

Section 4.6: Applying The Exponent Laws

Rational Exponent Law 1: $x^{\frac{1}{n}} = \sqrt[n]{x}$

Rational Exponent Law 2: $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$

Negative Integer Exponent: $x^{-n} = \frac{1}{x^n}$ or $\frac{1}{x^{-n}} = x^n$

Fraction Negative Exponent: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Negative Rational Exponent: $x^{-\frac{m}{n}} = \frac{1}{x^{\frac{m}{n}}}$

Fraction Neg. Rational Exponent: $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$

EXPONENT LAWS YOU LEARNED LAST YEAR

② Product of Powers: $a^m \cdot a^n = a^{m+n}$
 Quotient of Powers: $a^m \div a^n = a^{m-n}$ or $\frac{a^m}{a^n} = a^{m-n}$

① Power of a Power: $(a^m)^n = a^{mn}$

Power of a Product: $(ab)^m = a^m b^m$

Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Zero Exponent: $x^0 = 1$

Simplifying Using the Law of Exponents

Order:

× − ÷

Power of Powers, then Quotient or Product, then all exponents must be positive.

Example 1: Simplifying Numerical Expressions with Integer Exponents

Simplify each expression by writing it as a power with a positive exponent. Then evaluate if numbers are not too large.

$$A) 3^{-7} \cdot 3^3$$

$$\left[3^{-7+3} \right]$$

$$3^{-4}$$

$$\frac{1}{3^4}$$

$$\frac{1}{81}$$

$$B) (5^{-7})^2 (5^{-2})^3$$

$$(5^{-14})(5^{-6})$$

$$5^{-20}$$

$$\frac{1}{5^{20}}$$

$$C) \frac{(6)^{-8}(6)^4}{6^{-3}}$$

$$\left[= \frac{6^{-8+4}}{6^{-3}} \right]$$

$$\frac{6^{-4}}{6^{-3}}$$

$$\left[6^{-4-(-3)} \right]$$

$$6^{-1}$$

$$\frac{1}{6}$$

$$D) 2^{-4} \cdot 2^{-3}$$

$$\left[2^{-4-3} \right]$$

$$2^{-7}$$

$$\frac{1}{2^7}$$

$$\frac{1}{128}$$

$$E) (3^{-2}3^4)^2$$

$$(3^2)^{-2} \quad 3^4 \cdot 3^{-8}$$

$$3^{-4} \quad 3^{-4}$$

$$\frac{1}{3^4} \quad \frac{1}{3^4}$$

$$\frac{1}{81} \quad \frac{1}{81}$$

$$F) \frac{11^{-2}}{(11^{-4})(11^{-5})}$$

$$\frac{11^{-2}}{11^{-9}}$$

$$11^7$$

$$G) 0.8^2 \cdot 0.8^{-7}$$

$$0.8^{-5}$$

$$\left(\frac{8}{10} \right)^{-5}$$

$$\left(\frac{4}{5} \right)^{-5}$$

$$\left(\frac{5}{4} \right)^5$$

$$\frac{3125}{1024}$$

$$H) \left[\left(-\frac{4}{5} \right)^2 \right]^3 \div \left[\left(-\frac{4}{5} \right)^4 \right]^5$$

$$\left(-\frac{4}{5} \right)^{-6} \div \left(-\frac{4}{5} \right)^{-20}$$

$$\left(-\frac{4}{5} \right)^{14}$$

$$I) \frac{(1.5^{-3})^5}{1.5^5}$$

$$\frac{1.5^{15}}{1.5^5}$$

$$1.5^{10}$$

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Example 2: Simplifying Numerical Expressions with Rational Exponents

Simplify each expression by writing it as a power with a positive exponent.

