

Chapter 2: Trigonometry

What You Will Learn

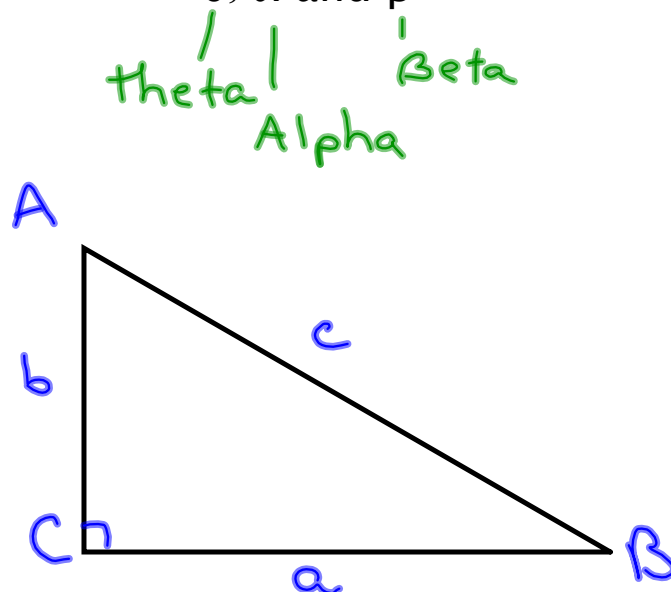
In a right triangle,

- The ratio of any two sides remains constant even if the triangle is enlarged or reduced.
- You can use the ratio of the lengths of two sides to determine the measure of one of the acute angles.
- You can use the length of one side and the measure of an acute angle to determine the length of another side of the triangle.
- You will solve problems that involve one or more right triangles.

Trigonometry is a branch of mathematics that enable us to determine the measures of the sides and angles of a triangle.

Conventions

Uppercase Letters are used to label the vertices of a triangle. Corresponding lower case letters are used to label the sides. Greek letters such as θ , α and β are often used to label angles.

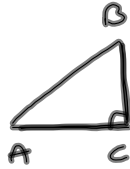


Section 2.1: The Tangent Ratio

Dynamic Activity

Recall: Similar Triangles

- the measures of corresponding angles are equal
- the ratios of corresponding sides are equal



| Measure of $\angle A$ | Value of $\frac{CB}{BA}$ $\frac{CB}{CA}$ | | |
|-----------------------|--|-------------------------------|-------------------------------|
| | Original Triangle | Similar Triangle 1 | Similar Triangle 2 |
| 10° | $1.02 \div 5.77$ 0.1768 | $2.21 \div 12.53$ 0.1764 | $2.86 \div 16.21$ 0.1764 |
| 30° | $6.14 \div 10.64$ 0.5771 | $7.69 \div 13.32$ 0.5773 | $9.20 \div 15.93$ 0.5775 |
| 45° | $7.04 \div 7.04$ 1 | $11.57 \div 11.57$ 1 | $15.89 \div 15.89$ 1 |
| 60° | $8.40 \div 4.85$ 1.7320 | $12.58 \div 7.26$ 1.7328 | $15.74 \div 9.09$ 1.7316 |
| 75° | $11.74 \div 3.15$ 3.7270 | $15.20 \div 4.07$ 3.7346 | |
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What do you notice about the value of the ratio CB/CA for each angle A ?

The ratios were the same.

What do you think the value of each ratio depends on?

The angle.

What is the ratio of CB/CA for angles less than 45° ?

$$0 \leq \tan A < 1$$

$$0 \leq \frac{CB}{CA} < 1$$

What is the ratio of CB/CA for angles greater than 45° ?

$$\frac{CB}{CA} > 1$$

$$\tan A > 1$$

What is the ratio of CB/CA for 45° ?

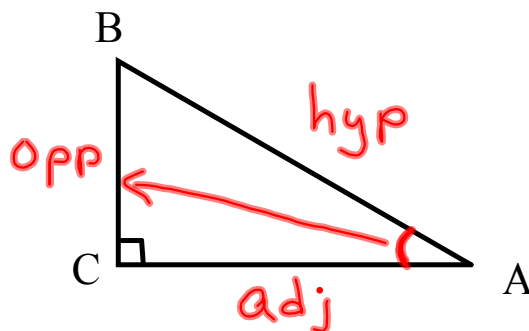
$$\frac{CB}{CA} = 1$$

$$\tan 45^\circ = 1$$

Labelling Right Triangles

We name the sides of a right triangle in relation to one of its acute angles. (Acute angle is less than 90°)

