

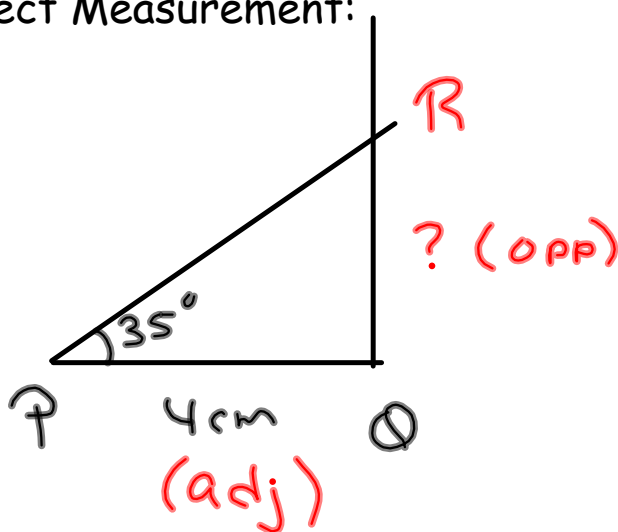
## Section 2.2: Using the Tangent Ratio To Calculate Lengths

- ✓ **Direct Measurement** - we use a measuring instrument to determine a length or angle in a polygon.
- ✓ **Indirect Measurement** - We use mathematical reasoning to calculate a length or an angle.

### Example:

In Triangle PQR, Angle Q =  $90^\circ$  and Angle P =  $35^\circ$  and PQ = 4 cm. Determine the length of RQ to the nearest centimeter.

Using direct Measurement:



$$\tan \theta = \frac{o}{a}$$

$$\frac{\tan 35^\circ}{1} = \frac{RQ}{4}$$

$$RQ = 4 \tan 35^\circ \\ = 2.8$$

If we know the measure of an acute angle and the length of a leg in a right triangle, we can use the tangent ratio to determine the length of the other leg.

### Finding the Length of a Side Opposite a Given Angle

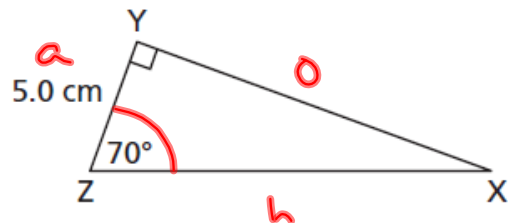
- Determine the length of XY to the nearest tenth of a centimetre.

$$\tan \theta = \frac{o}{a}$$

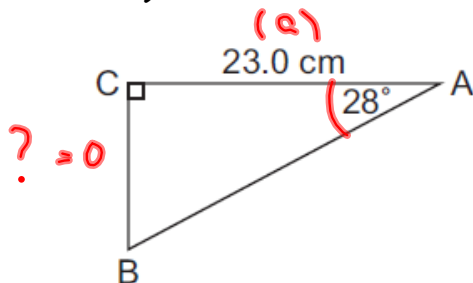
$$\frac{\tan 70^\circ}{1} = \frac{XY}{5}$$

$$XY = 5 \tan 70^\circ$$

$$XY = 13.7 \text{ cm}$$



- a) Find BC.



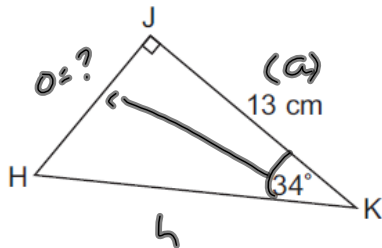
$$\tan A = \frac{o}{a}$$

$$\frac{\tan 28^\circ}{1} = \frac{BC}{23}$$

$$BC = 23 \tan 28^\circ$$

$$BC = 12.2 \text{ cm}$$

b) Side HJ



$$\tan \theta = \frac{o}{a}$$

$$\frac{\tan 34^\circ}{1} = \frac{HJ}{13}$$

$$HJ = 13 \tan 34^\circ$$

$$HJ = 8.8 \text{ cm}$$

Finding the Length of a Side Adjacent to a Given Angle

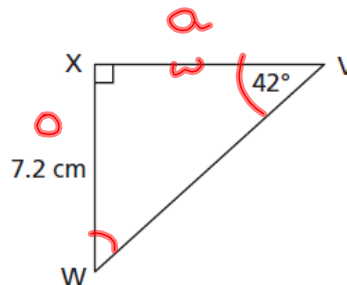
- Determine the length of VX to the nearest tenth of a centimetre.

$$\tan \theta = \frac{o}{a}$$

$$\frac{\tan 42^\circ}{1} = \frac{7.2}{VX}$$

$$\frac{VX \tan 42^\circ}{\tan 42^\circ} = \frac{7.2}{\tan 42^\circ}$$

$$VX = 8.0 \text{ cm}$$



- Find the length of PQ to the nearest tenth of a centimetre.

