
\* Each string supports half the weight (30N)



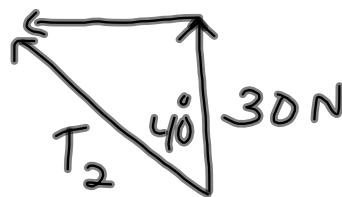
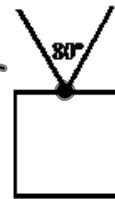
$$\cos 40^\circ = \frac{30\text{N}}{T}$$

$$T = \frac{30\text{N}}{\cos 40^\circ} = 39.2\text{N}$$

5. Find the tension in the string if the picture is hung as shown below. The weight is still 60.0 N and the angle between the strings is  $80.0^\circ$ .




Each string supports  $\frac{1}{2}$  the weight



$$\cos 40^\circ = \frac{30\text{N}}{T_2}$$

$$T_2 = \frac{30\text{N}}{\cos 40^\circ} = 39.2\text{N}$$

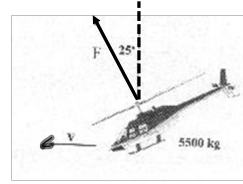
6. What is the force,  $F$ , provided by the rotor, if the helicopter shown is travelling in level flight?

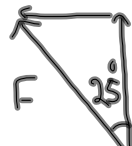


$$F_g = mg$$

$$= (5500 \text{ kg})(9.8 \text{ m/s}^2)$$

$$= 53900 \text{ N}$$



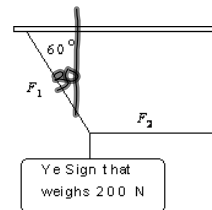
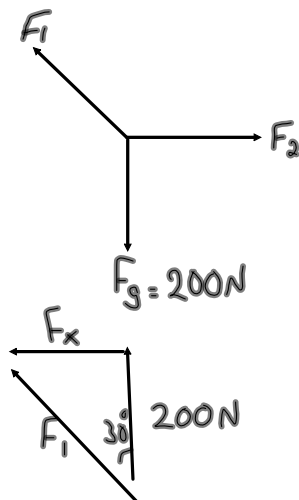


$$\cos 25^\circ = \frac{53900 \text{ N}}{F}$$

$$F = \frac{53900 \text{ N}}{\cos 25^\circ}$$

$$F = 59500 \text{ N}$$

7. A sign weighing 200.0 N is held in place by two wires as shown in the picture. One wire is horizontal and the other makes an angle of  $60.0^\circ$  with the top support. Find the tensional forces,  $F_1$  and  $F_2$  in the wires.



$F_1$  is the force responsible for supporting the weight of the sign.  $F_2$  just holds it in position.

$$\cos 30^\circ = \frac{200 \text{ N}}{F_1}$$

$$F_1 = \frac{200 \text{ N}}{\cos 30^\circ}$$

$$F_1 = 231 \text{ N}$$

\* Now find  $F_x$ .  $F_2$  is equal but in the opposite direction to  $F_x$

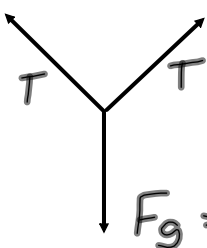
$$\tan 30^\circ = \frac{F_x}{200 \text{ N}}$$

$$F_x = 200 \text{ N} \tan 30^\circ \quad \text{so } F_2 = 115 \text{ N}$$

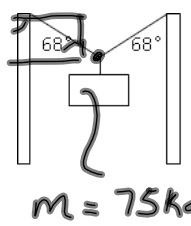
$$\approx 115 \text{ N}$$

8 A sign is held by 2 wires as shown. What is the tension in each wire?

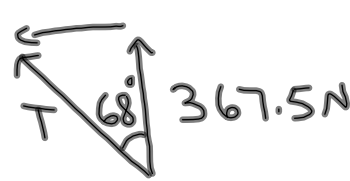
\* Each string supports  $\frac{1}{2} F_g$  (weight)



