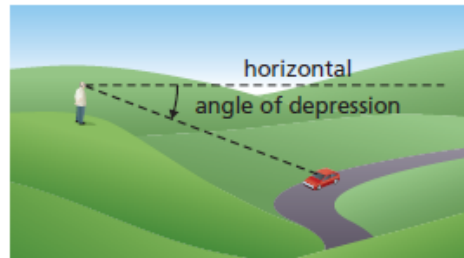
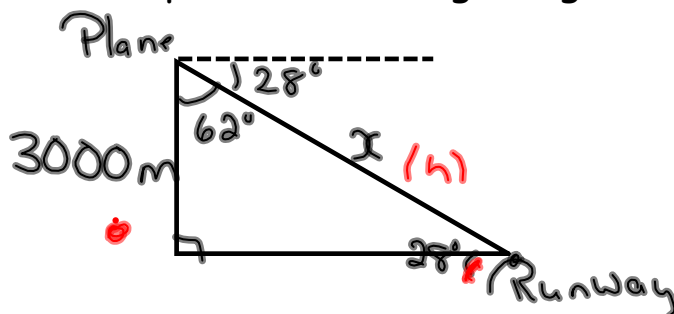


Section 2.7: Solving Problems Involving More than One Right Triangle

The angle of depression is the angle between the horizontal and the line of sight to an object beneath the horizontal (i.e. below eye level)



- The angle of depression from an airplane to the beginning of the runway is 28° . If the altitude of the airplane is 3000 m, determine the distance from the airplane to the beginning of the runway.



Altitude = height

2 methods

$$1. \quad 90^\circ - 28^\circ = 62^\circ$$

$$\frac{\cos 62^\circ}{1} = \frac{3000}{x}$$

$$\frac{x \cos 62^\circ}{\cos 62^\circ} = \frac{3000}{\cos 62^\circ}$$

$$\underline{x = 6390 \text{ m}}$$

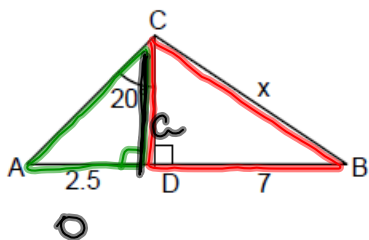
or 2. Angle of depression:
Angle of elevation.

$$\frac{\sin 28^\circ}{1} = \frac{3000}{x}$$

$$\frac{x \sin 28^\circ}{\sin 28^\circ} = \frac{3000}{\sin 28^\circ}$$

$$x = 6390 \text{ m}$$

2. Find side BC.



* To find "x" we need 2 pieces of information and we only have 1.

* In $\triangle ACD$ we know one angle and one side which means we can find everything in the triangle.

* We will only find CD b/c that side is in both triangles

$$\underline{\triangle ACD}: \frac{\tan 20^\circ}{1} = \frac{2.5}{a}$$

$$\frac{a \tan 20^\circ}{\tan 20^\circ} = \frac{2.5}{\tan 20^\circ}$$

$$a = 6.87$$

Find x using Pythagorean theorem.

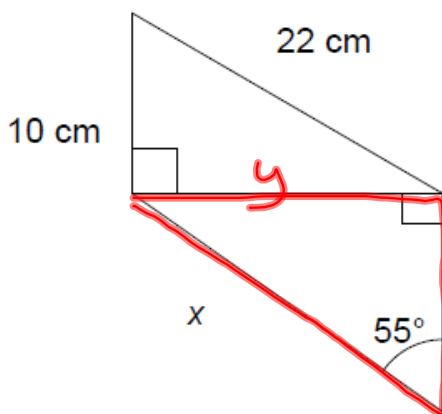
$$c^2 = a^2 + b^2$$

$$c^2 = (6.87)^2 + (7)^2$$

$$c^2 = 95.24$$

$$c = 9.8$$

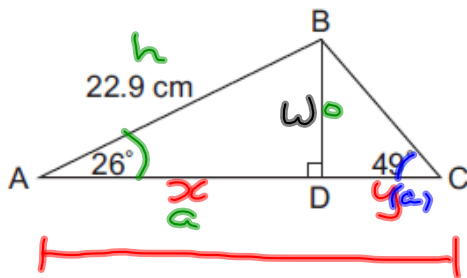
3. Find x .



