Unit 2: Quadratics

Section 2.1: Linear Versus Non-Linear Function Applications
Introduction to Parabolic Curves

Example: A ball is shot from a cannon into the air and its path can be recorded according to the data in the table below.

	t Time (s)	1	2	3	4	5
x						
y	H(t) Height (m)	35	60	75	80	75

D_1: 25, 15, 5, −5

D_2: 10, 10, 10

A) Using your knowledge of sequences and patterns determine algebraically whether a linear, quadratic, or cubic equation should describe the pattern.

Quadratic - D_2 is constant

B) Determine the equation of the curve of best fit. (TI-83 Plus)

$y = ax^2 + bx + c$ $\quad a = -5$

$y = -5x^2 + 40x$ $\quad b = 40$

$H(t) = -5t^2 + 40t$

height after t seconds

time

C) Using your curve of best fit, determine the height the cannon ball at 2.5 s? 5.5 s? (Two ways)

$H(t) = -5t^2 + 40t$

$H(2.5) = -5(2.5)^2 + 40(2.5)$

$H(2.5) = -5(6.25) + 100$

$H(2.5) = -31.25 + 100$

$\underline{H(2.5) = 68.75\,m}$

This means that the height at 2.5 s is 68.75m

$H(5.5) = -5(5.5)^2 + 40(5.5)$

$H(5.5) = -5(30.25) + 220$

$H(5.5) = -151.25 + 220$

$H(5.5) = 68.75\,m$

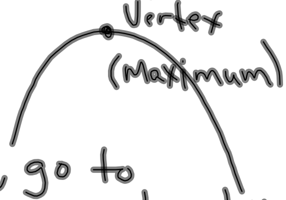
Use "Trace" on your calculator to find the height at 2.5 s & 5.5 s. You will get the same answer

<u>Steps</u>

$\boxed{\text{Trace}}$ $\boxed{\downarrow}$ ($y = -5x^2 + 40x$ should be at the top of the screen). Then type $x = 2.5$ s $\boxed{\text{enter}}$ you will see $y = 68.75$. This is your height at 2.5 s. Repeat for 5.5 s.

D) Use your calculator to determine the vertex of the parabola. (Recall, the vertex is the highest or lowest point on the graph.)

$\boxed{\text{2nd}}$ $\boxed{\text{trace}}$ V(4, 80)



$\boxed{\text{4: maximum}}$ → go to the left of the maximum $\boxed{\text{enter}}$, go to the right of maximum and enter

Guess? enter

when min/max occurs

V(4, 80)

The value of the max/min

The vertex means:

At 4s, the maximum height is 80m

E) When is the ball at its maximum height ? (use vertex)

↓

x-coordinate

(1st one)

4 seconds

F) What is the maximum height of the ball? (Use Vertex)

↓

y-coordinate

(2nd one)

80 m

G) When will the ball hit the ground?

↳ height is 0 m

↳ Need to find when the height of the ball is 0 m.

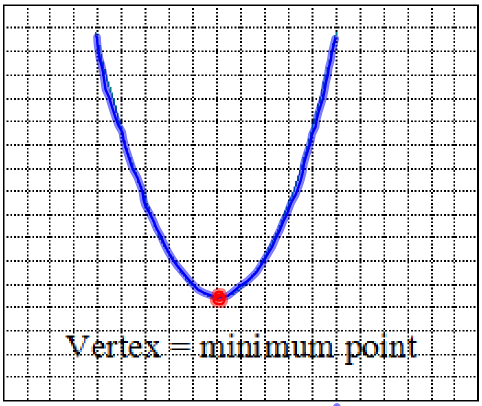
→ use $\boxed{\text{trace}}$ on the calculator

find the value of "X" when y=0.

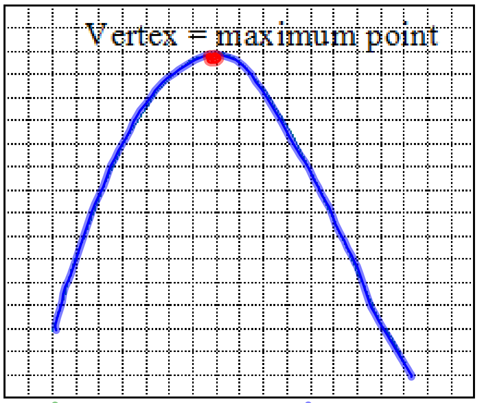
Answer: 8 s

All of these questions lead to the discussion of the Parabolic Curve often known as the PARABOLA= graph of a quadratic equation.

