Section 4.4 Fractional Exponents and Radicals

1. A. Use a calculator to complete the following tables.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$
36	$36^{1/2} = 6$
49	$49^{1/2} = 7$
64	$64^{1/2} = 8$

x	$x^{\frac{1}{3}}$
1	$1^{1/3} = 1$
8	$8^{1/3} = 2$
27	$27^{1/3} = 3$
64	$64^{1/3} = 4$
125	$125^{1/3} = 5$
216	$216^{1/3} = 6$
343	$343^{1/3} = 7$
512	$512^{1/3} = 8$

$1^{1/2} = 1 \wedge (1 \div 2)$

$\boxed{x^y}$

$\boxed{y^x}$

* fractional exponents must be put in a bracket.

$16^{\frac{1}{2}} = \sqrt[2]{16}$

index of the radical

What do you notice about the numbers in the first column?

Compare the numbers in the first and second columns.
What conclusions can you make?

What do you think the exponent $\frac{1}{2}$ means?

What do you think the exponent $\frac{1}{3}$ means?

What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?

What does $a^{\frac{1}{n}}$ mean? Explain your reasoning.

Recall: In grade 9 you learned that for powers with integral bases and whole number exponents:

Product of Powers $(a^m)(a^n) = a^{m+n}$

We can use this property for fractional exponents with numerator 1.

Consider:

$\sqrt{3} \times \sqrt{3}$ $= \sqrt{3 \times 3}$ $= \sqrt{9}$ $= 3$	$3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$ $= 3^{\frac{1}{2}+\frac{1}{2}}$ $= 3^1$ $= 3$

This means $\sqrt{3} = 3^{\frac{1}{2}}$.

Note the index in $\sqrt{3}$ and the location of the index in the exponent.

This means: $a^{\frac{1}{n}} = \sqrt[n]{a}$ OR $\sqrt[n]{a} = a^{\frac{1}{n}}$

Fractional Exponent Law 1

* The denominator of the fractional exponent is the index of the radical

Fractional Exponent Law 1:

$50^{\frac{1}{3}}$ $\quad a^{\frac{1}{n}} = \sqrt[n]{a}$ OR $\sqrt[n]{a} = a^{\frac{1}{n}}$

Example 1

Example: Express the radicals below as a power.

A) $\sqrt[2]{217}$ B) $\sqrt[5]{30}$ C) $\sqrt[3]{50}$ D) $\sqrt[4]{25}$

$217^{1/2}$ $\quad 30^{1/5}$ $\quad 50^{1/3}$ $\quad 25^{1/4}$

Example: Express the powers below as a radical and simplify if possible.

A) $9^{\frac{1}{2}}$ B) $81^{\frac{1}{4}}$ C) $200^{\frac{1}{3}}$ D) $(-64)^{\frac{1}{3}}$ E) $-4^{\frac{1}{5}}$

A) $= \sqrt{9}$ $= 3$

B) $= \sqrt[4]{81}$ $= 3$

C) $\sqrt[3]{200}$ $= \sqrt[3]{8 \cdot 25}$ $= 2\sqrt[3]{25}$

D) $= \sqrt[3]{-64}$ $= -4$

E) $= -1 \times 4^{1/5}$ $= -\sqrt[5]{4}$

Example: Evaluate each power without a calculator.

A) $1000^{\frac{1}{3}}$ B) $0.25^{\frac{1}{2}}$ C) $(-8)^{\frac{1}{3}}$ D) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

A) $= \sqrt[3]{1000}$ $= 10$

B) $= \sqrt{0.25}$ $= 0.5$

C) $= \sqrt[3]{-8}$ $= -2$

D) $\sqrt[4]{\frac{16}{81}} = \frac{2}{3}$

Fractional Exponent Law 2

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \left(\sqrt[n]{a^m}\right)$$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

depends on you & the question

$16^{\frac{3}{4}}$	$16^{\frac{3}{4}}$
$= \left(16^{\frac{1}{4}\times 3}\right)$	$= \left(16^{3\times\frac{1}{4}}\right)$
$= \left(16^{\frac{1}{4}}\right)^3$	$= \left(16^{3}\right)^{\frac{1}{4}}$
$= \left(\sqrt[4]{16}\right)^3$	$= \sqrt[4]{(16^3)}$
$= 2^3$	$= 2^3$
$= 8$	$= 8$

Example: Write each power as a radical.

