Section 1.4: Quadratic Sequences

As you have seen **arithmetic** sequences have a level one common difference (D_1) and the graph of an arithmetic sequence is linear. Its equation has the form $t_n = mn + b$.

The next type of sequence is a **quadratic sequence**. These sequences have a level two common difference.

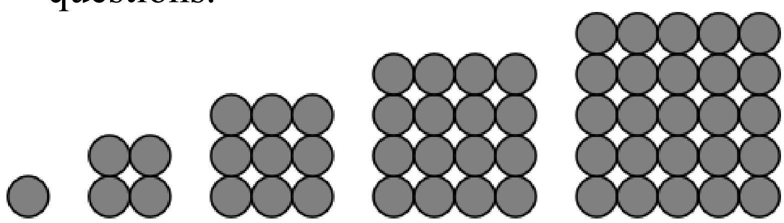
Example {7, 11, 17, 25, 35, 47, . . . }

D_1 4 6 8 10 12

D_2 2 2 2 2

A Quadratic sequence has an equation of the form $t_n = ax^2 + bx + c$. We will find the equation of a quadratic sequence using our calculator.

Example: Use the following diagram to answer the following questions.



A) List the first 5 terms of the sequences.

{1, 4, 9, 16, 25 . . . }

3, 5, 7, 9

2, 2, 2

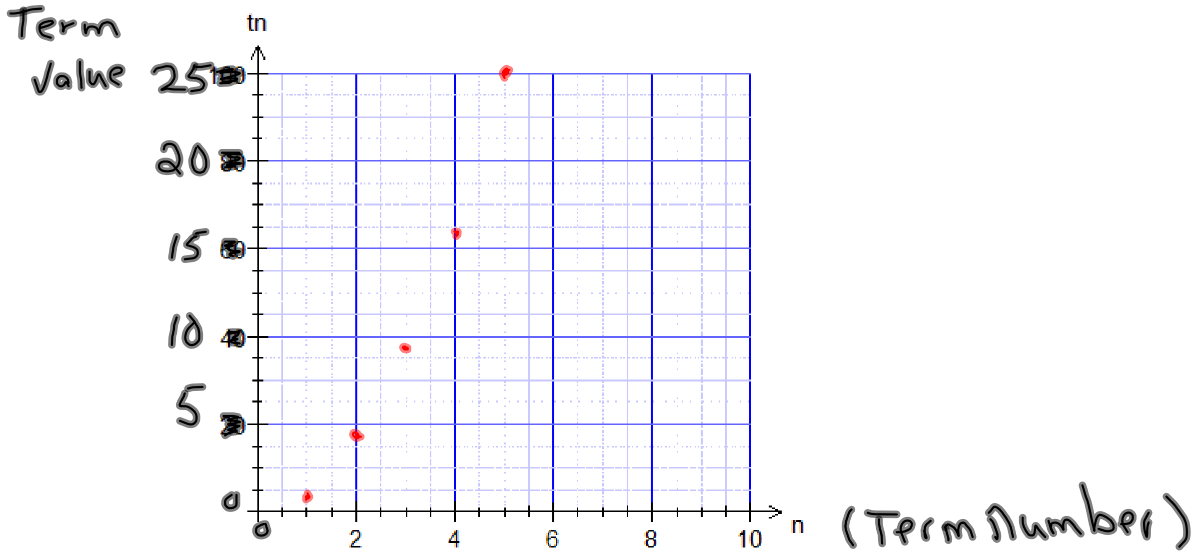
B) Determine if the sequence generated is arithmetic or quadratic.

Quadratic because D2 is constant.

C) Create a table of values for the sequence.

Term Number (n)	**Sequence Value (tn)**
1	1
2	4
3	9
4	16
5	25

D) Plot the graph of n vs. t_n.



E) Describe the shape of the graph.

The graph is a parabola.

F) What is the equation of the sequence graphed?

$\{1, 4, 9, 16, 25, \ldots\}$

$1^2, 2^2, 3^2, 4^2, 5^2, \ldots$

$\boxed{t_n = n^2}$

Using your calculator to generate the equation of a quadratic sequence.

1. Press **Stat 1** to access the list creator/editor.
2. Enter the term number in L1.
3. Enter the term values in L2.
4. To determine the equation of the line of best fit for the data in the table press S**tat** ⇒**Calc 5 Enter 2nd 1 , 2nd 2, vars y-vars enter enter enter.**
5. The equation of a quadratic function is y = ax^2 + bx + c. As a quadratic sequence we would instead express it as $t_n = an^2 + bn + c$

If we want to view the graph on our calculator we perform the following steps.

1. Press **2nd Y =** and select **ON** with the type of graph set to the first graph (a scatter plot). This will create a scatter plot of the data. Make sure the x-list is L1 and y-list is L2.
2. Press **2nd Mode** to begin the calculation.
3. Press **zoom**, then down arrow key until you reach **9: zoomstat** enter to see the graph.
3. To automatically place the equation of the line of best fit into Y1 to draw its graph, press **Stat Calc 5 2nd 1 , 2nd 2, vars y-vars Enter Enter Enter**.
4. Press G**raph** to see what the line of best fit looks like.

Example: Use the equation below to generate the first 4 terms of the sequence. Then determine if the sequence is ~~linear, quad~~ratic or neither by creating a sequence of differences.

A) $t_n = 4n^2 - 2n + 3$

Sub. 1, 2, 3 & 4 into the equation to find t_1, t_2, t_3 and t_4.

