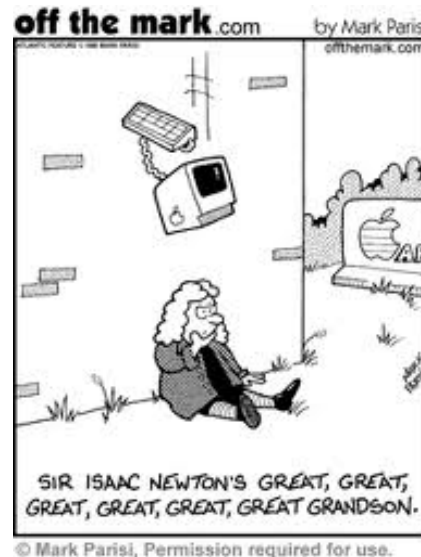


## Section 2.17: Newton's Law of Universal Gravitation



Gravity is an attractive force that exists between all objects that have mass. It is the force that keeps us stuck to the earth and the moon orbiting the earth. Gravity is a very weak force and it is only noticeable when the masses are very large.

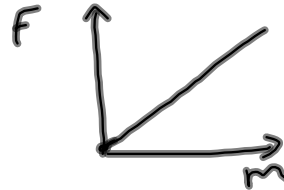
Gravity is a force that acts at a distance. Typical forces act through contact – your hand pushes a cart or pulls a wagon and so on. But gravity acts without contact: the Earth exerts a force on a falling apple and on the moon even though there is no contact, and the two objects may even be very far apart.

Newton reasoned that gravitational attraction exists between all objects in the universe and the strength of that attraction depends on the masses of the objects and the distance between them.

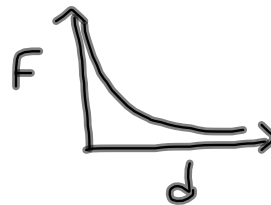
## Newton's Law of Universal Gravitation

Every object in the universe attracts every other particle with a **force** that is **directly proportional** to the **product** of their **masses** and **inversely proportional** to the **square of the distance between their centers**. This force acts along a line joining their centers.

Proportion  $F \propto \frac{m_1 m_2}{r^2}$



Equation  $F = G \frac{m_1 m_2}{r^2}$



where:

F is the gravitational force (ie. force of gravity) in Newtons (N)

$m_1$  and  $m_2$  are the masses in kilograms (kg)

r is the distance between their centers in meters (m)

G is the universal gravitational constant =  $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

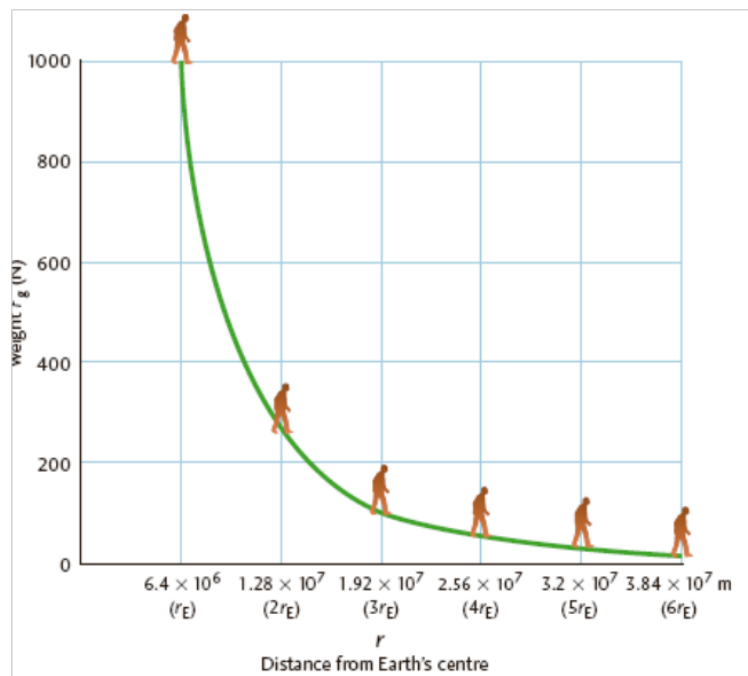
There are several important points here:

- First, there are two equal but opposite force present. The Earth pulls down on the on the moon and the moon pulls up on the Earth with an equal force. At the Earth's surface, the Earth pulls down on a 1 kg mass with a force of 9.8 N and the 1kg mass pulls up on the Earth with a force of 9.8 N.

