Section 2.3: Applying Quadratic Relationships and Maximum and Minimum Problems

We now begin applications of graphing parabolas.

By the end of this section you should be able to:

1 Determine if a relationship is quadratic (D2)
2 Find the equation of the parabola (curve of best fit) in your TI-83
3 Find the equation of the quadratic function.
4 Find the vertex of the quadratic function
5 Interpret your vertex as a solution to a maximum or minimum problem

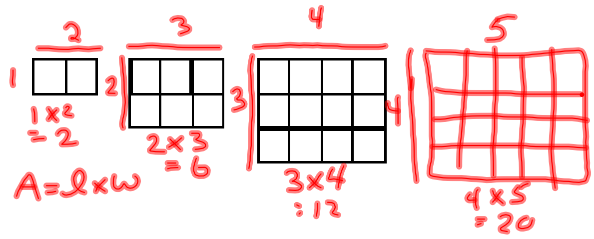
Recall,

To find the vertex, use $\frac{-b}{2a}$ to find the x-co-ordinate and then substitute this value into $y = ax^2 + bx + c$ to find the y-co-ordinate.

Applying Quadratic Relationships

1. Squares are joined together to make rectangles whose areas are recorded in the table. Each square's length and width are consecutive whole numbers.

A) Complete the table and extend the pattern by drawing rectangles on graph paper.



L1 Rectangle #	1	2	3	4	5	6	7	8
L2 Area (cm^2)	2	6	12	20	30	42	56	72

B) Find the sequence of first and second level differences, D_1 and D_2, for the area section of the table.

D_1: 4, 6, 8, 10, 12, 14, 16

D_2: 2, 2, 2, 2, 2, 2

C) Explain whether there is a linear or quadratic relationship.

Quadratic because D_2 is constant.

D) Using your TI-83 create a scatter plot and find the equation of the curve of best fit.

$a = 1$
$b = 1$
$c = 0$

$y = ax^2 + bx + c$

$y = 1x^2 + 1x$

$y = x^2 + x$

E) Use your equation of the curve of best fit to determine the area of rectangle 25. $25 = x$

$y = (25)^2 + 25$

$y = 625 + 25$

$y = 650$

Maximum and Minimum Problems Given the Function

2. A ball is thrown into the air and its path can be described by the function $y = -4.9x^2 + 29.4x + 1$, where x is the time in seconds and y is height in meters.

A) What kind of function is displayed above? How do you know?

quadratic b/c you have "x^2" in the equation.

B) What kind of graph will be displayed? What kind of vertex?

Parabola  maximum

C) Determine the initial height of the ball? (The initial height of the ball occurs at time, i.e x = 0.)

$$y = -4.9x^2 + 29.4x + 1$$
$$y = -4.9(0)^2 + 29.4(0) + 1$$
$$y = 1$$

MC

* In the equation $y = ax^2 + bx + c$ "c" is the initial height.

D) Determine, algebraically, the maximum height reached by the ball. (Note: this is a max/min problem because it used the term maximum...must find vertex

$$x = \frac{-b}{2a} \qquad x = \frac{-29.4}{2(-4.9)} = \frac{-29.4}{-9.8} = 3$$

Therefore, $y = -4.9x^2 + 29.4x + 1$

$$y = -4.9(3)^2 + 29.4(3) + 1$$
$$y = -4.9(9) + 88.2 + 1$$
$$y = -44.1 + 88.2 + 1$$
$$y = 45.1$$

(3, 45.1)

when it occurs ← x → time s

y → min/max height m

Statement: the maximum height was y = 45.1 m at x = 3 s

3. An arrow is shot into the air and its path can be described by the function $h(t) = -5t^2 + 30t$, where t is the time in seconds and h(t) is the height of the arrow in meters.

A) How do you know the arrow was fired from ground level? (In other words, how do you know that the initial height was 0?)

"c" in the equation is 0. ($h(t) = -5t^2 + 30t + 0$)
Since the initial height was 0, the arrow was fired from the ground.

B) Determine algebraically the maximum height reached by the arrow. (Note: this is a max/min problem because it used the term maximum...must find vertex)

$$t = \frac{-b}{2a} = \frac{-30}{2(-5)} = \frac{-30}{-10} = 3$$

h(t) is a fancy name for y therefore:

$h(3) = -5(3)^2 + 30(3)$
$h(3) = -5(9) + 90$
$h(3) = -45 + 90$
$h(3) = 45$

(This means: the height at 3s is 45m)

45 = y -coordinate of vertex.

The maximum height was 45 m at t = 3 s