Section 4.3: Mixed and Entire Radicals

Reminder: A fraction can be written in many different ways.

Ex. Write 5 other fractions that are equivalent to $\frac{2}{3}$.

$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$, $\frac{20}{30}$, $\frac{40}{60}$

Just as with fractions, equivalent expressions for any number have the same value.

Consider the following:

$$\begin{aligned}\sqrt{196} &= \sqrt{4 \cdot 49} \\ &= \sqrt{4} \cdot \sqrt{49} \\ &= 2 \cdot 7 \\ &= 14\end{aligned}$$

$$\begin{aligned}\sqrt[3]{216} &= \sqrt[3]{8 \cdot 27} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{27} \\ &= 2 \cdot 3 \\ &= 6\end{aligned}$$

✱ Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

where n is a natural number, and a and b are real numbers.

We can use this property to simplify square roots and cube roots that are not perfect squares or perfect cubes, but have factors that are perfect squares or perfect cubes.

Simplifying Radicals that are not Perfect Squares (Index = 2)

Examples

1. Simplify the following irrational numbers if possible. (exact value)

   A) $\sqrt{20}$

   $\sqrt{20} \doteq 4.47$ this is an approximation not an exact value

   Method 1: Extract the largest perfect square. (preferred method)

$$\sqrt{20} = \sqrt{4 \times 5}$$
$$= \sqrt{4} \times \sqrt{5}$$
$$= 2 \times \sqrt{5}$$
$$= 2\sqrt{5}$$

entire radical

— mixed radical

   Method 2: Prime Factorization

$$\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5}$$
$$= \sqrt{2^2 \cdot 5}$$
$$= \sqrt[2]{2^2} \cdot \sqrt{5}$$
$$= 2\sqrt{5}$$

B) $\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

C) $\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$ (4 times $\sqrt{5}$)
↳ exact value

D) $\sqrt{81} = 9$

E) $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$

F) $\sqrt{54} = \sqrt{9 \cdot 6} = \sqrt{9} \cdot \sqrt{6} = 3\sqrt{6}$

G) $\sqrt{63} = \sqrt{9 \cdot 7} = \sqrt{9} \cdot \sqrt{7} = 3\sqrt{7}$

H) $\sqrt{125} = \sqrt{25 \cdot 5} = \sqrt{25} \times \sqrt{5} = 5\sqrt{5}$

I) $\sqrt{288} = \sqrt{144 \cdot 2} = \sqrt{144} \cdot \sqrt{2} = 12\sqrt{2}$

J) $\sqrt{21}$ = cannot be simplified any further.

K) $\sqrt{338} = \sqrt{169 \cdot 2} = \sqrt{169} \cdot \sqrt{2} = 13\sqrt{2}$

Simplifying Radicals with indexes higher than 2 (cubed, fourth and fifth roots)

1. Write each radical in simplest form (if possible).

   Check on calculator to see if they are perfect cubes first!

A) $\sqrt[3]{24}$

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

B) $\sqrt[3]{128}$

$= \sqrt[3]{64 \times 2}$

$= \sqrt[3]{64} \times \sqrt[3]{2}$

$= 4\sqrt[3]{2}$

C) $\sqrt[3]{32}$

$= \sqrt[3]{8 \cdot 4}$

$= \sqrt[3]{8} \times \sqrt[3]{4}$

$= 2\sqrt[3]{4}$

D) $\sqrt[3]{10}$

$\rightarrow$ Cannot be simplified.

E) $\sqrt[3]{25}$

$\rightarrow$ cannot be simplified

F) $\sqrt[3]{250}$

$= \sqrt[3]{125} \times \sqrt[3]{2}$

$= 5\sqrt[3]{2}$

G) $\sqrt[3]{1000}$

$= 10$

H) $\sqrt[3]{108}$

$= \sqrt[3]{27} \times \sqrt[3]{4}$

$= 3\sqrt[3]{4}$

Higher Indexes (index of 4 and 5)

2. Write each radical in simplest form (if possible).

A) $\sqrt[4]{24}$

- cannot be simplified.