$A)\ 32^{\frac{1}{4}}$ $B)\ 5^{\frac{3}{4}}$ $C)\ 26^{\frac{2}{5}}$ $D)\ 15^{\frac{5}{3}}$

$= \sqrt[4]{32}$

$(\sqrt[4]{5})^3$ or $\sqrt[4]{5^3}$

$(\sqrt[5]{26})^2$ $\sqrt[5]{26^2}$

$(\sqrt[3]{15})^5$ $\sqrt[3]{15^5}$

$E)\ 8^{3.5}$ $F)\ 12^{0.75}$

$8^{7/2}$ $12^{3/4}$

$(\sqrt{8})^7$ $(\sqrt[4]{12})^3$

$\sqrt{8^7}$

Example: Write each power as a radical. Then evaluate.

$A)\ 81^{\frac{3}{4}}$ $B)\ 0.01^{\frac{3}{2}}$ $C)\ 27^{\frac{4}{3}}$ $D)\ (-32)^{0.4}$

$= (\sqrt[4]{81})^3$
$= (3)^3$
$= 27$

$= (\sqrt{0.01})^3$
$= (0.1)^3$
$= 0.001$

$= (\sqrt[3]{27})^4$
$= 3^4$
$= 81$

$(-32)^{4/10}$
$(-32)^{2/5}$
$(\sqrt[5]{-32})^2$
$(-2)^2$
4

$E)\ (-4)^{\frac{3}{2}}$ $F)\ -16^{\frac{3}{4}}$

$(\sqrt[2]{-4})^3$
impossible

$-1 \times 16^{3/4}$
$-(\sqrt[4]{16})^3$
$-(2)^3$
$= -8$

Example: Express each radical as a power.

A) $\sqrt[4]{4^3}$ $= 4^{3/4}$

B) $\sqrt[5]{6^3}$ $6^{3/5}$

C) $\sqrt[5]{(-6)^3}$ $(-6)^{3/5}$

D) $\sqrt[3]{\left(\frac{7}{8}\right)^2}$ $= \left(\frac{7}{8}\right)^{2/3}$

Applications of Fractional Exponents

Ex1 Video page 226

2. The area (*A*) of a face of a cube is given by, $A = V^{\frac{2}{3}}$ where *V* represents the volume of the cube. If $V = 64\ \text{cm}^3$, determine the value of *A*.

$A = V^{2/3}$

$A = (64)^{2/3}$

$A = (\sqrt[3]{64})^2$

$A = (4)^2$

$\boxed{A = 16\text{cm}^2}$

3. The value of a car (V) is given by the equation: $V = 32000(0.85)^{\frac{t}{2}}$ where t represents how old the vehicle is. Estimate the value of the car after 5 years.

$$V = 32000(0.85)^{t/2}$$
$$V = 32000(0.85)^{5/2}$$
$$V = 32000(0.6661)$$
$$V = \$21\,315.20$$

$$\left[\begin{array}{c} 0.85 \wedge (5 \div 2) \\ x^y \\ y^x \end{array}\right]$$

DOE

4. Ask students to respond to the following:

Is $-32^{\frac{2}{5}}$ equal to $(-32)^{\frac{2}{5}}$? Explain your reasoning.

$$-32^{2/5}$$
$$-1 \times 32^{2/5}$$
$$-1 \times \left(\sqrt[5]{32}\right)^2$$
$$-1 \times (2)^2$$
$$-1 \times 4$$
$$-4$$

$$(-32)^{2/5}$$
$$\left(\sqrt[5]{-32}\right)^2$$
$$(-2)^2$$
$$4$$

$\therefore$ $-32^{2/5} \neq (-32)^{2/5}$

as shown above.