

Summary of Quadratic Sequences

- Common difference occurs at D2
- The relation will be parabolic with graphed
- The equation has the form $t_n = an^2 + bn + c$
- The graph of the data is discrete

Section 1.5: Cubic Sequences

A Cubic Sequence

- Common Difference occurs at D3
- Its equation has the form $t_n = an^3 + bn^2 + cn + d$

Example: Determine if each sequence is arithmetic, quadratic, cubic, or neither. Then find the equation of each sequence. Finally, find t_8 .

A) $\{22, \underline{15}, \underline{10}, \underline{7}, \underline{6}, \underline{7}, \dots\}$

$$\begin{array}{cccccc} D_1 & -7 & -5 & -3 & -1 & 1 \\ D_2 & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\ & 2 & 2 & 2 & 2 & \end{array} \quad \text{Quadratic}$$

$$y = ax^2 + bx + c \quad a = 1$$

$$b = -10$$

$$c = 31$$

$$t_n = n^2 - 10n + 31$$

$$t_8 = (8)^2 - 10(8) + 31$$

$$t_8 = 64 - 80 + 31$$

$$t_8 = 15$$

B) $\{-4, \frac{3}{7}, \frac{22}{19}, \frac{59}{37}, \frac{120}{61}, \frac{211}{91}, \dots\}$

$$\begin{array}{ccccccccc} D_1 & 7 & 19 & 37 & 61 & 91 \\ D_2 & 12 & 18 & 24 & 36 & \\ D_3 & 6 & 6 & 6 & & \end{array} \rightarrow \text{Cubic}$$

$$y = ax^3 + bx^2 + cx + d \quad a = 1$$

$$t_n = n^3 + \cancel{bx^2} + \cancel{cx} - 5 \quad b = 0$$

$$t_n = n^3 - 5 \quad c = 2.5 \times 10^{-1} = 0$$

$$t_8 = (8)^3 - 5$$

$$t_8 = 512 - 5$$

$$t_8 = 507$$

C) $\{1, 2, 4, 8, 16, 32, 64, \dots\}$

Do on your own.

Example: Generate the first 4 terms of each cubic sequence.

A) $t_n = -3n^3 + 2n^2$

$$t_1 = -3(1)^3 + 2(1)^2 \quad t_2 = -3(2)^3 + 2(2)^2$$

$$t_1 = -3(1) + 2(1) \quad t_2 = -3(8) + 2(4)$$

$$t_1 = -3 + 2 \quad t_2 = -24 + 8$$

$$t_1 = -1 \quad t_2 = -16$$

$$t_3 = -3(3)^3 + 2(3)^2 \quad t_4 = -3(4)^3 + 2(4)^2$$

$$t_3 = -3(27) + 2(9) \quad t_4 = -3(64) + 2(16)$$

$$t_3 = -81 + 18 \quad t_4 = -192 + 32$$

$$t_3 = -63 \quad t_4 = -160$$

$$\{-1, -16, -63, -160, \dots\}$$

B) $t_n = 4n^3 + n^2 - 3n - 6$