

Summary of Quadratic Sequences

- Common difference occurs at D2
- The relation will be parabolic with graphed
- The equation has the form $t_n = an^2 + bn + c$
- The graph of the data is discrete

Section 1.5: Cubic Sequences

A Cubic Sequence

- Common Difference occurs at D3
- Its equation has the form $t_n = an^3 + bn^2 + cn + d$

Example: Determine if each sequence is arithmetic, quadratic, cubic, or neither. Then find the equation of each sequence. Finally, find t_8 .

A) $\{22, 15, 10, 7, 6, 7, \dots\}$

$$\begin{array}{cccccc} & & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ D_1 & -7 & -5 & -3 & -1 & 1 & \\ D_2 & & \checkmark & \checkmark & \checkmark & \checkmark & \\ & & 2 & 2 & 2 & 2 & \text{Quadratic} \end{array}$$

$$y = ax^2 + bx + c \quad \begin{array}{l} a = 1 \\ b = -10 \\ c = 31 \end{array}$$

$$t_n = 1n^2 - 10n + 31$$

$$t_8 = 1(8)^2 - 10(8) + 31$$

$$t_8 = 64 - 80 + 31$$

$$t_8 = 15$$

B) $\{-4, 3, 22, 59, 120, 211, \dots\}$

$$\begin{array}{r} \sqrt{} \\ \mathcal{D}_1 \quad 7 \quad 19 \quad 37 \quad 61 \quad 91 \\ \sqrt{} \\ \mathcal{D}_2 \quad 12 \quad 18 \quad 24 \quad 30 \\ \sqrt{} \\ \mathcal{D}_3 \quad 6 \quad 6 \quad 6 \end{array} \rightarrow \text{Cubic}$$

$$y = ax^3 + bx^2 + cx + d \quad a = 1$$

$$b = 0$$

$$t_n = 1n^3 + 0n^2 + 0n - 5 \quad c = 2.5 \quad e - 11 = 0$$

$$d = -5$$

$$t_n = n^3 - 5$$

$$t_8 = (8)^3 - 5$$

$$t_8 = 512 - 5$$

$$t_8 = 507$$

C) $\{1, 2, 4, 8, 16, 32, 64, \dots\}$

Do on your own.

Example: Generate the first 4 terms of each cubic sequence.

A) $t_n = -3n^3 + 2n^2$

$$t_1 = -3(1)^3 + 2(1)^2$$

$$t_2 = -3(2)^3 + 2(2)^2$$

$$t_1 = -3(1) + 2(1)$$

$$t_2 = -3(8) + 2(4)$$

$$t_1 = -3 + 2$$

$$t_2 = -24 + 8$$

$$t_1 = -1$$

$$t_2 = -16$$

$$t_3 = -3(3)^3 + 2(3)^2$$

$$t_4 = -3(4)^3 + 2(4)^2$$

$$t_3 = -3(27) + 2(9)$$

$$t_4 = -3(64) + 2(16)$$

$$t_3 = -81 + 18$$

$$t_4 = -192 + 32$$

$$t_3 = -63$$

$$t_4 = -160$$

$$\{-1, -16, -63, -160, \dots\}$$

B) $t_n = 4n^3 + n^2 - 3n - 6$