$$\begin{aligned}
 y^2 &= 22^2 - 10^2 \\
 y^2 &= 484 - 100 \\
 y^2 &= 384 \\
 y &= \sqrt{384} \\
 y &= 19.60 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\sin 55^\circ}{1} &= \frac{\sqrt{384}}{x} \\
 \frac{x \sin 55^\circ}{\sin 55^\circ} &= \frac{\sqrt{384}}{\sin 55^\circ} \\
 x &= 23.9 \text{ cm}
 \end{aligned}$$

4. Find the length of AC to the nearest tenth of a centimetre.



In $\triangle ABD$:

$$\frac{\sin 26^\circ}{1} = \frac{w}{22.9}$$

$$w = 22.9 \sin 26^\circ$$

$$w = 10.04 \text{ cm}$$

$$\frac{\cos 26^\circ}{1} = \frac{x}{22.9}$$

$$x = 22.9 \cos 26^\circ$$

$$x = 20.58 \text{ cm}$$

In $\triangle BCD$

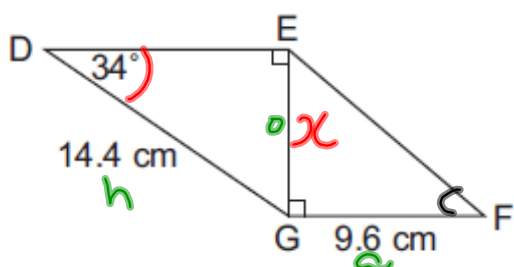
$$\frac{\tan 49^\circ}{1} = \frac{10.04}{y}$$

$$\frac{y \tan 49^\circ}{\tan 49^\circ} = \frac{10.04}{\tan 49^\circ}$$

$$y = 8.73 \text{ cm}$$

$$\begin{aligned} AC &= x + y \\ &= 20.58 + 8.73 \\ &= 29.3 \text{ cm} \end{aligned}$$