A) $3^{\frac{1}{2}} \cdot 3^{\frac{1}{4}}$
 $3^{\frac{2}{4}} \cdot 3^{\frac{1}{4}}$
 $3^{\frac{3}{4}}$

B) $2^{\frac{1}{3}} \cdot (2^{-2})^{\frac{1}{2}}$
 $2^{-\frac{1}{3}} \cdot 2^{-1}$
 $2^{-\frac{1}{3}} \cdot 2^{-\frac{2}{3}}$
 $2^{-\frac{4}{3}}$
 $(\frac{1}{2})^{\frac{4}{3}}$

C) $\frac{(5^{-0.5})(5^{1.5})}{5^{0.5}}$
 $\frac{5^1}{5^{0.5}}$
 $5^{0.5}$

D) $(4^{\frac{1}{2}} \cdot 4^{\frac{1}{4}})^3$
 $4^{\frac{3}{2}} \cdot 4^{-\frac{3}{4}}$
 $4^{\frac{6}{4}} \cdot 4^{-\frac{3}{4}}$
 $4^{\frac{3}{4}}$

E) $(2^{\frac{1}{3}})^3 \cdot (2^{-2})^{\frac{1}{3}}$
 $2^{-1} \cdot 2^{\frac{2}{3}}$
 $2^{-\frac{1}{3}} \cdot 2^{\frac{2}{3}}$
 $2^{-\frac{1}{3}}$
 $(\frac{1}{2})^{\frac{1}{3}}$

F) $\left[\frac{5^{-1.5}}{(5^{1.5})(5^{-0.25})} \right]^4$
 $\frac{5^{-6}}{5^6 \cdot 5^{-1}}$
 $\frac{5^{-6}}{5^5} \times$
 5^{-11}
 $(\frac{1}{5})^{11}$

G) $\left[\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right]^4$
 $\frac{7^{\frac{8}{3}}}{7^{\frac{4}{3}} \cdot 7^{\frac{20}{3}}}$
 $\frac{7^{\frac{8}{3}}}{7^{\frac{24}{3}}}$
 $7^{-\frac{16}{3}}$
 $(\frac{1}{7})^{\frac{16}{3}}$

H) $\frac{9^{\frac{5}{3}} \cdot 9^{\frac{-1}{4}}}{9^4}$
 $\frac{9^{\frac{4}{4}}}{9^{\frac{16}{4}}}$
 $9^{-\frac{12}{4}}$

I) $(-\frac{3}{4})^{\frac{2}{3}} (-\frac{3}{4})^{\frac{7}{6}}$
 $(-\frac{3}{4})^{\frac{2}{3} + \frac{7}{6}}$
 $(-\frac{3}{4})^{\frac{8}{6}}$

Example 3: Simplifying Algebraic Expressions with Integer Exponents

Simplify.

A) $3a^{-2} \cdot a^{-5} \cdot a^4$

$$3a^{-3}$$

$$\left[3 \times \frac{1}{a^3}\right]$$

$$\frac{3}{a^3}$$

B) $(2x^2 \cdot 3x^{-5})^3$

2 ways

$$\textcircled{1} (6x^{-3})^3$$

$$6^3 x^{-9}$$

$$216x^{-9}$$

$$\frac{216}{x^9}$$

C) $\frac{12a^2}{3a^{-3}}$ $\{-2+(+3)\}$

$4a^5$

D) $2x^3 \cdot x^4 \cdot x^{-9}$

$$2x^{-2}$$

$$\frac{2}{x^2}$$

E) $(2a^5 \cdot 3a^{-1})^2$

$$2^2 a^{10} \cdot 3^2 a^{-2}$$

$$4a^{10} \cdot 9a^{-2}$$

$$36a^8$$

F) $\frac{8y^2}{6y^{-4}}$ $\{2+(+4)\}$

$\frac{4}{3}y^6$

G) $m^4 n^{-2} \cdot m^2 n^3$

$m^6 n$

H) $\frac{6x^4 y^{-3}}{14xy^2}$

$\{-3-2\}$
 $\{-3+(-2)\}$

$\frac{3x^3 y^{-5}}{7}$

$\frac{3x^3}{7y^5}$

I) $\frac{(t^2)^3}{t^{-3}}$

$$\frac{t^6}{t^{-3}}$$

$$t^9$$

J) $\frac{2x(3xy)^2}{6xy^3}$

$$\frac{2x(3^2 x^2 y^2)}{6xy^3}$$

$$\frac{18x^3 y^2}{6xy^3}$$

$3x^2 y^{-1}$

$\frac{3x^2}{y}$

x	+
÷	-
(.)	x

Example 4: Simplifying Algebraic Expressions with Rational Exponents

Simplify. Explain the reasoning.