Consider $\triangle ABC$,

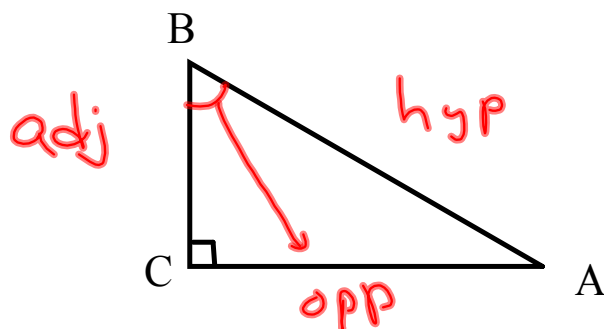


For angle A:

Hypotenuse - AB - the longest side

Opposite Side - BC - the side directly across (opposite) of angle A

Adjacent Side - AC - it is one of the sides that forms the angle (but not the hypotenuse)



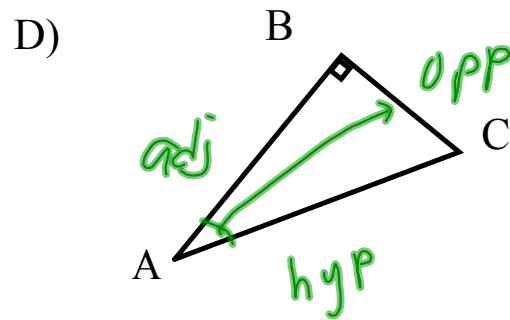
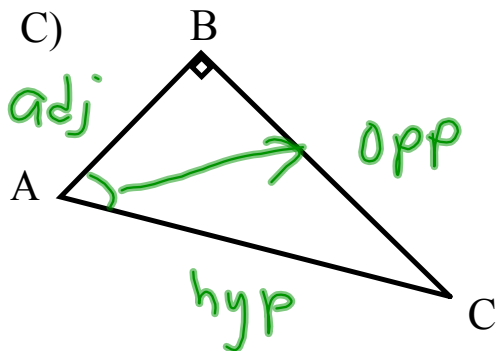
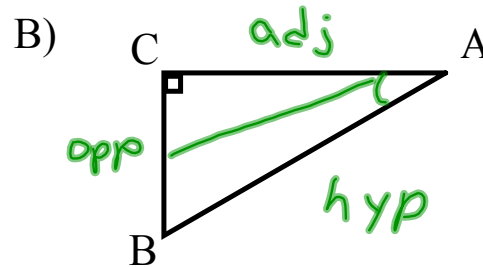
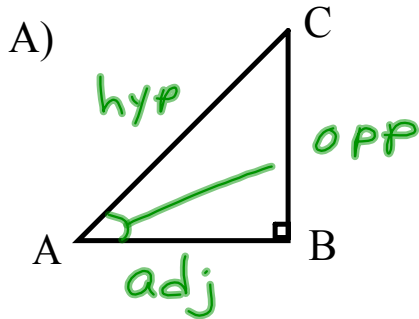
For angle B:

Hypotenuse - AB - the longest side

Opposite Side - AC - the side directly across (opposite) of angle A

Adjacent Side - BC - it is one of the sides that forms the angle (but not the hypotenuse)

Example: For each triangle, label the adjacent, opposite and hypotenuse with respect to angle A.

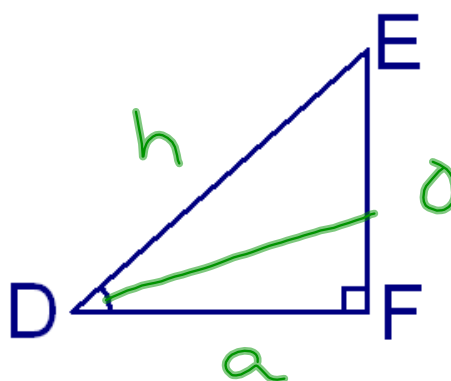


Which side is adjacent to $\angle D$?

A DE

B EF

C DF



Which side is opposite $\angle P$?

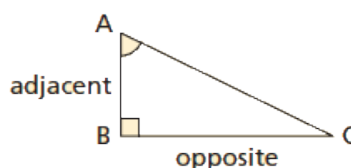
- A MN
- B MP
- C NP



The Tangent Ratio

If $\angle A$ is an acute angle in a right triangle, then

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$



$$\tan A = \frac{o}{a}$$

The tangent ratio depends only on the measure of the angle, not on how large or small the triangle is.

What can be said about the angles of a right triangle when the tangent ratio is larger than one or less than one?

If $\tan A > 1$, the angle is greater than 45° .

If $\tan A < 1$, the angle is less than 45° .

In a right triangle, the tangent of one of the acute angles is 1. Explain how the measures of the two legs are related.

