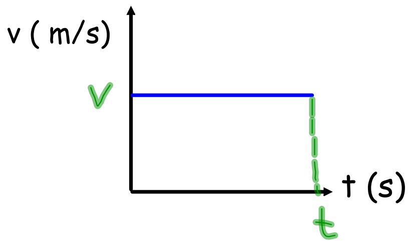
Section 2.2 Equations for Motion with Uniform Acceleration

Uniform Motion - means moving with a constant velocity. i.e. moving at a constant speed in a straight line.

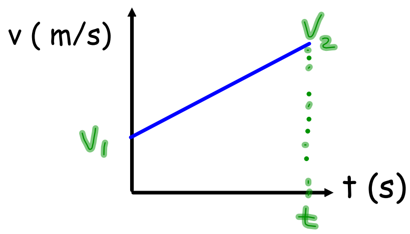
The formula, $\vec{v} = \frac{\vec{d}}{t}$ is actually derived from the velocity-time graph for uniform motion.



Recall: To find the displacement from a v-t graph, we need to find the area "under" the graph.

$\vec{d}$ = area
$\vec{d}$ = lw
$\vec{d}$ = vt
$v = \frac{\vec{d}}{t}$

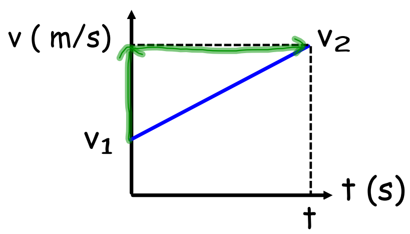
Non-Uniform Motion - means the object is accelerating; ie. changing speed and or direction.



Consider the v-t graph at the right. The graph indicates that the object is moving to the right at an increasing speed.

The formula, $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t}$,is actually derived from the above v-t graph for uniform acceleration.

To find the a from the v-t graph, we need to find the slope.



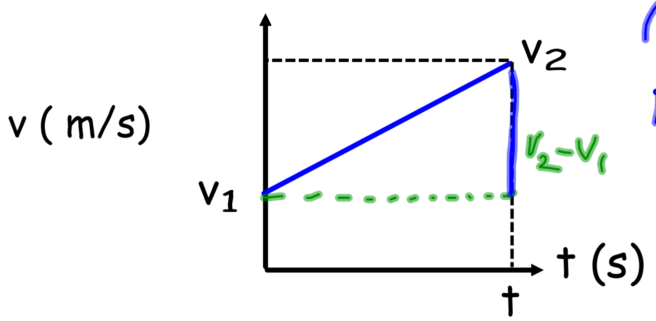
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{v_2 - v_1}{t}$$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t}$$

Similarly, we can use this graph for uniform acceleration to derive 4 formulas that can be used to calculate the displacement of any object.

1.



Recall: displacement is given by the area "under" the graph.

$$\text{Phy} \quad \vec{d} = A_{\square} + A_{\triangle}$$

$$\vec{d} = lw + \frac{1}{2}bh$$

$$\vec{d} = (t)(v_1) + \frac{1}{2}(t)(v_2 - v_1)$$

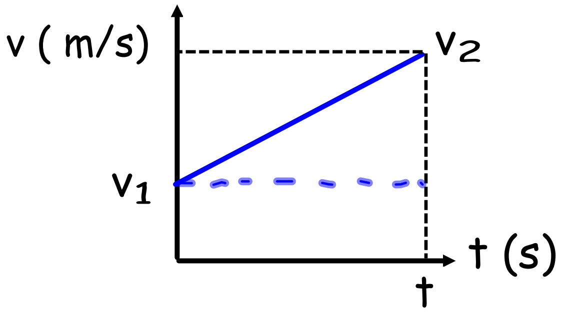
✓ but $a = \frac{v_2 - v_1}{t}$

$$\boxed{v_2 - v_1 = at}$$

$$\vec{d} = v_1 t + \frac{1}{2}t(at)$$

$$\vec{d} = v_1 t + \frac{1}{2}at^2$$

2.