B) $\sqrt[4]{48}$

$= \sqrt[4]{16} \times \sqrt[4]{3}$

$= 2\sqrt[4]{3}$

C) $\sqrt[4]{80}$

$= \sqrt[4]{16} \times \sqrt[4]{5}$

$= 2\sqrt[4]{5}$

D) $\sqrt[4]{81}$

$= 3$

E) $\sqrt[4]{243}$

$= \sqrt[4]{81} \times \sqrt[4]{3}$

$= 3\sqrt[4]{3}$

F) $\sqrt[5]{32}$

$= 2$

G) $\sqrt[5]{96}$

$= \sqrt[5]{32} \times \sqrt[5]{3}$

$= 2\sqrt[5]{3}$

H) $\sqrt[5]{486}$

$= \sqrt[5]{243} \times \sqrt[5]{2}$

$= 3\sqrt[5]{2}$

I) $\sqrt[5]{64}$

$= \sqrt[5]{32} \times \sqrt[5]{2}$

$= 2\sqrt[5]{2}$

Changing Mixed Radicals to Entire Radicals

$\sqrt{80}$ is an example of an entire radical.

$4\sqrt{3}$ is an example of a mixed radical.

↳ 4 is called the coefficient

To change a mixed radical to a entire radical, raise the coefficient (the number in front of the radical)to the index and move it back inside the radical sign and multiply it by the radicand.

1. Change the following mixed radicals to entire radicals.

A) $2\sqrt{5},\ 3\sqrt{3},\ 2\sqrt[3]{3},\ 3\sqrt[3]{-2}$

$2\sqrt{5} = \sqrt{2^2 \cdot 5} = \sqrt{4 \cdot 5} = \sqrt{20}$

$3\sqrt{3} = \sqrt{3^2 \cdot 3} = \sqrt{9 \cdot 3} = \sqrt{27}$

$2\sqrt[3]{3} = \sqrt[3]{2^3 \cdot 3} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{24}$

$3\sqrt[3]{-2} = \sqrt[3]{3^3 \cdot -2} = \sqrt[3]{27 \cdot -2} = \sqrt[3]{-54}$

To put these #'s in order, we would have to change each # to a decimal.

B) $3\sqrt[4]{5},\ 6\sqrt[4]{2},\ 9\sqrt[4]{3},\ 7\sqrt[5]{2},\ 4\sqrt[3]{-5},\ 10\sqrt[5]{200}$

$3\sqrt[4]{5} = \sqrt[4]{3^4 \cdot 5} = \sqrt[4]{81 \cdot 5} = \sqrt[4]{405}$

$6\sqrt[4]{2} = \sqrt[4]{6^4 \cdot 2} = \sqrt[4]{1296 \cdot 2} = \sqrt[4]{2592}$

$7\sqrt[5]{2} = \sqrt[5]{7^5 \cdot 2} = \sqrt[5]{16\,807 \cdot 2} = \sqrt[5]{33\,614}$

$4\sqrt[3]{-5} = \sqrt[3]{4^3 \cdot -5} = \sqrt[3]{64 \cdot -5} = \sqrt[3]{-320}$

$10\sqrt[5]{200} = \sqrt[5]{10^5 \cdot 200} = \sqrt[5]{100\,000 \cdot 200} = \sqrt[5]{20\,000\,000}$

2. Place the following radicals in order from least to greatest.

A) $2\sqrt{7},\ \sqrt{30},\ 5\sqrt{2},\ 3\sqrt{3}$

$2\sqrt{7} = \sqrt{2^2 \cdot 7} = \sqrt{4 \cdot 7} = \sqrt{28}$

$5\sqrt{2} = \sqrt{5^2 \cdot 2} = \sqrt{25 \cdot 2} = \sqrt{50}$

$3\sqrt{3} = \sqrt{3^2 \cdot 3} = \sqrt{9 \cdot 3} = \sqrt{27}$

$\sqrt{30} = \sqrt{30}$

Order: $3\sqrt{3},\ 2\sqrt{7},\ \sqrt{30},\ 5\sqrt{2}$

B) $2\sqrt[3]{5},\ 3\sqrt[3]{2},\ 4\sqrt[3]{5},\ 5\sqrt[3]{3}$

$2\sqrt[3]{5} = \sqrt[3]{2^3 \cdot 5} = \sqrt[3]{8 \cdot 5} = \sqrt[3]{40}$

$3\sqrt[3]{2} = \sqrt[3]{3^3 \cdot 2} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{54}$

$4\sqrt[3]{5} = \sqrt[3]{4^3 \cdot 5} = \sqrt[3]{64 \cdot 5} = \sqrt[3]{320}$

$5\sqrt[3]{3} = \sqrt[3]{5^3 \cdot 3} = \sqrt[3]{125 \cdot 3} = \sqrt[3]{375}$

Order: $2\sqrt[3]{5},\ 3\sqrt[3]{2},\ 4\sqrt[3]{5},\ 5\sqrt[3]{3}$

Exercises Page 218: 3, 4, 5, 6, 9, 10, 11, 12, 14, 15, 17, 18, 20, 21, 22