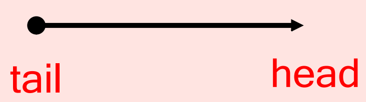
Section 1.3: Adding and Subtracting Linear and Perpendicular Vectors

Motion in two dimensions must use vectors and vector diagrams.

Vector Representation:



magnitude (size): given by the length of the straight line on some given scale

direction: given by arrowhead

tail: starting point

head: ending point

Direction is determined by reference to points on a compass. They are indicated by saying which direction the **resultant** is drawn with respect to the compass.

Vector Addition

One Dimension: If vectors are acting along the same line of action [E or W] or [N or S], the vector sum is equal to the algebraic sum (with the proper use of sign convention).

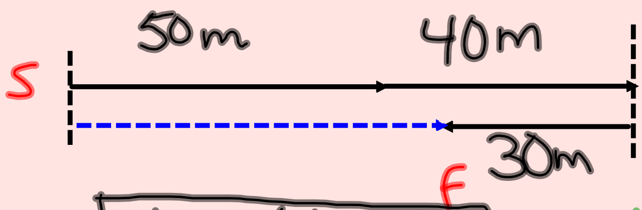
Example: Find the sum of vectors A, B and C graphically and algebraically. A = 50 m [E], B = 30 m [W], C = 40 m [E]

Algebraically:

$\vec{d}_T = 50m[E] + 30m[W] + 40m[E]$

$= 60m[E]$ — Algebraic Sum

Scale: 1cm = 10m



$\boxed{\vec{d}_T = 60m[E]}$ — Vector Sum

<u>Two Dimensions</u>: When vectors are not acting along the same line of action, the vector sum **<u>is not</u>** equal to the algebraic sum.

Let's say we have 2 vectors: one is 4 m long and the other is 3 m long.

What is the maximum length of the resultant?

Pull

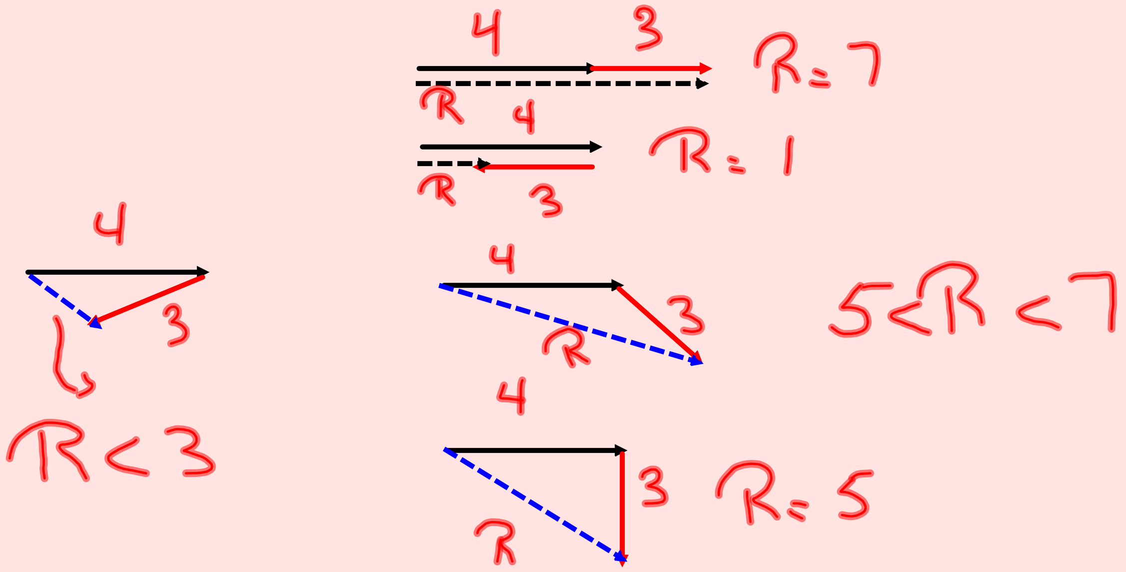
The maximum value for the resultant is 7 m if they are acting in the same direction.

What is the minimum length of the resultant?

Pull

The minimum value for the resultant is 1 m if they are acting in the opposite direction.

If these vectors have an angle between them, then any values between 1 m and 7 m are possible.



There are 2 ways to add perpendicular vectors:

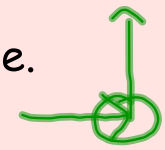
A. Graphically using a Scale Diagram

B. Algebraically using Pythagorean Theorem and Right Triangle Trigonometry

A. Scale Diagram

Step1: Pick a scale and a starting point.

Step 2: Draw the 1st vector using the scale.