A) $x^{\frac{3}{2}} \cdot x^{-1}$ $\left\{ \begin{array}{l} \frac{3}{2} + -1 \\ \frac{3}{2} + -\frac{2}{2} \\ \frac{1}{2} \end{array} \right.$ $x^{\frac{1}{2}}$

OR $x^{\frac{3}{2}} \cdot x^{-\frac{2}{2}}$
 $x^{\frac{1}{2}}$

B) $\frac{10a^{\frac{9}{4}}}{8a^3}$
 $\frac{5a^{-3/4}}{4}$
 $\frac{5}{4a^{3/4}}$
 $\frac{9}{4} - \frac{3}{1}$
 $\frac{9}{4} - \frac{12}{4} = -\frac{3}{4}$

C) $x^{\frac{5}{2}} x^{-\frac{3}{2}}$ $\left\{ \begin{array}{l} \frac{5}{2} + -\frac{3}{2} \\ \frac{2}{2} \\ 1 \end{array} \right.$
 x^1
 x

D) $\frac{8a^{\frac{1}{3}}}{6a^{\frac{2}{3}}}$
 $\frac{4}{3} a^{5/3}$
 $\frac{1}{3} + (\frac{4}{3})$

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 E) $(25a^4b^2)^{\frac{3}{2}}$
 $25^{3/2} a^6 b^3$
 $(\sqrt{25})^3 a^6 b^3$
 $125a^6b^3$
 $4 \times \frac{3}{2} = 2 \times \frac{3}{2}$
 $\frac{12}{2} = 6$
 $\frac{6}{2} = 3$

F) $(x^3y^{\frac{3}{2}})(x^{-1}y^{\frac{1}{2}})$
 x^2y^{-1}
 $\frac{x^2}{y}$
 $-\frac{3}{2} + \frac{1}{2} = -\frac{2}{2} = -1$

G) $\frac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{\frac{1}{2}}}$
 $= 4x^{-9/2}y^3$
 $= \frac{4y^3}{x^{9/2}}$
 $-\frac{5}{1} - (-\frac{1}{2})$
 $-\frac{10}{2} + (\frac{1}{2})$
 $-\frac{9}{2}$
 $\frac{5}{2} + (\frac{1}{2}) = \frac{6}{2} = 3$

H) $(\frac{50x^2y^4}{2x^4y^7})^{\frac{1}{2}}$
 $(25x^{-2}y^{-3})^{\frac{1}{2}}$
 $25^{1/2} x^{-1} y^{-3/2}$
 $\frac{\sqrt{25}}{xy^{3/2}} = \frac{5}{xy^{3/2}}$
 $-\frac{2}{1} \times \frac{1}{2} = -1$
 $-\frac{3}{1} \times \frac{1}{2} = -\frac{3}{2}$

I) $(x^{\frac{1}{10}}y^{-\frac{2}{5}})^{\frac{1}{2}} (x^{\frac{1}{4}}y^{\frac{1}{2}})^{-1}$
 $(x^{\frac{1}{20}}y^{-\frac{1}{5}})(x^{\frac{1}{4}}y^{-\frac{1}{2}})$
 $x^{\frac{7}{20}}y^{-\frac{7}{10}}$
 $\frac{x^{\frac{7}{20}}}{y^{\frac{7}{10}}}$
 $\frac{1}{20} + \frac{1}{4}$
 $\frac{2}{20} + \frac{5}{20}$
 $-\frac{1}{4} + -\frac{1}{2}$
 $-\frac{1}{4} + -\frac{2}{4}$