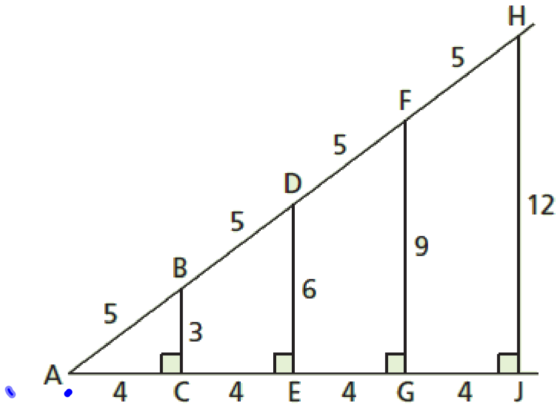
Section 2.4: The Sine and Cosine Ratio

We defined the tangent ratio for an acute angle in a right triangle. There are two other ratios we can form to compare the sides of the triangle; each ratio involves the hypotenuse.



Triangle	Measures of Sides			Ratios	
	Hypotenuse	Side opposite $\angle A$	Side adjacent to $\angle A$	$\frac{\text{Side opposite } \angle A}{\text{Hypotenuse}}$	$\frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$
ΔABC	5	3	4	$\frac{3}{5}$	$\frac{4}{5}$
ΔADE	10	6	8	$\frac{6}{10} = \frac{3}{5}$	$\frac{8}{10} = \frac{4}{5}$
ΔAFG	15	9	12	$\frac{9}{15} = \frac{3}{5}$	$\frac{12}{15} = \frac{4}{5}$
ΔAHJ	20	12	16	$\frac{12}{20} = \frac{3}{5}$	$\frac{16}{20} = \frac{4}{5}$

The Sine Ratio

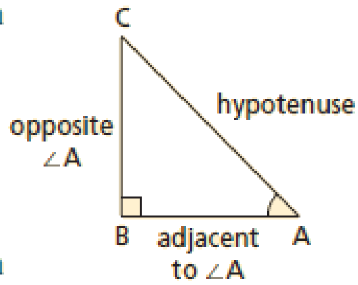
If $\angle A$ is an acute angle in a right triangle, then

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

The Cosine Ratio

If $\angle A$ is an acute angle in a right triangle, then

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$



$$\text{Sin } A = \frac{\text{opposite}}{\text{hypotenuse}}$$

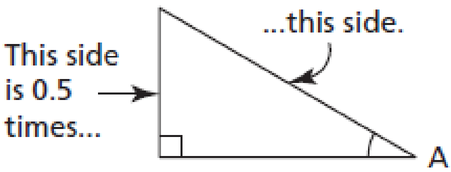
$$\text{Cos } A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The tangent, sine, and cosine are called the **primary trigonometric ratios**. The word **trigonometry** comes from three Greek words "tri + gonia + metron" that together mean "three angle measure."

Question: What does sin A = 0.5 mean? $\sin A = \frac{o}{h} = \frac{0.5}{1}$

Pull

It means that the length of the side opposite $\angle$A is 0.5 times the length of the hypotenuse.

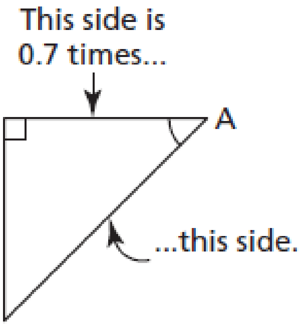


sin A = 0.5

Question: What does cos A = 0.7 mean? $\cos A = \frac{a}{h} = \frac{0.7}{1}$

Pull

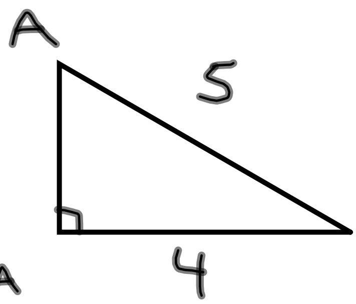
The length of the side adjacent to $\angle$A is 0.7 times the length of the hypotenuse.

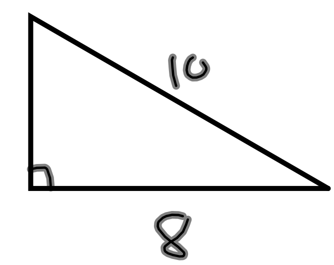


cos A = 0.7

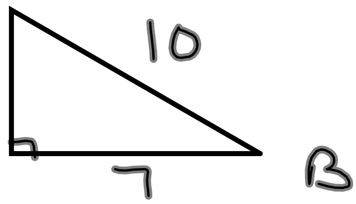
Question: For each ratio, sketch two different triangles and label their sides.

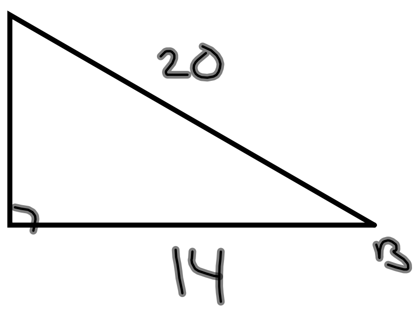
A) Sin A = 4/5 = o/h





B) Cos B = 7/10 = a/h





Example: Find each of the following to 4 decimal places.