$F_g = mg$   
 $= (75 \text{ kg})(9.8 \text{ m/s}^2)$   
 $= 735 \text{ N}$



$m = 75 \text{ kg}$



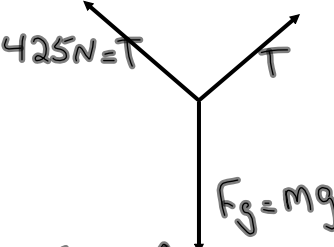
$\cos 68^\circ = \frac{367.5 \text{ N}}{T}$

$T = \frac{367.5 \text{ N}}{\cos 68^\circ}$

$T = 980 \text{ N}$

9 A sign hangs from 2 cables as shown. If the tension in each wire is 425 N, what is the mass of the sign?

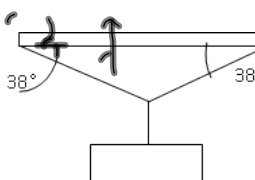
Each string supports  $\frac{1}{2} F_g$ .



$425 \text{ N} = T$

$F_g = mg$

$x$  ( $x = \frac{1}{2} F_g$ )

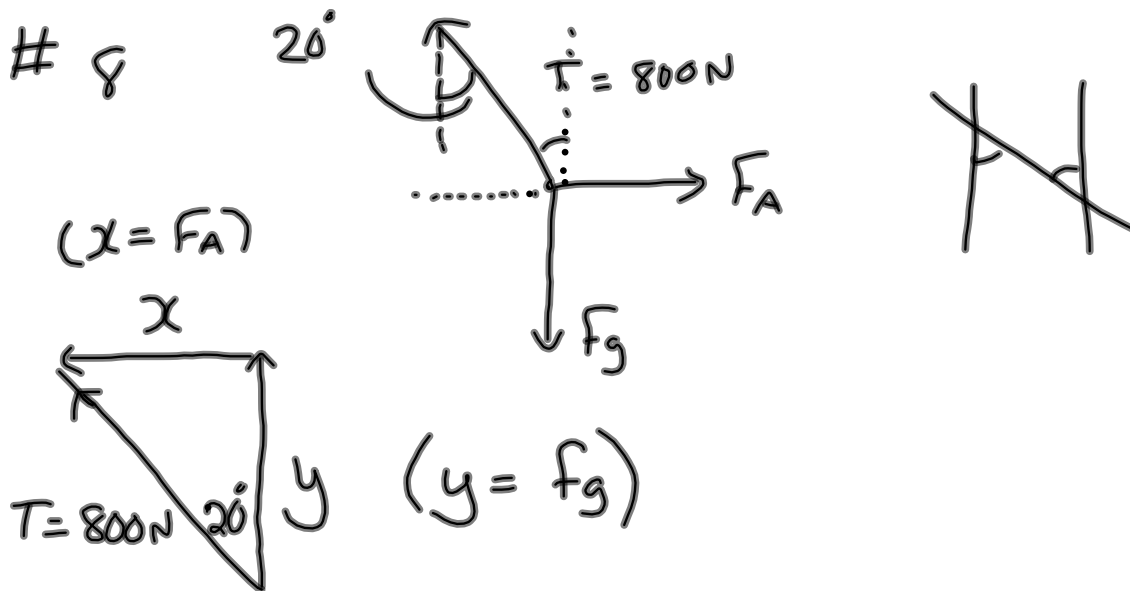


$\sin 38^\circ = \frac{x}{425 \text{ N}}$

$x = 261.7 \text{ N}$

$F_g = 2(261.7 \text{ N})$   
 $F_g = 523.4 \text{ N}$

$F_g = \frac{mg}{g}$   
 $\frac{523.4 \text{ N}}{9.8 \text{ m/s}^2} = m$   
 $53.4 \text{ kg}$



$$\cos 20^\circ = \frac{y}{800\text{ N}}$$

$$y = 751.8\text{ N}$$

$$\therefore F_g = 751.8\text{ N}$$

$$\sin 20^\circ = \frac{x}{800\text{ N}}$$

$$x = \underline{273.6\text{ N}}$$

$$F_A = 273.6\text{ N}$$