$$\vec{d} = A_{\square} + A_{\triangle}$$

$$\vec{d} = lw + \frac{1}{2}bh$$

$$\vec{d} = (t)(v_1) + \frac{1}{2}(t)(v_2 - v_1)$$

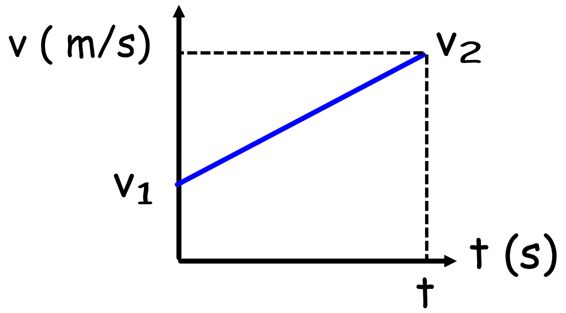
$$d = v_1 t + \frac{1}{2}v_2 t - \frac{1}{2}v_1 t$$

$$d = \frac{1}{2}v_1 t + \frac{1}{2}v_2 t$$

$$d = \frac{1}{2}t\,(v_1 + v_2)$$

$$d = \left(\frac{v_1 + v_2}{2}\right)t$$

3.



* From formula #2, the second last line states:

but $a = \dfrac{V_2 - V_1}{t}$

$\therefore t = \dfrac{V_2 - V_1}{a}$

$$d = \frac{1}{2}t(V_1 + V_2)$$

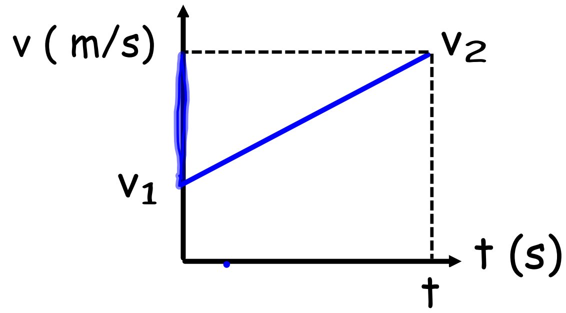
$$d = \frac{1}{2}\left(\frac{V_2 - V_1}{a}\right)(V_1 + V_2)$$

$$2ad = (V_2 - V_1)(V_1 + V_2)$$

$$2ad = \cancel{V_1V_2} + V_2^2 - V_1^2 - \cancel{V_1V_2}$$

$$2ad = V_2^2 - V_1^2$$

4.



$\vec{d} = A_{\text{large}\,\square} - A_{\triangle \text{ on top}}$

$d = lw - \frac{1}{2}bh$

$d = v_2 t - \frac{1}{2}t(v_2 - v_1)$

but $a = \frac{v_2 - v_1}{t}$

$v_2 - v_1 = at$

$d = v_2 t - \frac{1}{2}t(at)$

$d = v_2 t - \frac{1}{2}at^2$

Summary of Kinematics Formulae for Uniformly Accelerated Motion

Formulae	Variables
$a = \frac{v_2 - v_1}{t}$	a, v_1, v_2, t
$d = v_1 t + \frac{1}{2} a t^2$	a, v_1, d, t
$d = \left(\frac{v_1 + v_2}{2}\right) t$	d, v_1, v_2, t
$2ad = {v_2}^2 - {v_1}^2$	a, d, v_1, v_2
$d = v_2 t - \frac{1}{2} a t^2$	a, d, v_2, t

Note: Each formula uses 4 different variables. To determine which formula to use, identify the givens and what you want to find. As well, we must remember that these formulae are vector formulae. Hence, directions must be used.

Rearrangements:

1. $d = \underline{v_1 t} + \underbrace{\frac{1}{2}at^2}$: v_1 & a

Solve for v_1:

$$\frac{d - \frac{1}{2}at^2}{t} = \frac{v_1 t}{t}$$

$$\frac{d - \frac{1}{2}at^2}{t} = v_1$$