- Second, the forces of attraction are governed by an **inverse square law**.  $F \propto \frac{1}{r^2}$

If an object is moved 5 times as far from the **center of the Earth as the Earth's surface**, the force of gravity is reduced to  $(1/5)^2$  or  $1/25$  of what it was at the Earth's surface.

In other words, if your weight on the surface of the Earth is 650 N and you move 4 radii above the surface of the Earth (note your distance from the center of the Earth increases by a factor of 5) your weight will be 26 N.



The force of gravity between two objects decreases rapidly as the object's move apart. However, there is no value of  $r$ , no matter how great that would reduce the force of attraction to zero. Every object in the universe exerts a force on all other objects, near or far.

- Third, the value of the constant,  $G$ , is small. The force of gravity is significant only if one or both of the objects is massive. It takes the Earth, which has a mass of  $5.98 \times 10^{24}$  kg to exert a 9.8 N force on a 1 kg object.

Proportionality Exercises *MC*

- 1 The force of gravity between two objects is  $1.0 \times 10^{-4}$  N. What will be the new force if one of the masses is doubled?

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F_{\text{new}} = 2 \times 1.0 \times 10^{-4} \text{ N}$$

$$= 2.0 \times 10^{-4} \text{ N}$$

$$F \propto \frac{(2)(1)}{(1)^2}$$

$$F \propto 2$$

- 2 By what factor will the gravitational force between two objects change if the mass of one object is doubled and the other mass is tripled?

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\propto \frac{(2)(3)}{(1)^2}$$

$$\propto 6$$

$\therefore F$  is 6 times greater.

- 3 By what factor will the gravitational force between two objects change if the distance between their centers increases by a factor of 3?

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\propto \frac{(1)(1)}{(3)^2}$$

$$\propto \frac{1}{9}$$

Force decreases by a factor of 9.

- 4 By what factor will the gravitational force between two objects change if the distance between their centers is ~~decreased by a factor of 4?~~

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\propto \frac{(1)(1)}{(1/4)^2}$$

$$\propto \frac{1}{1/16}$$

$$\propto 16$$

Force is 16 times greater.

- 5 If the masses of 2 objects doubles while the distance between their centers also doubles, by what factor will the gravitational force between them change?

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F \propto \frac{(2)(2)}{(2)^2}$$

$$F \propto 1$$

$\therefore$  Force stays the same.

- 6 By what factor will the force of gravity between two objects change if the mass of one object decreases by a factor of 3, the mass of the other object doubles and the distance between their centers decreases by a factor of 4?

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F \propto \frac{(\frac{1}{3})(2)}{(\frac{1}{4})^2}$$

$$F \propto \frac{2/3}{1/16}$$

$$F \propto \frac{2}{3} \times 16$$

$$\propto \frac{32}{3}$$

$\therefore$  Force is  $\approx$  10.7 times larger.

### Calculations Involving Newton's Law of Universal Gravitation

- 1 <sup>ms</sup> What is the force of attraction between 2 apples, each with a mass of  $5.0 \times 10^2$  g, held so that their centers are 15 cm apart?

$$F = ?$$

$$m_1 = 500 \text{ g}$$

$$= 0.50 \text{ kg}$$

$$m_2 = 0.50 \text{ kg}$$

$$r = 15 \text{ cm}$$

$$= 0.15 \text{ m}$$

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{(6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2})(0.50 \text{ kg})(0.50 \text{ kg})}{(0.15 \text{ m})^2}$$

$$F = 7.4 \times 10^{-10} \text{ N}$$

- 2 Tom has a mass of 85 kg and Sally has a mass of 55 kg. If the gravitational force of attraction between Sally and Tom on the dance floor is  $7.84 \times 10^{-7}$  N, how far apart are they?