$t_1 = 4(1)^2 - 2(1) + 3$
$t_1 = 4(1) - 2(1) + 3$
$t_1 = 4 - 2 + 3$
$t = 5$

$t_2 = 4(2)^2 - 2(2) + 3$
$t_2 = 4(4) - 4 + 3$
$t_2 = 16 - 4 + 3$
$t_2 = 15$

$t_3 = 4(3)^2 - 2(3) + 3$
$t_3 = 4(9) - 6 + 3$
$t_3 = 36 - 6 + 3$
$t_3 = 33$

$t_4 = 4(4)^2 - 2(4) + 3$
$t_4 = 4(16) - 8 + 3$
$t_4 = 64 - 8 + 3$
$t_4 = 59$

$\{5, 15, 33, 59 \ldots\}$
D_1: 10, 18, 26
D_2: 8, 8

Quadratic

B) $t_n = 5 - 2n$

$t_n = -2n + 5$

$t_1 = -2(1) + 5$
$t_1 = -2 + 5$
$t_1 = 3$

$t_2 = -2(2) + 5$
$= -4 + 5$
$= 1$

$t_3 = -2(3) + 5$
$t_3 = -6 + 5$
$t = -1$

$t_4 = -2(4) + 5$
$t_4 = -8 + 5$
$t_4 = -3$

$\{3, 1, -1, -3 \ldots\}$
-2 -2 -2
Arithmetic

C) $t_n = -3n^2 + 5n - 2$

$t_1 = -3(1)^2 + 5(1) - 2$
$t_1 = -3(1) + 5 - 2$
$t_1 = -3 + 5 - 2$
$t_1 = 0$

$t_2 = -3(2)^2 + 5(2) - 2$
$t_2 = -3(4) + 10 - 2$
$t_2 = -12 + 10 - 2$
$t_2 = -4$

$t_3 = -3(3)^2 + 5(3) - 2$
$t_3 = -3(9) + 15 - 2$
$t_3 = -27 + 15 - 2$
$t_3 = -14$

$t_4 = -3(4)^2 + 5(4) - 2$
$t_4 = -3(16) + 20 - 2$
$t_4 = -48 + 20 - 2$
$t_4 = -30$

$\{0, -4, -14, -30 \ldots\}$
D_1: -4, -10, -16
D_2: -6, -6

Quadratic

D) $t_n = 2n^2 - 6$

How can you tell if the equation is linear or quadratic just by looking at it?

* If the highest exponent on an "n" term is "2" then it is quadratic.
ie. the equation has an n^2 term.

Example: Determine if the sequence is linear, quadratic, or neither. If it is linear or quadratic, find the equation for t_n. Then use the equation to find the 100th term.

Linear: $t_n = d(n-1) + t_1$
Quadratic : Calculator

A) {5, 15, 29, 47, 69, 95, . . .}

D_1 10 14 18 22 26
D_2 4 4 4 4 - Quadratic

$\boxed{\text{Stat}}$ $\boxed{\text{1: Edit}}$ $\boxed{\text{enter}}$

L_1: 1, 2, 3, 4, 5
L_2: enter your sequence

$\boxed{\text{Stat}}$ $\boxed{\rightarrow}$ $\boxed{\text{calc}}$ $\boxed{\downarrow}$ $\boxed{\text{5: Quadreg}}$ $\boxed{\text{enter}}$

$\boxed{\text{2nd}}$ $\boxed{1}$ $\boxed{,}$ $\boxed{\text{2nd}}$ $\boxed{2}$ $\boxed{,}$ $\boxed{\text{Vars}}$

$\boxed{\rightarrow}$ $\boxed{\text{Y-Vars}}$ $\boxed{\text{enter}}$ $\boxed{\text{enter}}$ $\boxed{\text{enter}}$

$y = ax^2 + bx + c$ $a = 2$
$b = 4$
$c = -1$

Ans: $t_n = 2n^2 + 4n - 1$

B) {1, 3, 5, 7, 9, 11, . . . }

t_1

D_1 d 2 2 2 2 2 Arithmetic (Linear)

$t_n = d(n-1) + t_1$
$t_n = 2(n-1) + 1$
$t_n = 2n - 2 + 1$
$t_n = 2n - 1$

C) {4, -1, -10, -23, -40, -61, . . . }

D_1 -5 -9 -13 -17 -21
D_2 -4 -4 -4 -4 quadratic

$t_n = -2n^2 + 1n + 5$

D) {5, 12, 25, 44, 69, 100, . . }

D_1 7, 13, 19, 25, 31

D_2 6, 6, 6, 6

$t_n = 3n^2 - 2n + 4$

E) {5, 13.75, 20, 23.75, 25, 23.75, . . . }

D_1 8.75, 6.25, 3.75, 1.25, −1.25

D_2 −2.5, −2.5, −2.5, −2.5

$t_n = 1.25n^2 + 12.5n - 6.25$

Example: Create a quadratic sequence with the following properties.

a) $t_1 = -4$ and $d_2 = 5$

{−4, −1, 7, 20, 38, ... }

D_1 3, 8, 13, 18

D_2 5, 5, 5

b) $t_1 = 96$ and $d_2 = -10$

{96, 100, 94, 78, 52, . . . }

D_1 4, −6, −16, −26

D_2 −10, −10, −10

{6, 7, 13, 20, 33, 53

$t_n = t_1 + d(n-1)$

$\boxed{t_n = d(n-1) + t_1}$