

Section 17: Newton's Law of Universal Gravitation

Besides developing the three laws of motion, Sir Isaac Newton also examined the motion of planets and the moon. In particular, he wondered about the nature of the force that must act to keep the moon in its nearly circular orbit around the Earth.

Newton was also thinking about the problem of gravity. Since falling bodies accelerate, Newton had concluded that they must have a force exerted on them, a force we call the force of gravity. Whenever a body has a force exerted on it, that force is exerted by some other body. But what exerts the force of gravity? Every object on the surface of the Earth feels the force of gravity and no matter where the object is, the force is directed toward the center of the Earth. Newton concluded that it must be the Earth itself that exerts the gravitational force on objects at its surface.



According to an early account, Newton was sitting in his garden and noticed an apple drop from a tree. He is said to have been struck by a sudden inspiration: if gravity acts at the top of trees and even at the top of mountains, then perhaps it acts all the way to the moon. With this idea that it is terrestrial gravity that holds the moon in its orbit, Newton developed his theory of Universal Gravitation.

Many great thinkers had trouble accepting the idea of a force acting at a distance. Typical forces act through contact – your hand pushes a cart or pulls a wagon and so on. But gravity acts without contact, said Newton: the Earth exerts a force on a falling apple and on the moon even though there is no contact, and the two objects may even be very far apart.

Newton reasoned that gravitational attraction exists between all objects in the universe and the strength of that attraction depends on the masses of the objects and the distance between them.

Newton's Law of Universal Gravitation states:

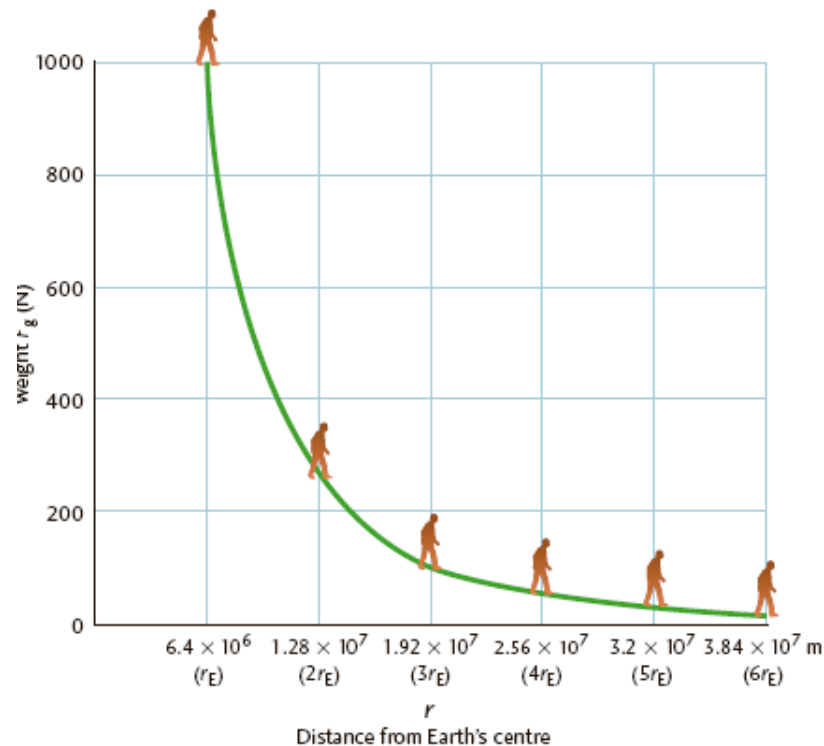
Every object in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. This force acts along a line joining their centers.

Proportion	$F \propto \frac{m_1 m_2}{r^2}$
Equation	$F = G \frac{m_1 m_2}{r^2}$

Where F is the gravitational force (ie. force of gravity) in Newtons (N)
 m_1 and m_2 are the masses in kilograms (kg)
 r is the distance between their centers in meters (m)
 G is the universal gravitational constant = $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

There are several important points here.

- First, there are two equal but opposite forces present. The Earth pulls down on the moon and the moon pulls up on the Earth with an equal force. At the Earth's surface, the Earth pulls down on a 1 kg mass with a force of 9.8 N and the 1 kg mass pulls up on the Earth with a force of 9.8 N.
- Second, the forces of attraction are governed by an **inverse square law**. ($F \propto \frac{1}{r^2}$)
If an object is moved 5 times as far from the **center of the Earth as the Earth's surface**, the force of gravity is reduced to $(\frac{1}{5})^2$ or 1/25 of what it was at the Earth's surface. In other words, if your weight on the surface of the Earth is 650 N and you move 4 radii above the surface of the Earth (note your distance from the center of the Earth increases by a factor of 5) your weight will be 26 N.



The force of gravity between two objects decreases rapidly as the object's move apart. However, there is no value of r, no matter how great that would reduce the force of attraction to zero. Every object in the universe exerts a force on all other objects, near or far.

- Third, the value of the constant, G, is small. The force of gravity is significant only if one or both of the objects is massive. It takes the Earth, which has a mass of 5.98×10^{24} kg, to exert a 9.8 N force on a 1 kg object.

Proportionality Exercises

- 1 The force of gravity between two objects is 1.0×10^{-4} N. What will be the new force if one of the masses is doubled?

- 2 By what factor will the gravitational force between two objects change if the mass of one object is doubled and the other mass is tripled?

- 3 By what factor will the gravitational force between two objects change if the distance between their centers increases by a factor of 3?

- 4 By what factor will the gravitational force between two objects change if the distance between their centers is decreased by a factor of 4?

- 5 If the masses of 2 objects doubles while the distance between their centers also doubles, by what factor will the gravitational force between them change?

- 6 By what factor will the force of gravity between two objects change if the mass of one object decreases by a factor of 3, the mass of the other object doubles and the distance between their centers decreases by a factor of 4?

Calculations Involving Newton's Law of Universal Gravitation

- 1 What is the force of attraction between 2 apples, each with a mass of 5.0×10^2 g, held so that their centers are 15 cm apart?

- 2 Tom has a mass of 85 kg and Sally has a mass of 55 kg. If the gravitational force of attraction between Sally and Tom on the dance floor is 7.84×10^{-7} N, how far apart are they?
- 3 Two objects experience a force of attraction of 3.4×10^{-8} N when they are placed 850 cm apart. If one mass is 4 times greater than the other, find each mass.
- 4 Use two different methods to find the weight of a person who has a mass of 75 kg. Mass of the Earth is 5.98×10^{24} kg and the radius of the Earth is 6380 km.

From example number 4, you should be able to see that you can find a person's weight at the surface of the Earth by using $F = mg$ or $F = G \frac{m_1 m_2}{r^2}$.

This means $mg = \frac{m_1 m_2}{r^2}$

Let's say the m_1 is your mass and m_2 is the mass of Earth. The radius of the Earth is r . The above expression will become

$$m_1 g = G \frac{m_1 m_{\text{earth}}}{r_{\text{earth}}^2}$$

$$g = G \frac{m_{\text{earth}}}{r_{\text{earth}}^2}$$

More generally, the gravitational field strength of any planet can be found using the formula

$$g = G \frac{m_{\text{planet}}}{r_{\text{planet}}^2}$$

Example: Find the gravitational field strength of Earth.

$$m = 5.98 \times 10^{24} \text{ kg} \qquad r = 6.38 \times 10^6 \text{ m}$$

Example: Find the gravitational field strength of Mars.

$$m = 6.37 \times 10^{23} \text{ kg} \qquad r = 3.43 \times 10^6 \text{ m}$$