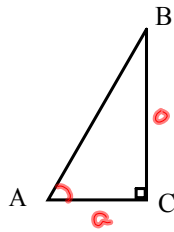
$$\tan A = 1$$

$$\angle A = 45^\circ$$



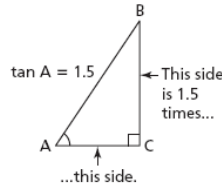
If $\tan A = 1$, then $\angle A = 45^\circ$ which means the triangle is an isosceles right triangle. This means the legs are equal.

What does it mean if we say $\tan A = 1.5$?



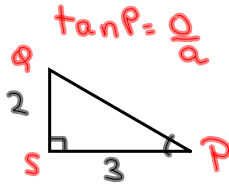
$\frac{\text{opp}}{\text{adj}} = \frac{1.5}{1}$
 $\text{opp} = 1.5\text{adj}$

Pull

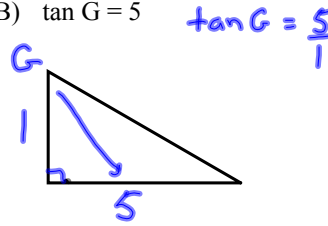


Example: Illustrate each tangent ratio by sketching a right triangle, then labelling the measures of its legs.

A) $\tan P = 2/3$

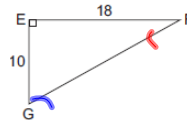


B) $\tan G = 5$



Determining The Tangent Ratios for Angles

1. Find $\tan F$ and $\tan G$.



$\tan F = \frac{o}{a}$

$\tan F = \frac{10}{18}$

$\tan F = \frac{5}{9}$

$\tan F = 0.5556$

$\tan G = \frac{o}{a}$

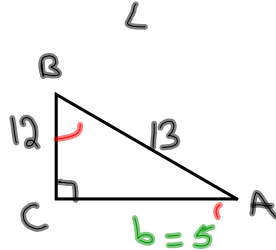
$\tan G = \frac{18}{10}$

$\tan G = \frac{9}{5}$

$\tan G = 1.8$

Note $\tan F$ & $\tan G$ are reciprocals.

2. In triangle ABC, $C = 90^\circ$, the hypotenuse is 13 and the side opposite A is 12cm. Find $\tan A$ and $\tan B$.



need the adjacent side \Rightarrow use pythagorean theorem

$b^2 = c^2 - a^2$

$b^2 = 13^2 - 12^2$

$b^2 = 169 - 144$

$b^2 = 25$

$b = 5$

$\tan A = \frac{o}{a}$

$\tan A = \frac{12}{5}$

$\tan B = \frac{o}{a}$

$\tan B = \frac{5}{12}$

If we know the tangent ratio, then we can use our calculator to determine the measure of an acute angle. The \tan^{-1} or InvTan calculator does this. **NOTE: Your calculator must be in degree mode.**

Using the Tangent Ratio to Determine the Measure of an Angle

1. To the nearest degree, determine the measure of angle X for each value of $\tan X$.

$$\begin{aligned} \text{a) } \tan X &= 0.4236 \\ \angle X &= \tan^{-1}(0.4236) \\ \angle X &= 23^\circ \end{aligned}$$

$$\begin{aligned} \text{b) } \tan X &= 1.7268 \\ \angle X &= \tan^{-1}(1.7268) \\ \angle X &= 60^\circ \end{aligned}$$

2. Find the measure of $\angle A$ and $\angle C$ to the nearest degree.

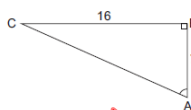
$$\tan A = \frac{o}{a}$$

$$\tan A = \frac{16}{7}$$

$$\angle A = \tan^{-1}\left(\frac{16}{7}\right) \quad \text{calc: } \tan^{-1}(16 \div 7)$$

$$\angle A = 66^\circ$$

$$\angle C = 90^\circ - 66^\circ = 24^\circ \quad (\angle A \text{ \& } \angle C \text{ are complementary angles ie they add up to } 90^\circ)$$



$$\begin{aligned} \text{or } \tan C &= \frac{o}{a} \\ \tan C &= \frac{7}{16} \\ \angle C &= \tan^{-1}\left(\frac{7}{16}\right) \\ \angle C &= 24^\circ \end{aligned}$$

2. Determine the measures of $\angle K$ and $\angle N$ to the nearest tenth of a degree.

$$\tan K = \frac{o}{a}$$

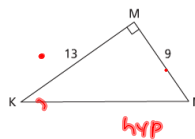
$$\tan K = \frac{9}{13}$$

$$\begin{aligned} K &= \tan^{-1}(9/13) \\ K &= 34.7^\circ \end{aligned}$$

$$\tan N = \frac{o}{a}$$

$$\tan N = \frac{13}{9}$$

$$\begin{aligned} \angle N &= \tan^{-1}(13/9) \\ \angle N &= 55.3^\circ \end{aligned}$$



Homework:

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3, 4, 5, 7, 8, 9.