Section 2.4 Maximum and Minimum Problems (Not Given the Function)

Type Two Max and Min Problems

the answer when you multiply 2 numbers together.

add

1. Find two numbers whose sum is 84 and whose product is a maximum.
(To do this we need to set up a quadratic function and find the maximum of the function - ie. the vertex).

Step 1: Create a Table

L1 First Number (x)	1	2	3	4
Second Number	83 (84-1=83)	82 (84-2=82)	81 (84-3=81)	80 (84-4=80)
L2 Product (y) (1st x 2nd)	83	164	243	320

Step 2: Determine what type of relationship is between the first number (x) and the Product (y). Find Differences

D1= 81, 79, 77

D2= −2 −2

Since D 2 is constant...relationship is quadratic.

Step 3: Use TI-83 to determine the equation for the curve of best fit (the parabola).
[Enter x in L1 and y in L2]. Perform a quadratic regression.

$y = ax^2 + bx + c$

$a = -1$ $c = 0$

$b = 84$

The equation of the curve of best fit is:

$y = -1x^2 + 84x$

$y = -x^2 + 84x$

Step 4: Use your curve of best fit to find the vertex of the parabola.

$x = \frac{-b}{2a} = \frac{-84}{2(-1)} = \frac{-84}{-2} = 42$

Therefore y = $-x^2 + 84x$

$y = -(42)^2 + 84(42)$

$y = -1764 + 3528$

$y = 1764$

The vertex is: (42 , 1764)

when it occurs → max/min value

Step 5: Interpret your vertex.
Complete the statement:

The maximum product is 1764 (y -value of vertex) and one of the numbers is 42 (x-coordinate of vertex).

How do I find the other number? The sum of the 2 numbers is 84. So, if one number is 42, the other number is 84-42=42.

Step 6: State your answer.

The two numbers are 42 & 42 & the maximum product is 1764.

difference (subtract)

2. Two numbers differ by 30. Set up a quadratic function and determine the two numbers if their product is a minimum.

Step 1: Create a Table

First No (x)	1	2	3	4
Second Number	31 (30+1=31) (31−1=30)	32	33	34
Product (y) 1st × 2nd	31	64	99	136

Step 2: Determine what type of relationship is between the first number (x) and the Product (y).

Sequence for Minimum product:

{31, 64, 99, 136,..} Is this a quadratic relationship? Show!

D1= 33, 35, 37

D2= 2 2 – Since D_2 is constant, it is quadratic.

Step 3: Use TI-83 to determine the equation for the curve of best fit (the parabola).
[Enter *x* in L1 and *y* in L2]. Perform a quadratic regression.

$a = 1$, $b = 30$

Therefore, the equation of the curve of best fit is: y = $1x^2 + 30x$

$y = x^2 + 30x$

Step 4: Use your curve of best fit to find the vertex of the parabola.

$$x = \frac{-b}{2a} = \frac{-30}{2(1)} = \frac{-30}{2} = -15$$

$$y = x^2 + 30x$$
$$y = (-15)^2 + 30(-15)$$
$$y = 225 - 450$$
$$y = -225$$

$$(-15, \underline{-225})\ \text{min}$$

Step 5: Interpret your vertex. **Step 6: State your answer.**

Complete the statement:

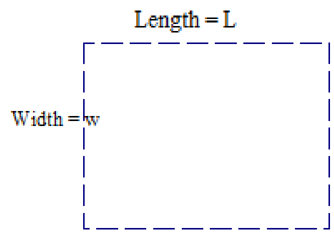
The minimum product is $\underline{-225}$ and it occurs at x = $\underline{-15}$.
Therefore, one number is $\underline{-15}$ and the other number is $\underline{15}$.

$$(-15 + 30 = 15)$$

3. Zach has 150 m of fencing to fence a rectangular region. Set up a quadratic function to determine the dimensions that will yield the largest possible area.

Recall: Area $= l \times w$

Perimeter $= 2l + 2w$



150 m does 4 sides, 2 lengths & 2 widths.

