Section 2.6: Finding the x-intercepts of a Quadratic Functions By Factoring

III. Factoring Trinomials of the Form $x^2 + bx + c$

To determine the factors of a trinomial of the form $x^2 + bx + c$, first determine two numbers whose sum is b and whose product is c. These numbers are the constant terms in two binomial factors, each of which has x as its first term.

Example: Factor:

A) $x^2+7x+10$ $2 \times 5 = 10$, $2 + 5 = 7$

$(x+2)(x+5)$

B) $x^2 + 3x - 10$ $-2 \times 5 = -10$, $-2 + 5 = 3$

$(x-2)(x+5)$

10 → 1, 10 → 2, 5

Example: Solve the following Quadratics Equations

a. $x^2 + 12x + 20 = 0$

$2 \times 10 = 20$, $2 + 10 = 12$

20 → 1, 20 → 2, 10 → 4, 5

$(x+2)(x+10) = 0$

$x+2=0$ $x+10=0$

$x=-2$ $x=-10$

b. $x^2 - 2x - 8 = 0$

$2 \times -4 = -8$, $2 + -4 = -2$

-8 → 1, 8 → 2, -4

$(x+2)(x-4) = 0$

$x+2=0$ $x-4=0$

$x=-2$ $x=4$

c. $a^2 + 7a - 18 = 0$

$-2 \times 9 = -18$, $-2 + 9 = 7$

18 → 1, 18 → -2, 9 → 3, 6

$(a-2)(a+9) = 0$

$a-2=0$ $a+9=0$

$a=2$ $a=-9$

d. $x^2+15x+14 = 0$

$1 \times 14 = 14$, $1 + 14 = 15$

14 → 1, 14 → 2, 7

$(x+1)(x+14) = 0$

$x+1=0$ $x+14=0$

$x=-1$ $x=-14$

e. $x^2+8x-48 = 0$

$-4 \times 12 = -48$, $-4 + 12 = 8$

-48 → 1, 48 → 2, 24 → 3, 16 → 4, 12 → 6, 8

$(x-4)(x+12) = 0$

$x-4=0$ $x+12=0$

$x=4$ $x=-12$

f. $x^2 - x - 72 = 0$

$8 \times -9 = -72$, $8 + -9 = -1$

-72 → 1, 72 → 2, 36 → 3, 24 → 4, 18 → 6, 12 → 8, -9

$(x+8)(x-9) = 0$

$x+8=0$

$x=-8$

$x-9=0$

$x=9$

Example: Solve $x^2 + bx + c$ (should be +)

A) $30 = 7m + m^2$

$0 = m^2 + 7m - 30$

$-3 \times 10 = -30$, $-3 + 10 = 7$

-30: $1\ 30$, $2\ 15$, $3\ 10$, $5\ 6$

$0 = (m-3)(m+10)$

$m-3=0$ $m+10=0$

$m=3$ $m=-10$

B) $-5d + d^2 = -6$

$d^2 - 5d + 6 = 0$

$-2 \times -3 = 6$, $-2 + -3 = -5$

6: $1\ 6$, $2\ 3$

$(d-2)(d-3)=0$

$d-2=0$ $d-3=0$

$d=2$ $d=3$

Example:

A) $-5h^2 - 20h = -60$

$-5h^2 - 20h + 60 = 0$

$5h^2 + 20h - 60 = 0$

$\div 5 \quad \div 5 \quad \div 5$

*Take out GCF = 5

$5(h^2 + 4h - 12) = 0$

$+6 \times -2 = -12$, $6 + -2 = 4$

$5(h+6)(h-2)=0$

$h+6=0$ $h-2=0$

$h=-6$ $h=2$

B) $-4t^2 - 16t = -128$

$-4t^2 - 16t + 128 = 0$

$4t^2 + 16t - 128 = 0$ GCF = 4

$\div 4 \quad \div 4 \quad \div 4$

$4(t^2 + 4t - 32) = 0$

$-4 \times 8 = -32$, $-4 + 8 = 4$

-32: $1\ 32$, $2\ 16$, $4\ 8$

$4(t-4)(t+8)=0$

$t-4=0$ $t+8=0$

$t=4$ $t=-8$

C) $a^2 = 48 - 2a$

$a^2 + 2a - 48 = 0$

$-6 \times 8 = -48$, $-6 + 8 = 2$

48: $1\ 48$, $2\ 24$, $3\ 16$, $4\ 12$, $6\ 8$

$(a-6)(a+8)=0$

$a-6=0$ $a+8=0$

$a=6$ $a=-8$