5. Find the measure of angle F to the nearest degree.



$\triangle DEG$

$$\frac{\sin 34^\circ}{1} = \frac{x}{14.4}$$

$$x = 14.4 \sin 34^\circ$$

$$x = 8.05 \text{ cm}$$

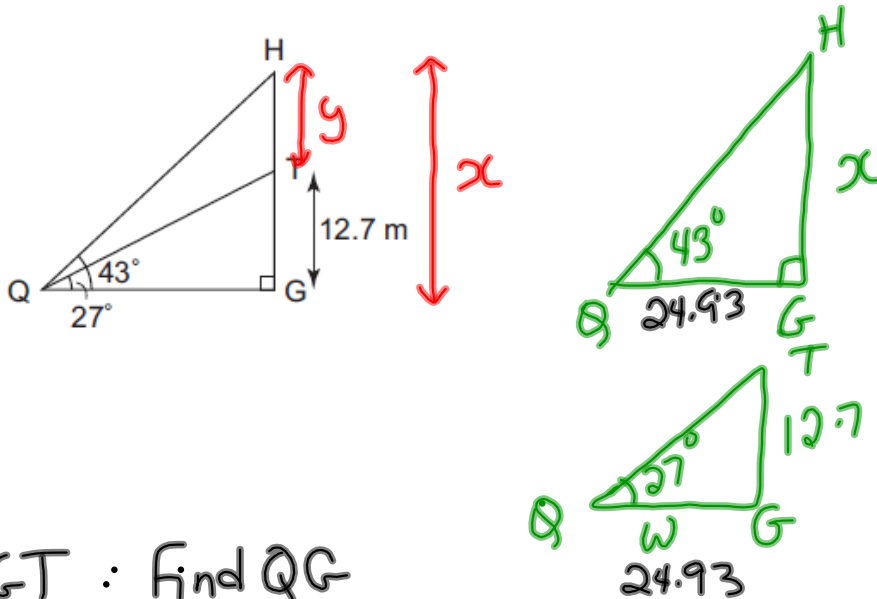
$\triangle EGF$

$$\tan F = \frac{8.05}{9.6}$$

$$F = \tan^{-1} \left(\frac{8.05}{9.6} \right)$$

$$F = 40^\circ$$

6. The angle of elevation of the top of a tree, T, is 27° . From the same point on the ground, the angle of elevation of a hawk, H, flying directly above the tree is 43° . The tree is 12.7 m tall. How high is the hawk above the tree? Give your answer to the nearest tenth of a metre.



$\triangle QGT$: Find QG

$$\frac{\tan 27^\circ}{1} = \frac{12.7}{w}$$

$$\frac{w \tan 27^\circ}{\tan 27^\circ} = \frac{12.7}{\tan 27^\circ}$$

$$w = 24.93 \text{ m}$$

$\triangle HQG$

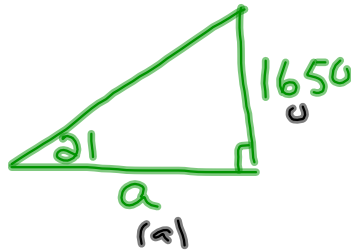
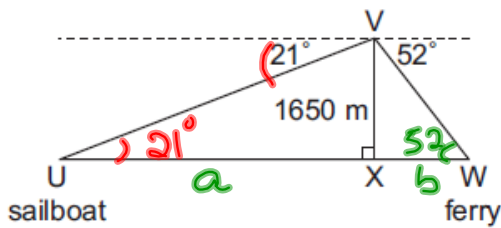
$$\frac{\tan 43^\circ}{1} = \frac{x}{24.93}$$

$$x = 24.93 \tan 43^\circ$$

$$x = 23.25 \text{ m}$$

$$\text{So } y = 23.25 - 12.7 = 10.6 \text{ m}$$

7. From a small plane, V, the angle of depression of a sailboat is 21° . The angle of depression of a ferry on the other side of the plane is 52° . The plane is flying at an altitude of 1650 m. How far apart are the boats, to the nearest metre?



$$\frac{\tan 21^\circ}{1} = \frac{1650}{a}$$

$$\frac{\tan 52^\circ}{1} = \frac{1650}{b}$$

$$\frac{a \tan 21^\circ}{\tan 21^\circ} = \frac{1650}{\tan 21^\circ}$$

$$\frac{b \tan 52^\circ}{\tan 52^\circ} = \frac{1650}{\tan 52^\circ}$$

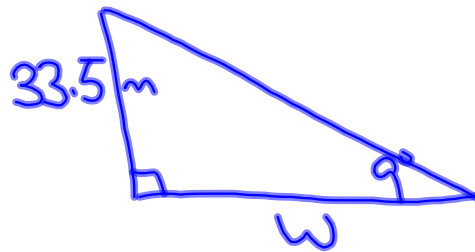
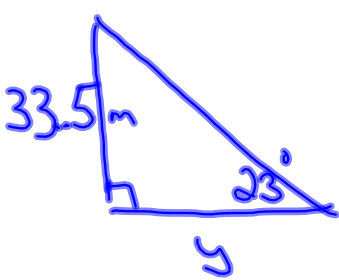
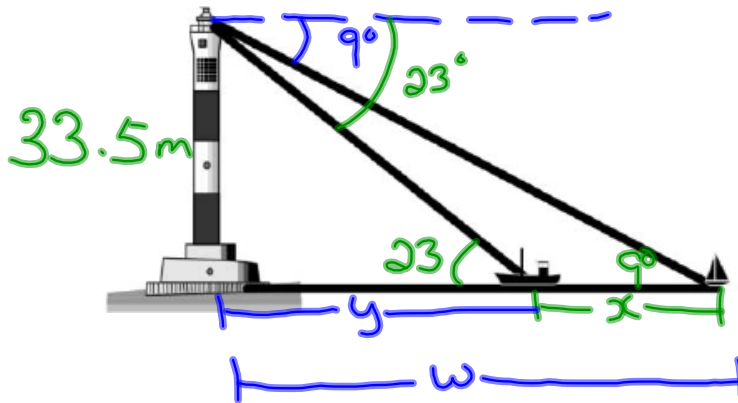
$$a = 4298.40 \text{ m}$$