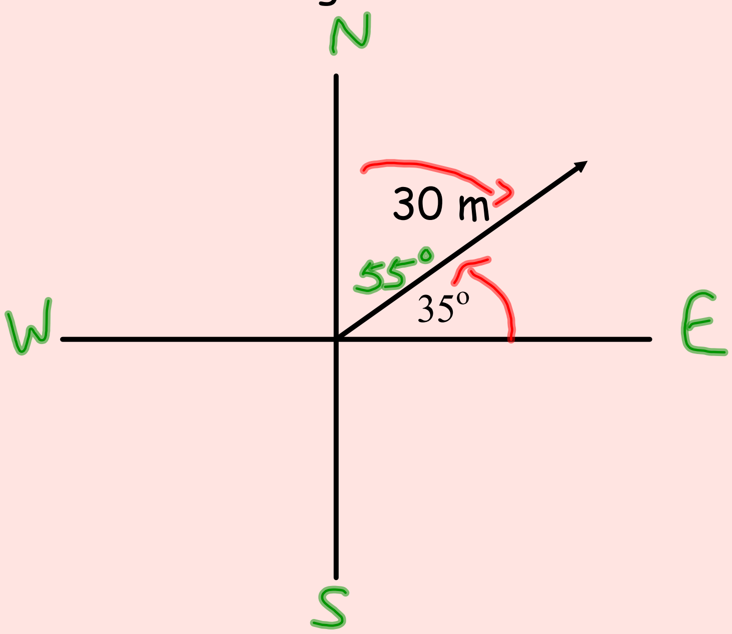
Step 3: Draw the remaining vectors head-to-tail, that is the tail of the 2nd vector joins to the head of the 1st, and so on.

Step 4: Draw the **resultant vector** from the tail of the 1st to the tip of the last (i.e. from the starting point to the ending point). The direction of the resultant vector is toward the head of the last vector (i.e. its direction points toward the final position).

Step 5: Measure the resultant vector with a ruler and convert back using scale. Measure the angle with a protractor and indicate direction appropriately.

Determining the Direction of Vectors

Consider the following vector:



Write the above vector in as many different ways as possible.

a) 30m [E 35° N]

c) 30m [35° N E]

North of the East Direction

b) 30m [N 55° E]

d) 30m [55° E N]

For each of the vectors above, state its opposite.

a) 30m [W 35° S]

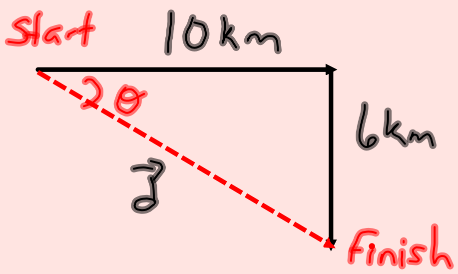
c) 30m [35° S W]

b) 30m [S 55° W]

d) 30m [55° W S]

Example: While on a moose hunting trip, a hunter travels 10.0 km [E] and then turns and goes a further 6.0 km [S]. Determine his displacement.

1cm = 2km



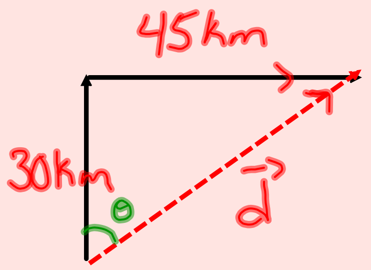
$\vec{d} = 5.8\,cm \times \frac{2\,km}{cm}$

$\vec{d} = 11.6\,km$

$\vec{d} = 12\,km$ [E 30° S]

What is the total displacement of a trip in which you travel 30.0 km North and then 45.0 km East?

1cm = 10 km



$\vec{d} = 5.5\,cm \times 10\frac{km}{cm}$

$= 55\,km$ [N 56° E]

B: Pythagorean Theorem and Right Triangle Trigonometry

- Used to add perpendicular vectors.

Pythagorean Theorem: $c^2 = a^2 + b^2$

Right Triangle Trigonometry: $\sin\theta = \text{opp}/\text{hyp}$

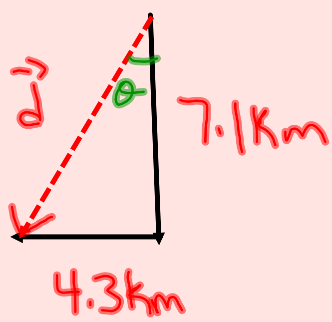
$\cos\theta = \text{adj}/\text{hyp}$

$\tan\theta = \text{opp}/\text{adj}$

1. (a) What is the total displacement of a trip of 4.8 km [N], 4.3 km [W] and 11.9 km [S]?

