

Summary of Quadratic Sequences

- Common difference occurs at D2
- The relation will be parabolic with graphed
- The equation has the form $t_n = an^2 + bn + c$
- The graph of the data is discrete

Section 1.5: Cubic Sequences

A Cubic Sequence

- Common Difference occurs at D3
- Its equation has the form $t_n = an^3 + bn^2 + cn + d$

Example: Determine if each sequence is arithmetic, quadratic, cubic, or neither. Then find the equation of each sequence. Finally, find t_8 .

A) {22, 15, 10, 7, 6, 7, ...}

$$\begin{array}{l} D_1: \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\ \quad \quad -7 \quad -5 \quad -3 \quad -1 \quad 1 \\ D_2: \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\ \quad \quad \quad 2 \quad 2 \quad 2 \quad 2 \end{array} \quad \text{Quadratic}$$

$$y = ax^2 + bx + c \quad \begin{array}{l} a = 1 \\ b = -10 \\ c = 31 \end{array}$$

$$t_n = 1n^2 - 10n + 31$$

$$t_8 = 1(8)^2 - 10(8) + 31$$

$$t_8 = 64 - 80 + 31$$

$$t_8 = 15$$

B) {-4, 3, 22, 59, 120, 211, ...}

$$\begin{array}{l} D_1: \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\ \quad \quad 7 \quad 19 \quad 37 \quad 61 \quad 91 \\ D_2: \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \\ \quad \quad \quad 12 \quad 18 \quad 24 \quad 30 \\ D_3: \quad \checkmark \quad \checkmark \quad \checkmark \\ \quad \quad \quad \quad 6 \quad 6 \quad 6 \end{array} \rightarrow \text{Cubic}$$

$$y = ax^3 + bx^2 + cx + d \quad \begin{array}{l} a = 1 \\ b = 0 \\ c = 2.5 \\ d = -5 \end{array}$$

$$t_n = 1n^3 + 0n^2 + 2.5n - 5$$

$$t_n = n^3 - 5$$

$$t_8 = (8)^3 - 5$$

$$t_8 = 512 - 5$$

$$t_8 = 507$$

C) {1, 2, 4, 8, 16, 32, 64, ...}

Do on your own.

Example: Generate the first 4 terms of each cubic sequence.

A) $t_n = -3n^3 + 2n^2$

$$t_1 = -3(1)^3 + 2(1)^2 \quad t_2 = -3(2)^3 + 2(2)^2$$

$$t_1 = -3(1) + 2(1) \quad t_2 = -3(8) + 2(4)$$

$$t_1 = -3 + 2 \quad t_2 = -24 + 8$$

$$t_1 = -1 \quad t_2 = -16$$

$$t_3 = -3(3)^3 + 2(3)^2 \quad t_4 = -3(4)^3 + 2(4)^2$$

$$t_3 = -3(27) + 2(9) \quad t_4 = -3(64) + 2(16)$$

$$t_3 = -81 + 18 \quad t_4 = -192 + 32$$

$$t_3 = -63 \quad t_4 = -160$$

$$\{-1, -16, -63, -160, \dots\}$$

B) $t_n = 4n^3 + n^2 - 3n - 6$