$150\text{m} \div 2 = \boxed{75\text{m}}$

So 75m will do 1 length & 1 width.

$\boxed{l + w = 75}$ → use to complete the table.

Step 1: Create a Table

L_1	Length (x)	1 ~~(twice)~~	2	3	4	5
	Width	74 (75-1=74)	73 (75-2=73)	72 (75-3=72)	71 (75-4=71	70 (75-5=70)
L_2	Area (A) ($l \times w$) m^2	74	146	216	284	350

Step 2: Check to see if Area and length are a quadratic relationship

Sequence for Area: {74, 146, 216, 284, 350,...}

D1 = 72 70 68 66

D2 = −2 −2 −2

Quadratic because D_2 is constant.

Step 3: Use TI-83 to determine the equation for the curve of best fit (the parabola).
[Enter *x* in L1 and *y* in L2]. Perform a quadratic regression.

Equation of the curve of best fit is:

$y = ax^2 + bx + c$ $a = -1$

$y = -1x^2 + 75x$ $b = 75$

$y = -x^2 + 75x$ $c = 0$

Step 4: Use your curve of best fit to find the vertex of the parabola.

$x = \frac{-b}{2a} = \frac{-75}{2(-1)} = \frac{-75}{-2} = 37.5$

$y = -x^2 + 75x$

$y = -(37.5)^2 + 75(37.5)$

$y = -1406.25 + 2812.5$

$y = 1406.25$

The vertex is : (37.5, 1406.25)

Step 5: Interpret your vertex.

Complete the statement:

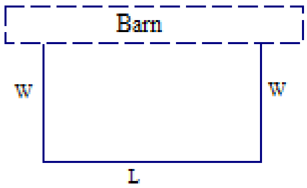
The maximum AREA is y = 1406.25 m² and it occurs at a length of x = 37.5m. What is the width? 37.5m

$l + w = 75$m

Width = 75 − 37.5m = 37.5m

Step 6: State your answer.

4. A farmer has 400 m with which to enclose a rectangular region along side of his barn. He plans on using one side of the barn as a length in his rectangle. What are the dimensions of the largest possible lot he can enclose? What is the maximum area?



$P = 2w + l$

$400 = 2w + l$

$\boxed{400 - 2w = l}$

Step 1: Create a Table

L_1	~~Length~~ width (x)	1	2	3	4	5
	~~Width~~ length	398	396	394	392	390
L_2	Area (A)	398	792	1182	1568	1950

if $w = 1$

$l = 400 - 2(1)$
$= 400 - 2$
$= 398$

if $w = 2$

$l = 400 - 2(2)$
$= 400 - 4$
$= 396$

Step 2: Quadratic Equation of Curve of Best Fit =

$y = ax^2 + bx + c$

$y = -2x^2 + 400x$

$a = -2$

$b = 400$

$c = 0$

Step 3: Vertex =

$x = \frac{-b}{2a} = \frac{-400}{2(-2)} = \frac{-400}{-4} = 100$

$y = -2x^2 + 400x$

$y = -2(100)^2 + 400(100)$

$y = -2(10\,000) + 40\,000$

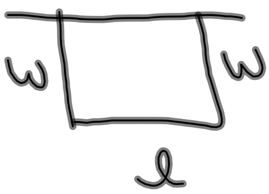
$y = -20\,000 + 40\,000$

$y = 20\,000$

Vertex $(\underline{\underline{100}}, 20\,000)$

Step 4: Interpret vertex.

The maximum AREA is y = <u>20000 m^2</u> and it occurs at a width of x = <u>100 m</u>. What is the length?



$l = 400 - 2w$

$l = 400 - 2(100)$

$= 400 - 200$

$\boxed{l = 200\text{m}}$

Format Sheet for answering Maximum and Minimum Problems

1 Table:

x-variable					
y-variable					

2 Equation of Curve of Best Fit (Quadratic Function)

3 Vertex of Quadratic Function

4 Interpret your vertex.