$$\begin{aligned}
 m_1 &= 85 \text{ kg} \\
 m_2 &= 55 \text{ kg} \\
 F &= 7.84 \times 10^{-7} \text{ N} \\
 r &= ?
 \end{aligned}$$

$$F = \frac{G m_1 m_2}{r^2}$$

$$r^2 = \frac{G m_1 m_2}{F}$$

$$r = \sqrt{\frac{G m_1 m_2}{F}}$$

$$r = \sqrt{\frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(85 \text{ kg})(55 \text{ kg})}{7.84 \times 10^{-7} \text{ N}}}$$

$$r = \sqrt{0.3977 \text{ m}^2}$$

$$r = 0.63 \text{ m}$$

- 3 Two objects experience a force of attraction of  $3.4 \times 10^{-8}$  N when they are placed 850 cm apart. If one mass is 4 times greater than the other, find each mass.

$$F = 3.4 \times 10^{-8} \text{ N}$$

$$r = 850 \text{ cm} = 8.5 \text{ m}$$

$$m_1 = x$$

$$m_2 = 4x$$

$$\frac{F}{1} = \frac{G m_1 m_2}{r^2}$$

$$\frac{G m_1 m_2}{G} = \frac{F r^2}{G}$$

$$m_1 m_2 = \frac{F r^2}{G}$$

$$(x)(4x) = \frac{(3.4 \times 10^{-8} \text{ N})(8.5 \text{ m})^2}{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2}$$

$$4x^2 = \frac{36829.1 \text{ kg}^2}{4}$$

$$x^2 = 9207.3 \text{ kg}^2$$

$$x = 96 \text{ kg}$$

$$\therefore m_1 = 96 \text{ kg}$$

$$m_2 = 4 \times 96 \text{ kg} = 384 \text{ kg} = 380 \text{ kg}$$

$F_g$

- 4 Use two different methods to find the weight of a person who has a mass of 75 kg. Mass of the Earth is  $5.98 \times 10^{24}$  kg and the radius of the Earth is 6380 km.

$$F_g = (75 \text{ kg})(9.8 \text{ m/s}^2) = 735 \text{ N}$$

need  $m_1 = 75 \text{ kg}$   
 $m_2 = m_e = 5.98 \times 10^{24} \text{ kg}$   
 $r = r_e = 6380 \text{ km} = 6380 \times 10^3 \text{ m} = 6.38 \times 10^6 \text{ m}$

$$F = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(75 \text{ kg})}{(6.38 \times 10^6 \text{ m})^2}$$

$$F = 735 \text{ N}$$

From example number 4, you should be able to see that you can find a person's weight at the surface of the Earth by using

$$F = mg \quad \text{or} \quad F = G \frac{m_1 m_2}{r^2}$$

This means  $mg = \frac{G m_1 m_2}{r^2}$

Let's say the  $m_1$  is your mass and  $m_2$  is the mass of Earth. The radius of the Earth is  $r$ . The above expression will become

$$m_1 g = G \frac{m_1 m_{\text{earth}}}{r_{\text{earth}}^2}$$

$$g = G \frac{m_{\text{earth}}}{r_{\text{earth}}^2}$$

More generally, the gravitational field strength of any planet can be found using the formula

$$F = \frac{G m_1 m_2}{r^2}$$

$$g = G \frac{m_{\text{planet}}}{r_{\text{planet}}^2}$$

Example: Find the gravitational field strength of Earth.

$$m = 5.98 \times 10^{24} \text{ kg} \quad r = 6.38 \times 10^6 \text{ m}$$

$$g = \frac{G m_{\text{planet}}}{r_{\text{planet}}^2} = \frac{(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}) (5.98 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2}$$

$$= 9.8 \text{ N/kg}$$

$$= 9.8 \text{ m/s}^2$$

Example: Find the gravitational field strength of Mars.

$$m = 6.37 \times 10^{23} \text{ kg} \quad r = 3.43 \times 10^6 \text{ m}$$