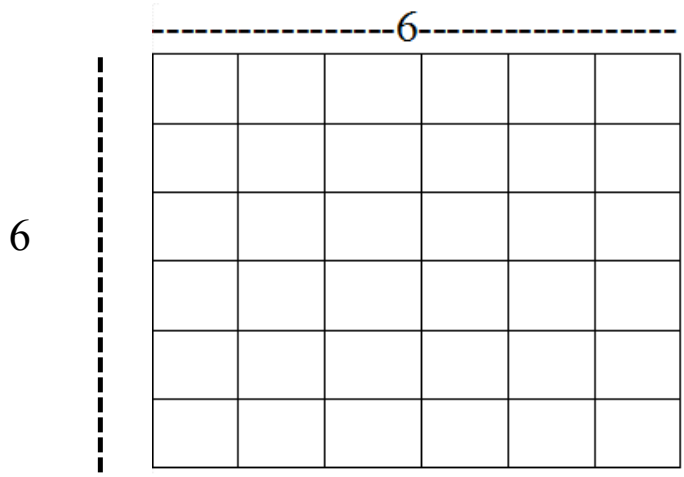
Section 3.2: Perfect Squares (Review), Perfect Cubes (New) and Their Roots

Perfect Square: Any whole number that can be represented as the area of a square with a whole number side length.

Concretely: 6 tiles by 6 tiles in the room



Area = 36 units^2

Side length = $\sqrt{36}$ = 6 units

Symbolically,

$36 = 4 \times 9$
$= (2 \times 2) \times (3 \times 3)$ Prime factors are grouped in pairs
$= (2 \times 3) \times (2 \times 3)$ Rearrange the factors in two equal
$= 6 \times 6$ groups

$\sqrt{36} = 6$

$\sqrt{n}$ - the means the positive (or principal) square root of a number.

Know

Perfect Squares	Their Square Roots
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
169	13
196	14
225	15
256	16
289	17
324	18
361	19
400	20
625	25

Example: Find the square root of

A) 324 B) 1296

Method 1

A) $324 = 2 \cdot 162$

$= 2 \cdot 2 \cdot 81$

$= 2 \cdot 2 \cdot 9 \cdot 9$

$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= (2 \cdot 2) \cdot (3 \cdot 3) \cdot (3 \cdot 3)$

$= (2 \cdot 3 \cdot 3) \cdot (2 \cdot 3 \cdot 3)$

$= 18 \cdot 18$

So, $\sqrt{324} = 18$

Method 2

$\sqrt{324} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

$= \sqrt{2^2 \cdot 3^2 \cdot 3^2}$

$= 2 \cdot 3 \cdot 3$

$= 18$

B) $1296 = 2 \cdot 648$

$= 2 \cdot 2 \cdot 324$

$= 2 \cdot 2 \cdot 2 \cdot 162$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 81$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 9 \cdot 9$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= (2 \cdot 2)(2 \cdot 2)(3 \cdot 3)(3 \cdot 3)$

$= (2 \cdot 2 \cdot 3 \cdot 3)(2 \cdot 2 \cdot 3 \cdot 3)$

$= 36 \cdot 36$

$\sqrt{1296} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

$= \sqrt{2^2 \cdot 2^2 \cdot 3^2 \cdot 3^2}$

$= 2 \cdot 2 \cdot 3 \cdot 3$

$= 36$

So, $\sqrt{1296} = 36$

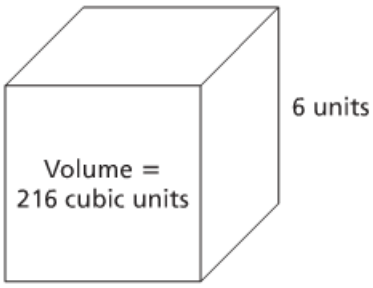
Perfect Cubes

Perfect Square: Any whole number that can be represented as the area of a square with a whole number side length. (Area = lw)

Perfect Cube: Any whole number that can be represented as the volume of a cube with a whole number edge length. (Volume = lwh)

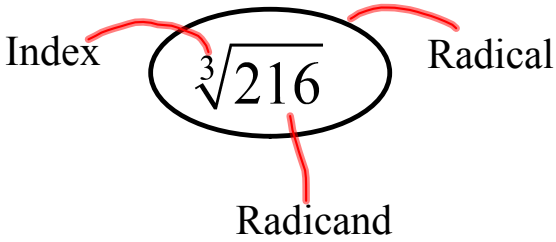
Volume = 3 x 3 x 3 = 27 cubes
therefore 27 is a perfect cube

So, we write $\sqrt[3]{27} = 3$



A 6 x 6 x 6 cube has 216 cubes
therefore 216 is a perfect cube

So, we write $\sqrt[3]{216} = 6$



The index is not written for square roots but it is always taken to be two.

In $\sqrt{144}$ the index is 2.

P. 146 4, 7

P. 149 1 - 6

Example: Evaluate (without a calc).

A) $\sqrt[3]{512}$

$512 = 2 \times 256$
$= 2 \times (16 \times 16)$
$= 2 \times (4 \times 4) \times (4 \times 4)$
$= 2 \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$ Group Factors into set of 3
$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$= 8 \times 8 \times 8$

So, $\sqrt[3]{512} = 8$

B) $\sqrt[3]{1728}$

Example: Compute the following using technology:

A) $\sqrt{900}$

B) $\sqrt[3]{900}$

C) $\sqrt[3]{512\ 000}$

D) $\sqrt[3]{-729}$

E) $\sqrt[4]{1296}$

What does each mean?

Example: Compute.

A) $\sqrt[3]{343}$

B) $\sqrt{121} - \sqrt[3]{216}$

C) $\sqrt[3]{64} + \sqrt[3]{1000} \div \sqrt{25}$

Examples

1. What is the length of the side of a square farm which contains 1764 yd.^{2}? How far apart are its opposite corners?

2. If the volume of a cube is 125m^3, what is the expression for the length of each side?

3. A right rectangular prism measures 9 in. x 8 in. x 24 in. What are the dimensions of a cube with the same volume?

4. Determine the cube root of 3375 in a variety of ways. This could include the use of prime factorization, the use of benchmarks and/or the use of a calculator.