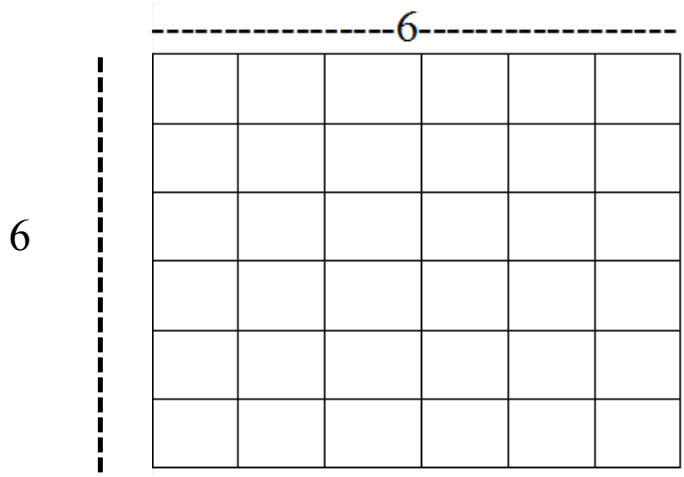
Section 3.2: Perfect Squares (Review), Perfect Cubes (New) and Their Roots

Perfect Square: Any whole number that can be represented as the area of a square with a whole number side length.

Concretely: 6 tiles by 6 tiles in the room



Area = 36 units^2 Perfect Square

Side length = $\sqrt{36}$ = 6 units Square Root

Symbolically,

36 = 4 x 9
= (2 x 2) x (3 x 3) Prime factors are grouped in pairs
= (2 x 3) x (2 x 3) Rearrange the factors in two equal
= 6 x 6 groups

$\sqrt{36} = 6$

$\sqrt{n}$ - the means the positive (or principal) square root of a number.

Know

Perfect Squares	Their Square Roots
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
169	13
196	14
225	15
256	16
289	17
324	18
361	19
400	20
625	25

Example: Find the square root of

A) 324 B) 1296

Method 1

A) $324 = 2 \cdot 162$

$= 2 \cdot 2 \cdot 81$

$= 2 \cdot 2 \cdot 9 \cdot 9$

$= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= (2 \cdot 2) \cdot (3 \cdot 3) \cdot (3 \cdot 3)$ * Group into pairs

$= (2 \cdot 3 \cdot 3) \cdot (2 \cdot 3 \cdot 3)$ * Rearrange the factors in 2 equal groups.

$= 18 \cdot 18$

So, $\sqrt{324} = 18$

Method 2

$\sqrt{324} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

$= \sqrt{2^2 \cdot 3^2 \cdot 3^2}$

$= 2 \cdot 3 \cdot 3$

$= 18$

B) $1296 = 2 \cdot 648$

$= 2 \cdot 2 \cdot 324$

$= 2 \cdot 2 \cdot 2 \cdot 162$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 81$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 9 \cdot 9$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= (2 \cdot 2)(2 \cdot 2)(3 \cdot 3)(3 \cdot 3)$

$= (2 \cdot 2 \cdot 3 \cdot 3)(2 \cdot 2 \cdot 3 \cdot 3)$

$= 36 \cdot 36$

So, $\sqrt{1296} = 36$

$\sqrt{1296} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

$= \sqrt{2^2 \cdot 2^2 \cdot 3^2 \cdot 3^2}$

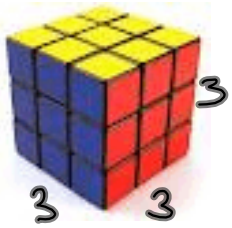
$= 2 \cdot 2 \cdot 3 \cdot 3$

$= 36$

Perfect Cubes

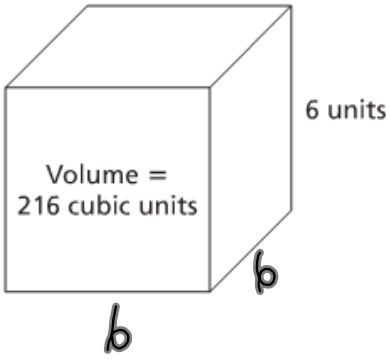
Perfect Square: Any whole number that can be represented as the area of a square with a whole number side length. (Area = lw)

Perfect Cube: Any whole number that can be represented as the volume of a cube with a whole number edge length. (Volume = lwh)



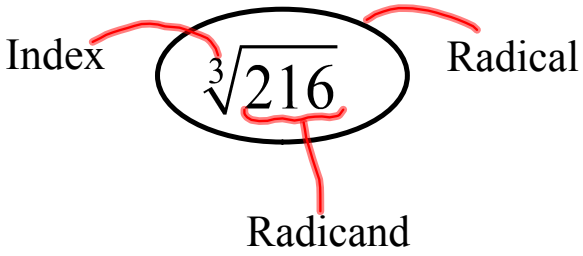
Volume = 3 x 3 x 3 = 27 cubes
therefore 27 is a perfect cube

So, we write $\sqrt[3]{27} = 3$



A 6 x 6 x 6 cube has 216 cubes
therefore 216 is a perfect cube

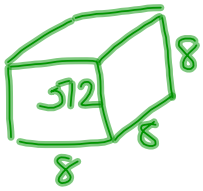
So, we write $\sqrt[3]{216} = 6$



The index is not written for square roots but it is always taken to be two.