$$b = 1289.12 \text{ m}$$

$$UW = 4298.40 \text{ m} + 1289.12 \text{ m}$$

$$= 5588 \text{ m}$$

8. A tourist at Point Amour sees a fishing boat at an angle of depression of 23° and a sailboat at an angle of depression of 9° . If the tourist is 33.5 m above the water, determine how far apart the two vessels are.



$$\tan 23^\circ = \frac{33.5}{y}$$

$$\tan 9^\circ = \frac{33.5}{w}$$

$$\frac{y \tan 23^\circ}{\tan 23^\circ} = \frac{33.5}{\tan 23^\circ}$$

$$\frac{w \tan 9^\circ}{\tan 9^\circ} = \frac{33.5}{\tan 9^\circ}$$

$$y = 78.92 \text{ m}$$

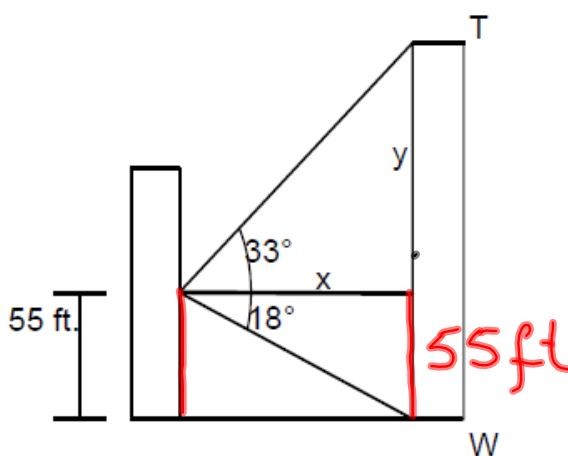
$$w = 211.51 \text{ m}$$

$$x = 211.51 \text{ m} - 78.92 \text{ m}$$

$$x = 132.59 \text{ m}$$

$$x = 132.6 \text{ m}$$

9. Two buildings are separated by an alley. Joan is looking out of a window 55 feet above the ground in one building. She estimates that the measurement of the angle of depression to the base of the second building to be 18° and the angle of elevation to the top to be 33° . How tall is the second building?