Case I



Case II



opens upward (orientation) opens downward

The **General Form** of any Quadratic Equation is:

$$y = ax^2 + bx + c$$

Case I = a > 0

In other words, **a** is positive.

graph opens upward
vertex is a minimum

Case II = a < 0

In other words, **a** is negative.

graph opens downward
vertex is a maximum

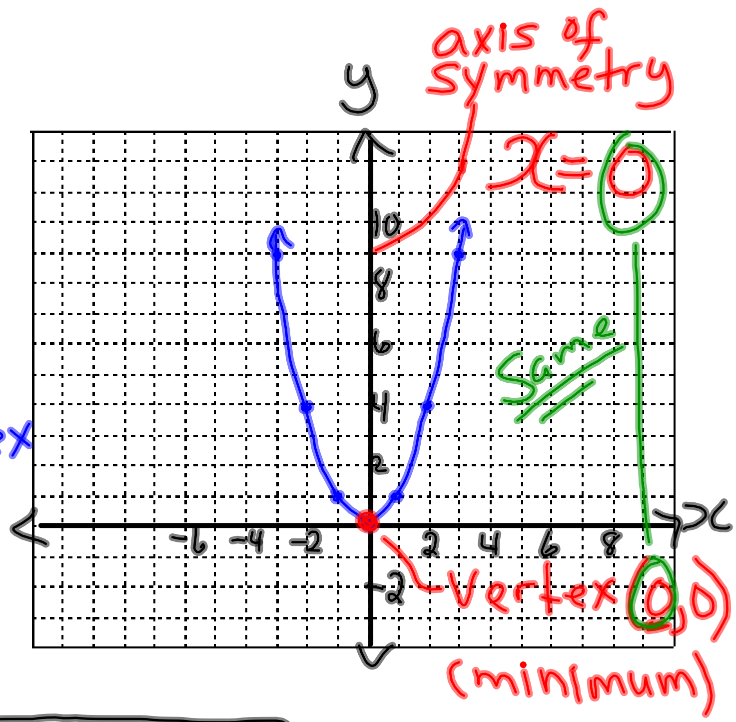
Graphing Quadratic Functions

The Base Graph is always $y = x^2$. All parabolas are compared to this graph.

The graph of $y = x^2$ has the following Table of Values:

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Vertex



* $(-3)^2$

$\boxed{\text{Zoom}}$ $\boxed{\text{6: Standard}}$

→ This will give you an x-yplane

*

Important notes on $y = ax^2 + bx + c$

1 if a > 0, the parabola opens upwards (minimum vertex) (+)

2 if a < 0, the parabola opens downwards (maximum vertex) (−)

3 the form is called general form

4 the vertex is always found by using the formula $x = \frac{-b}{2a}$

This is always the *x*-coordinate of the vertex. To get the *y*-coordinate of the vertex you substitute the *x*-coordinate of the vertex into the quadratic equation and solve for *y*.

Example: For each quadratic equation determine

A) the direction of the opening of the parabola
B) the coordinates of the vertex
C) the type of vertex

i) $y = 3x^2 -12x + 1$

A) a=3 ∴ it opens upward (U)

C) Vertex is a minimum

B) $x = -\frac{b}{2a}$ (−b means use the opposite of "b")

$x = \frac{12}{2(3)}$

$x = \frac{12}{6}$

$x = 2$

To find the y-coordinate, sub x=2 into equation.

$y = 3x^2 - 12x + 1$

$y = 3(2)^2 - 12(2) + 1$

$y = 3(4) - 24 + 1$

$y = 12 - 24 + 1$

$y = -11$

Vertex (2, −11)

ii) $y = \overset{a}{8}x^2 + \overset{b}{16}x$

A) $a = 8$, it opens upward (∪)

c) Vertex is a minimum

B) $x = -\frac{b}{2a} = \frac{-16}{2(8)} = \frac{-16}{16} = -1$

$y = 8x^2 + 16x$

$y = 8(-1)^2 + 16(-1)$

$y = 8(1) - 16$

$y = -8$

$V(-1, -8)$

iii) $y = -x^2 + 10x - 2$

A) $a = -1$, ∴ it opens downward ∩

c) Vertex is a maximum.

B) $x = -\frac{b}{2a} = \frac{-10}{2(-1)} = \frac{-10}{-2} = 5$

$y = -x^2 + 10x - 2$

$\boxed{y = -(5)^2 + 10(5) - 2}$

$y = -(25) + 50 - 2$

$y = -25 + 50 - 2$

$y = 23$

$V(5, 23)$

iv) $y = -4x^2 + 16x + 3$

A) $a = -4$, opens down ∩

C) Vertex is a maximum

B) $x = -\frac{b}{2a} = \frac{-16}{2(-4)} = \frac{-16}{-8} = 2$

$y = -4x^2 + 16x + 3$

$y = -4(2)^2 + 16(2) + 3$

$y = -4(4) + 32 + 3$

$y = -16 + 32 + 3$

$y = 19$

$V(2, 19)$