In $\sqrt{144}$ the index is 2.

Example: Evaluate (without a calc).



A) $\sqrt[3]{512}$

$512 = 2 \times 256$
$= 2 \times (16 \times 16)$
$= 2 \times (4 \times 4) \times (4 \times 4)$
$= \underline{2 \times (2 \times 2)} \times \underline{(2 \times 2) \times (2} \times \underline{2) \times (2 \times 2)}$ Group Factors into set of 3
$= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$= 8 \times 8 \times 8$

So, $\sqrt[3]{512} = 8$

B) $\sqrt[3]{2744}$

$2744 = 2 \times 1372$
$= 2 \times 2 \times 686$
$= 2 \times 2 \times 2 \times 343$
$= 2 \times 2 \times 2 \times 7 \times 49$
$= (2 \times 2 \times 2) \times (\square \times 7 \times 7)$

* Group the factors into sets of 3

$= (2 \times 7)(2 \times 7)(2 \times 7)$
$= 14 \times 14 \times 14$

* Rearrange the factors into 3 equal groups

$\sqrt[3]{2744} = 14$

method 2: $\sqrt[3]{2744} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7}$

$= \sqrt[3]{2^3 \cdot 7^3}$
$= 2 \cdot 7$
$= 14$

Example: Compute the following using technology:

what does it mean?

A) $\sqrt{900}$ = 30 , $(30)^2 = 900$

B) $\sqrt[3]{900}$ ≈ 9.65 $(9.65)^3 \approx 900$

C) $\sqrt[3]{512\,000}$ = 80 $(80)^3 = 512\,000$

D) $\sqrt[3]{-729}$ = −9 $(-9)^3 = -729$

E) $\sqrt[4]{1296}$ = 6 $(6)^4 = 1296$

F) $\sqrt{-16}$ • error

What does each mean?

* TI83: Math #4 $\sqrt[3]{(900)}$

To get 4th, 5th, or 6th root:

4 Math # 5 $\sqrt[x]{}$ enter

$\sqrt[4]{1296}$ =

4 2nd $\sqrt[x]{}$ 1296

$-4^2 = -1 \times 4^2 = -1 \times 16 = -16$

$(-4)^2 = -4 \times -4 = 16$

$-(4)^2 = -1 \times 4 \times 4 = -16$

Example: Compute.

BEDMAS
↑
Radical

A) $\sqrt[3]{343} = 7$

B) $\sqrt{121} - \sqrt[3]{216}$

$11 - 6$

5

C) $\sqrt[3]{64} + \sqrt[3]{1000} \div \sqrt{25}$

$4 + 10 \div 5$

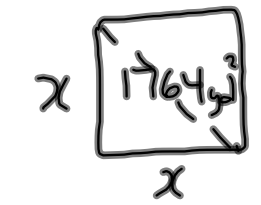
$4 + 2$

6

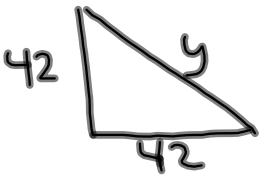
Examples

Area → Perfect Squares
Volum → Perfect Cubes.

1. What is the length of the side of a square farm which contains 1764 yd.^{2}? How far apart are its opposite corners?

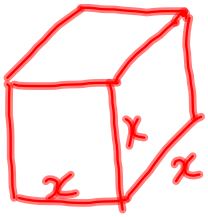


$x = \sqrt{1764}$
$= 42$



$y^2 = 42^2 + 42^2$
$y^2 = 1764 + 1764$
$y^2 = 3528$
$y = \sqrt{3528} \doteq 59.4\ m$

2. If the volume of a cube is 125m^3, what is the expression for the length of each side?



$V = 125\ cm^3$

$x = \sqrt[3]{125}$
$x = 5m$

3. A right rectangular prism measures 9 in. x 8 in. x 24 in. What are the dimensions of a cube with the same volume?

$V_{rectangle} = 9 \times 8 \times 24 = 1728\ in^3$
$V_{cube} = 1728\ in^3$
So $x = \sqrt[3]{1728} = 12\ in^3$

4. Determine the cube root of 3375 in a variety of ways. This could include the use of prime factorization, the use of benchmarks and/or the use of a calculator.