

Section 4.5 Negative Exponents and Reciprocals

Consider the function $y = 2^x$. The table below shows the descending powers of 2. Complete the table down to 2^1 . What pattern do you see?

Power	Number
2^4	
2^3	
2^2	
2^1	
2^0	
2^{-1}	
2^{-2}	
2^{-3}	
2^{-4}	

Do the same for $y = 3^x$.

Power	Number
3^4	
3^3	
3^2	
3^1	
3^0	
3^{-1}	
3^{-2}	
3^{-3}	
3^{-4}	

Negative Integer Exponent Law

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

i.e. x^n is the reciprocal of x^{-n}

Examples

1. Evaluate without technology.

A) 4^{-2} B) 8^{-3} C) 5^{-4} D) 7^{-2}

$$\frac{1}{4^2} = \frac{1}{16} \quad \frac{1}{8^3} = \frac{1}{512} \quad \frac{1}{5^4} = \frac{1}{625} \quad \frac{1}{7^2} = \frac{1}{49}$$

E) 3^{-2} **F** -3^{-2} **D** $(-3)^{-2}$

$$\frac{1}{3^2} = \frac{1}{9}$$

$$-1 \times 3^{-2} = -1 \times \frac{1}{3^2} = -\frac{1}{9}$$

$$= \frac{1}{(-3)^2} = \frac{1}{9}$$

Fraction negative integer exponent

2. Simplify $\left(\frac{2}{3}\right)^{-2}$ (Two ways)

$$\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\frac{4}{9}} = 1 \div \frac{4}{9} = 1 \times \frac{9}{4} = \frac{9}{4}$$

Smart Cut: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Exponent Law for a fraction to the power of a negative exponent.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

3. Simplify.

$$A) \left(\frac{5}{4}\right)^{-3} = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$$

$$B) \left(\frac{3}{7}\right)^{-2} = \left(\frac{7}{3}\right)^2 = \frac{49}{9}$$

Negative Rational Exponents (Exponents that are fractions)

$$(a)^{\frac{m}{n}} = \frac{1}{(a)^{\frac{m}{n}}}$$

$$\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$$

* Recall Rational Exponents
 $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

4. Evaluate: $8^{\frac{2}{3}}$ $\xrightarrow{\text{Square}}$
 $\xrightarrow{\text{Cube root}}$
 \downarrow
 Reciprocal

$$\frac{1}{8^{2/3}} = \left(\frac{1}{\sqrt[3]{8}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

5. Simplify.

A) $9^{-1/2}$

$$\frac{1}{9^{1/2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

B) $8^{-1/3}$

$$\frac{1}{8^{1/3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$$

C) $8^{\frac{-2}{3}}$

$$\frac{1}{8^{2/3}} = \left(\frac{1}{\sqrt[3]{8}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

D) $64^{\frac{-3}{2}}$

$$\frac{1}{64^{3/2}} = \left(\frac{1}{\sqrt{64}}\right)^3 = \left(\frac{1}{8}\right)^3 = \frac{1}{512}$$

E) $64^{\frac{4}{3}}$

$$\left(\sqrt[3]{64}\right)^4 = (4)^4 = 256$$

E) $\left(\frac{9}{16}\right)^{\frac{-3}{2}}$

$$\left(\frac{16}{9}\right)^{3/2} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

F) $\left(\frac{8}{27}\right)^{\frac{-2}{3}}$

$$\left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

G) $\left(\frac{125}{64}\right)^{\frac{-2}{3}}$

$$\left(\frac{64}{125}\right)^{2/3} = \left(\sqrt[3]{\frac{64}{125}}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

6. Evaluate without a calculator.

$$(-5)^2 = 25$$

A) 5^{-2}
 $\frac{1}{5^2}$
 $\frac{1}{25}$

B) -5^{-2}
 $-\frac{1}{5^2}$
 $-\frac{1}{25}$

C) $(-5)^{-2}$
 $\frac{1}{(-5)^2}$
 $\frac{1}{25}$

D) -5^2
 -1×5^2
 -25

E) $\frac{1}{5^{-2}}$
 5^2
 25

F) 0.3^{-2}
 $\left(\frac{3}{10}\right)^{-2}$
 $\left(\frac{10}{3}\right)^2$
 $\frac{100}{9}$

G) $\left(\frac{5}{3}\right)^{-3}$
 $\left(\frac{3}{5}\right)^3$
 $\frac{27}{125}$

H) $\left(-\frac{2}{3}\right)^{-4}$
 $\left(-\frac{3}{2}\right)^4$
 $\frac{81}{16}$

I) $\left(\frac{5}{4}\right)^{-2}$
 $-\left(\frac{4}{5}\right)^2$
 $-\frac{16}{25}$

J) $16^{\frac{3}{4}}$
 $16^{\frac{3}{4}}$
 $(\sqrt[4]{16})^3$
 $(2)^3$
 8

K) $0.25^{\frac{3}{2}}$
 $\left(\frac{25}{100}\right)^{\frac{3}{2}}$
 $\left(\frac{1}{4}\right)^{\frac{3}{2}}$
 $(4)^{\frac{3}{2}}$
 $(\sqrt{4})^3$
 $(2)^3$
 8

L) $\left(\frac{64}{27}\right)^{\frac{4}{3}}$
 $\left(\frac{27}{64}\right)^{\frac{4}{3}}$
 $\left(\sqrt[3]{\frac{27}{64}}\right)^4$
 $\left(\frac{3}{4}\right)^4$
 $\frac{81}{256}$

M) $\left(-\frac{25}{49}\right)^{\frac{3}{2}}$
 $\left(-\frac{49}{25}\right)^{\frac{3}{2}}$
 $\left(\sqrt{\frac{-49}{25}}\right)^3$

* Cannot be simplified b/c you cannot get $\sqrt{\text{of a neg. number}}$.

N) $\left(\frac{81}{256}\right)^{\frac{3}{4}}$

O) $625^{\frac{3}{4}}$
 $-\frac{1}{625^{\frac{3}{4}}}$
 $= \frac{1}{(\sqrt[4]{625})^3}$
 $= \frac{1}{(5)^3}$
 $= \frac{1}{125}$

$$11. P = 3000(1.025)^{-5}$$

$$P = 3000 \left(\frac{1}{1.025} \right)^5$$

$$P = 3000(0.9756\dots)^5$$

$$P = 3000(0.8838\dots)$$

$$P = \$2651.40$$

$$12/ \left(-\frac{64}{125} \right)^{-5/3} = \left(\frac{64}{125} \right)^{3/5}$$

$$= \left(\sqrt[3]{\frac{64}{125}} \right)^5$$

$$= \left(\frac{4}{5} \right)^5$$

$$= \frac{1024}{3125}$$

$$\left(-\frac{64}{125} \right)^{-5/3} = \left(-\frac{125}{64} \right)^{5/3}$$

$$3^x < 1$$

$$3^x > 1$$

$$= \left(\sqrt[3]{-\frac{125}{64}} \right)^5$$

$$= \left(-\frac{5}{4} \right)^5 = -\frac{3125}{1024}$$

$$3^x = 1$$