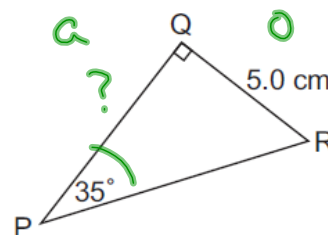
$$\tan \theta = \frac{o}{a}$$

$$\frac{\tan 35^\circ}{1} = \frac{5}{PQ}$$

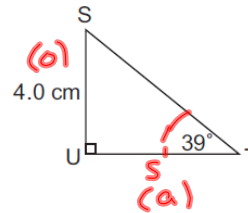
$$\frac{PQ \tan 35^\circ}{\tan 35^\circ} = \frac{5}{\tan 35^\circ}$$

$$PQ = 7.1 \text{ cm}$$

$$P. 82 \text{ 3-5}$$



3. Find side s.



$$\tan \theta = \frac{o}{a}$$

$$\frac{\tan 39^\circ}{1} = \frac{4}{s}$$

$$\frac{s \tan 39^\circ}{\tan 39^\circ} = \frac{4}{\tan 39^\circ}$$

$$s = \frac{4 \times 1}{\tan 39^\circ}$$

$$s = 4.9 \text{ cm}$$

Or  $\tan 39^\circ = \frac{4}{s}$

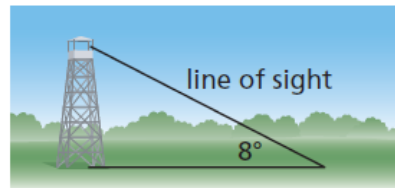
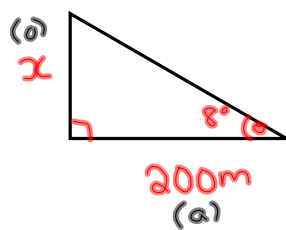
$$\frac{0.8098}{1} = \frac{4}{s}$$

$$\frac{0.8098 s}{0.8098} = \frac{4}{0.8098}$$

$$s = 4.9 \text{ cm}$$

Using Tangent to solve an indirect Measurement Problem

1. At a horizontal distance of 200 m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is  $8^\circ$ . How high is the tower to the nearest metre? The diagram is not drawn to scale.



$$\tan \theta = \frac{o}{a}$$

$$\frac{\tan 8^\circ}{1} = \frac{x}{200}$$

$$x = 200 \tan 8^\circ$$

$$x = 28 \text{ m}$$

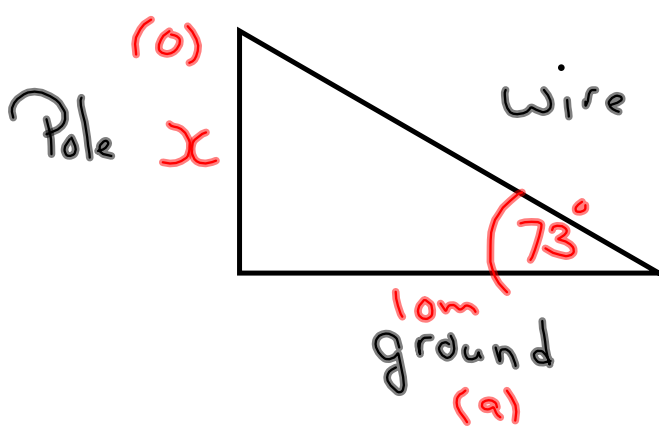
$$\tan 8^\circ = \frac{x}{200}$$

$$\frac{0.1405}{1} = \frac{x}{200}$$

$$x = (0.1405)(200)$$

$$x = 28 \text{ m}$$

2. A wire supports a flagpole. The angle between the wire and the level ground is  $73^\circ$ . The wire is anchored to the ground 10 m from the base of the pole. How high up the pole does the wire reach? Give the answer to the nearest tenth of a metre.



$$\tan \theta = \frac{o}{a}$$

$$\frac{\tan 73^\circ}{1} = \frac{x}{10}$$

$$x = 10 \tan 73^\circ$$

$$x = 32.7 \text{ m}$$

$$\tan 73^\circ = \frac{x}{10}$$

$$\frac{3.3709}{1} = \frac{x}{10}$$

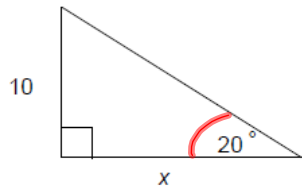
$$x = (3.3709)(10)$$

$$x = 32.7 \text{ m}$$

Other Problems

1. A) Find and correct the error in the following calculation:

0



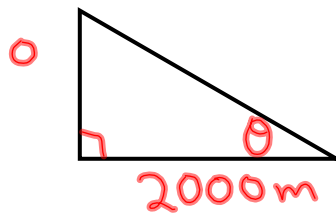
a

$$\begin{aligned} \frac{\tan 20^\circ}{1} &= \frac{10}{x} \\ x \tan 20^\circ &= 10 \\ \frac{x \tan 20^\circ}{\tan 20^\circ} &= \frac{10}{\tan 20^\circ} \\ x &= 27.5 \end{aligned}$$

$$\begin{aligned} \tan 20^\circ &= \frac{x}{10} \\ 10 \tan 20^\circ &= x \\ 3.64 &= x \end{aligned}$$

B) By examining the side lengths and angle measures of the right triangle above, explain why  $x=3.64$  must be incorrect.

2. Given a right triangle with acute angle  $\theta$  and  $\tan \theta = 1.875$ , find the ~~length of the side opposite~~ angle  $\theta$  if the side adjacent to angle  $\theta$  is 2000 m.



$$\tan \theta = \frac{o}{a}$$

$$\frac{1.875}{1} = \frac{o}{2000}$$

$$o = (1.875)(2000)$$

$$o = 3750 \text{ m}$$

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