A) $\sin 35^{o} = 0.5735$

B) $\cos 73^{o} = 0.2924$

C) $\tan 79^{o} = 5.1446$

Example: Given the following ratios, find the angle to the nearest tenth of a degree. $(\sin^{-1}, \cos^{-1}, \tan^{-1})$

A) $\sin A = 0.2378$

$A = \sin^{-1}(0.2378)$

$A = 13.8^{\circ}$

B) $\cos B = 0.9276$

$B = \cos^{-1}(0.9276)$

$B = 21.9^{\circ}$

C) $\tan C = 5.9278$

$C = \tan^{-1}(5.9278)$

$C = 80.4^{\circ}$

D) $\sin \theta = 38/45$

$\theta = \sin^{-1}\left(\frac{38}{45}\right)$

$\theta = 57.6^{\circ}$

Example: Complete the following table.

90°

θ	sin θ	cos θ	tan θ
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5000	0.8660	0.5773
45°	0.7071	0.7071	1.000
60°	0.8660	0.5000	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	undefined

Test What do you notice about the relationship between the angle and each ratio?

As θ increases sin θ increases b/c the opposite side gets larger and closer to the value of the hypotenuse. Hence,the ratio o/h gets larger.

$$0 \le \sin\theta \le 1$$

As θ increases cos θ decreases b/c the adjacent side gets smaller while the hypotenuse gets larger. Hence,the ratio a/h decreases.

$$0 \le \cos\theta \le 1$$

As θ increases tan θ increases b/c the opposite side gets larger while the adjacent side gets smaller. Hence,the ratio o/a keeps increasing.

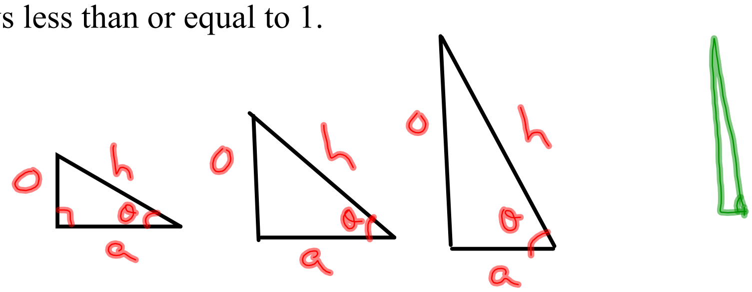
Note

15° and 75° and 30° and 60° are complementary angles (ie. they add up to 90°.)

sin 15° = cos 75° cos 15° = sin 75°
sin 30° = cos 60° cos 30° = sin 60°

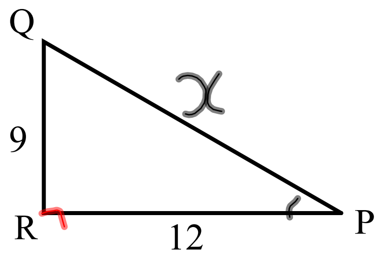
The sine of angle is equal to the cosine of its complement, and vice versa. (complement = 90 − angle)

Note: The sine and cosine ratios will never be greater than one because the adjacent side and the opposite side cannot be greater than the hypotenuse. Thus the ratios o/h and a/h are always less than or equal to 1.



Determining the Primary Trig Ratios of an Angle

Example: Find the primary trigonometric ratios for angle P and angle Q.



$c^2 = a^2 + b^2$
$x^2 = 9^2 + 12^2$ ✓
$x^2 = 81 + 144$
$x^2 = 225$
$x = 15$ ✓

Angle P

$\sin P = \frac{o}{h} = \frac{9}{15} = \frac{3}{5}$

$\cos P = \frac{a}{h} = \frac{12}{15} = \frac{4}{5}$

$\tan P = \frac{o}{a} = \frac{9}{12} = \frac{3}{4}$

Angle Q.

$\sin Q = \frac{o}{h} = \frac{12}{15} = \frac{4}{5}$

same

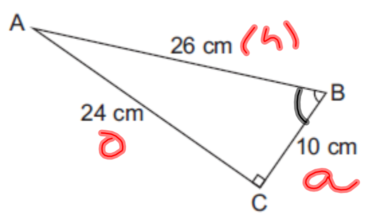
$\cos Q = \frac{a}{h} = \frac{9}{15} = \frac{3}{5}$

$\tan Q = \frac{o}{a} = \frac{12}{9} = \frac{4}{3}$

Note P and Q are complementary angles (ie. they add up to 90). So,

sin P = cos Q
cos P = sin Q

Example: Find the primary trig ratios of angle B to the nearest hundredth.



$\sin B = \frac{o}{h} = \frac{24}{26} = 0.92$

$\cos B = \frac{a}{h} = \frac{10}{26} = 0.38$

$\tan B = \frac{o}{a} = \frac{24}{10} = 2.40$