$\vec{d}_1 = 11.9\text{ km [S]} + 4.8\text{ km [N]} = 7.1\text{ km [S]}$

$\vec{d}_2 = 4.3\text{ km [W]}$



✓ $\vec{d}^{\,2} = (7.1\text{ km})^2 + (4.3\text{ km})^2$

$d^2 = 50.41\text{ km}^2 + 18.49\text{ km}^2$

$d^2 = 68.9\text{ km}^2$

✓ $d = 8.3\text{ km [S } 31^\circ \text{ W]}$

$\tan\theta = \dfrac{4.3\text{ km}}{7.1\text{ km}}$

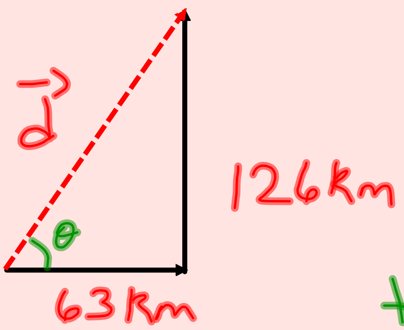
$\theta = 31^\circ$

$\theta = \tan^{-1}\left(\dfrac{4.3\text{ km}}{7.1\text{ km}}\right)$

$\theta = 31^\circ$

(b) In which direction should he travel to get back to where he started? [N 31° E]

2. (a) What is the total displacement of a trip in which a person travels 63 km [E] and 126 km [N]?



$$\vec{d}^{\,2} = (63\,\text{km})^2 + (126\,\text{km})^2$$

$$\vec{d} = 140.9\,\text{km}$$

$$\vec{d} = 140\,\text{km}\ [E\ 63^\circ N]$$

$$\tan\theta = \frac{126\,\text{km}}{63\,\text{km}} \qquad \theta = \tan^{-1}\left(\frac{126\,\text{km}}{63\,\text{km}}\right)$$

$$\theta = 63^\circ \qquad = 63^\circ$$

(b) In which direction should he travel to get back to where he started?

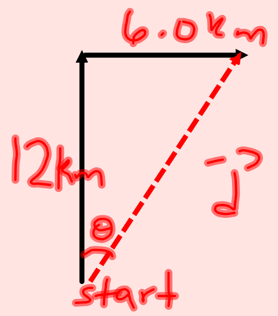
[W 63° S]

Example: A hiker first hikes 10.0 km [N], then 6.0 km [E] and finally 2.0 km [N] in a time of 4.0 h. Determine distance, displacement, average speed and average velocity.

a) $d_T = 18.0\,km$

b) $\vec{d}_1 = 12\,km\,[N]$

$\vec{d}_2 = 6.0\,km\,[E]$



$\vec{d}^{\,2} = (6.0\,km)^2 + (12\,km)^2$

$\vec{d} = 13\,km\,[N\,27^\circ\,E]$

or

$[27^\circ\,E\,N]$

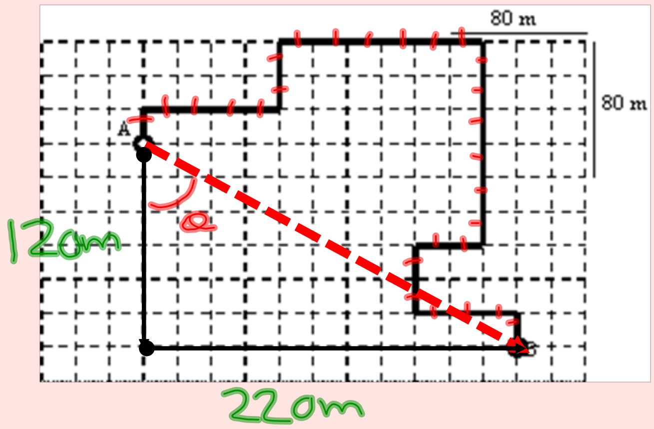
$\theta = \tan^{-1}\left(\frac{6.0\,km}{12.0\,km}\right)$

$\theta = 27^\circ$

c) $V_{AV} = \frac{d_T}{t_T} = \frac{18\,km}{4.0\,h} = 4.5\,km/h$

d) $\vec{V}_{AV} = \frac{\vec{d}_T}{t_T} = \frac{13\,km\,[N\,27^\circ\,E]}{4.0\,h} = 3.3\,km/h\,[N\,27^\circ\,E]$

Example: The grid below shows the path taken by Jim on his way to school. Each square on the grid is 20.0 m long.



a. Calculate Jim's final displacement from his starting position.

$\vec{d}_1 = 6 \times 20m = 120m\ [S]$

$\vec{d}_2 = 11 \times 20m = 220m\ [E]$

$\vec{d}^2 = (120m)^2 + (220m)^2$

$d = 251km\ [S61°E]$

or $[61°ES]$

$\tan\theta = \dfrac{220m}{120m}$

$\theta = 61°$

b. The entire trip took 5.0 minutes. Calculate the average speed in m/s.

$t = 5min$
$= 300s$

$d = 27 \times 20m = 540m$

$V_{AV} = \dfrac{d}{t} = \dfrac{540m}{300s} = 1.8\ m/s$

c. Calculate the average velocity in m/s.

$\vec{V}_{AV} = \dfrac{\vec{d}_T}{t_T} = \dfrac{251km\ [S61°E]}{300s} = 0.83\ m/s\ [S61°E]$