$$\frac{\tan 18^\circ}{1} = \frac{55}{x}$$

$$\frac{\tan 33^\circ}{1} = \frac{y}{169.27}$$

$$\frac{x \tan 18^\circ}{\tan 18^\circ} = \frac{55}{\tan 18^\circ}$$

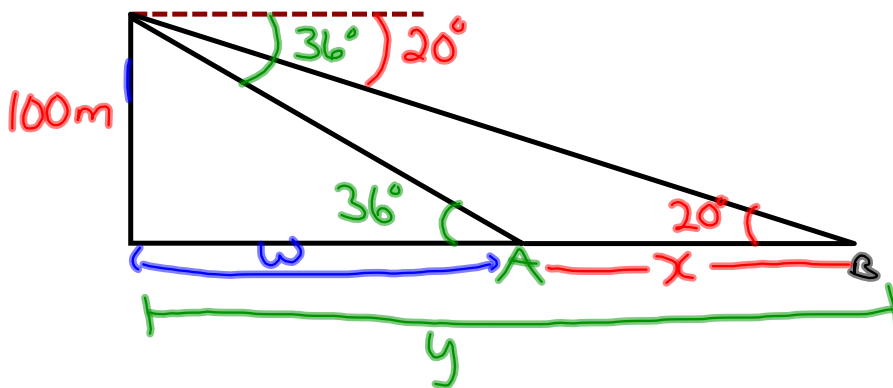
$$x = 169.27 \text{ ft}$$

$$y = 169.27 \tan 33^\circ$$

$$y = 109.93 \text{ ft}$$

$$\begin{aligned} TW &= 55 \text{ ft} + 109.93 \text{ ft} \\ &= 165 \text{ ft.} \end{aligned}$$

10. From a hotel window 100 m above street level, David observes a bus moving away from the hotel. The angle of depression of the bus changes from 36° to 20° . Determine the distance the bus travels during this change to the nearest metre.



$$\frac{\tan 20^\circ}{1} = \frac{100 \text{ m}}{y}$$

$$\frac{y \tan 20^\circ}{\tan 20^\circ} = \frac{100 \text{ m}}{\tan 20^\circ}$$

$$y = 274.75 \text{ m}$$

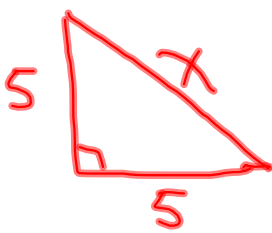
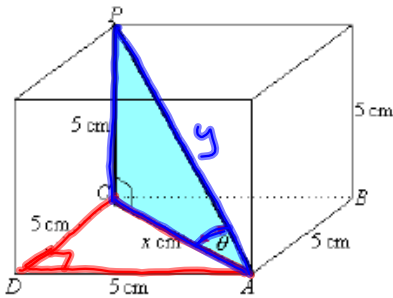
$$\tan 36^\circ = \frac{100}{w}$$

$$\frac{w \tan 36^\circ}{\tan 36^\circ} = \frac{100}{\tan 36^\circ}$$

$$w = 137.64$$

$$\begin{aligned} x &= 274.75 - 137.64 \\ &= 137.11 \text{ m} \\ &= 137 \text{ m} \end{aligned}$$

11. Find x and θ .



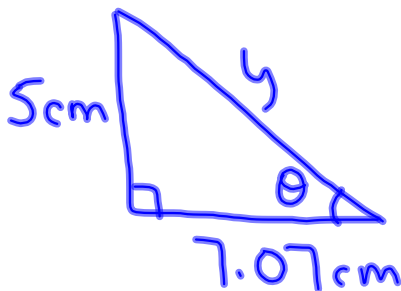
$$x^2 = 5^2 + 5^2$$

$$x^2 = 25 + 25$$

$$x^2 = 50$$

$$x = \sqrt{50}$$

$$x = 7.07 \text{ cm}$$



$$y^2 = 5^2 + 7.07^2$$

$$y^2 = 25 + 50$$

$$y^2 = 75$$

$$y = \sqrt{75} \text{ cm}$$

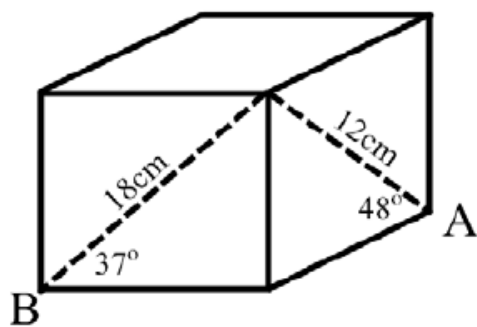
$$y = 8.7$$

$$\tan \theta = \frac{5}{7.07}$$

$$\theta = \tan^{-1}\left(\frac{5}{7.07}\right)$$

$$\theta = 35.3^\circ$$

12. Determine the shortest distance between A and B in the rectangular prism shown.



13. Given the following diagram, find the length of BC.

