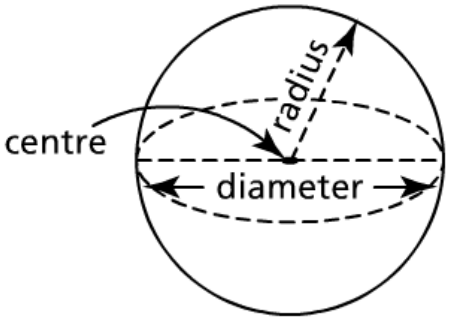
Section 1.6 Surface Area and Volume of a Sphere

A **sphere** is the set of points in space that are the same distance from a fixed point, which is the center.

A line segment that joins the center to any point on the sphere is the **radius**.



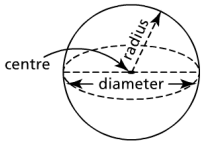
A line that joins two points on a sphere and passes through the **diameter**.

I. Surface Area of a Sphere

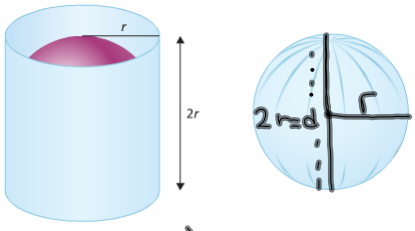
How can we find the surface area of a sphere?

So, the surface area of sphere with radius, r, is given by the formula:

$$SA = 4\pi r^2$$



The surface area of a sphere is related to the curved surface area of a cylinder that encloses it. The cylinder has the same diameter as the sphere, and height equal to its diameter.



(Lateral Area)

The curved surface area of a cylinder is given by
$SA = 2\pi rh$ but the height of the sphere is 2r.
So,
$SA = 2\pi r(2r)$
$SA = 4\pi r^2$

1. The diameter of a softball is approximately 4 in. Determine the surface area of a softball to the nearest square inch.

$SA = 4\pi r^2$
$= 4\pi(2)^2$
$= 50\ in^2$

d = 4in
r = 2in

2. The surfce area of a soccer ball is 250 square inches. What is the diameter of the soccer ball to the nearest tenth of an inch?

$SA = 4\pi r^2$
$250 = 4\pi r^2$
$\frac{250}{12.57} = \frac{12.57r^2}{12.57}$
$19.89 = r^2$
$\sqrt{19.89} = r$
$4.46\ in = r$

Diameter = 2 x 4.46 = 8.9 in

3. An official basketball has a radius of 12.5 cm and usually has a leather covering. Approximately how much leather is required to cover 12 official basketballs?

$SA = 4\pi r^2$
$SA = 4\pi(12.5)^2$
$SA = 1963.50\ cm^2$

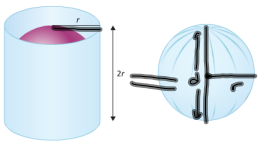
12 basketballs $= 12 \times 1963.50$
$= 23562\ cm^2$

Homework: P.51-52
Questions: 3, 7, 8, 9, 12a,

II. Volume of a Sphere

There are two ways to develop the formula for the volume of a sphere.

Consider a sphere with radius r, that is place inside a cylinder with radius r and height 2r.



The volume of the sphere is two-thirds the colume of the cylinder.

$$V = \frac{2}{3} \times volume\ of\ cylinder$$

$$V = \frac{2}{3}\pi r^2 h$$ But h = 2r, so

$$V = \frac{2}{3}\pi r^2 \times 2r$$

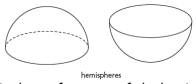
$$V = \frac{4}{3}\pi r^3$$

1. The moon approximates a sphere with diameter of 2160 mi. What is the approximate volume of the moon? (discuss scientific notation on a calculator)

$d = 2160\text{ mi}$
$r = 1080\text{ mi}$
$4\pi \times 1080 \wedge 3 \div 3$

$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi (1080)^3$
$= 5{,}276{,}669{,}286\text{ mi}^3$
$= 5.3 \times 10^9\text{ mi}^3$

2. A hemisphere has radius 5.0 cm. (A hemisphere is half of a sphere.



hemispheres

A) What is the surface area of the hemisphere to the nearest tenth of a square centimeter?

$SA_{\text{Hemi-Sphere}} = \frac{1}{2}SA_{\text{sphere}} + A_{\text{circular base}}$
$SA_{HS} = \frac{1}{2}(4\pi r^2) + \pi r^2$
$SA_{HS} = 2\pi r^2 + \pi r^2$
$SA_{HS} = 3\pi r^2$
$SA_{HS} = 3\pi (5)^2$
$SA = 235.6\text{ cm}^2$

B) What is the volume of the hemisphere tot he nearest tenth?

$V_{HS} = \frac{1}{2}V_{\text{sphere}}$
$= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$= \frac{2}{3}\pi r^3$
$= \frac{2}{3}\pi (5)^3$
$= 261.8\text{ cm}^3$

$\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

3. A carnival clown has 75 m^3 of helium (volume) compressed in a tank. How many spherical balloons with radius 0.25 m can be filled with the helium from the tank?

$V_{\text{Balloon}} = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi (0.25)^3$
$= 0.0654\text{ m}^3$

$\#\text{Balloons} = 75 \div 0.0654 = 1146.8$
$\therefore$ 1146 balloons can be filled.

4. Eight basketballs are put in a container. The radius of each basketball is 10 cm. If the container is shaped like a square based pyramid, approximately how much room will be left (volume space not occupied by a basketball) if each side of the base measures 40 cm and the height is 70 cm?

$V_{\text{pyramid}} = \frac{1}{3}lwh$
$V_{\text{Basketball}} = \frac{4}{3}\pi r^3$
$V_{8\text{ Basketball}} = 8 \times V_{\text{Basketball}}$
$V_{\text{left}} = V_{\text{pyramid}} - V_{8\text{ Basketballs}}$
$= 3823\text{ cm}^3$

5. A spherical Christmas ornament measures 12 cm in circumference. What is the approximate volume of the cubed box that will hold this ornament?

Key: diameter of sphere = length of cube

$C = 12\text{cm}$

$V_{box} = ?$

$C = \pi d$

$\frac{12}{\pi} = \frac{\pi d}{\pi}$

$3.82\text{cm} = d \Rightarrow$ length of box

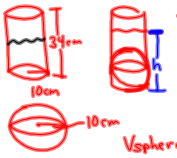
$V_{box} = lwh$

$= (3.82)(3.82)(3.82)$

$= 55.7\text{cm}^3$

6. A heavy sphere with diameter 20 cm is dropped into a right circular cylinder with a base radius of 10 cm and a height of 34 cm.

A) If the cylinder is half full of water, what is the total volume of the water and the sphere?



$V_{water} = \frac{1}{2} V_{cylinder}$

$= \frac{1}{2}\pi r^2 h$

$= \frac{1}{2}\pi(10)^2(34)$

$= 5340.7\text{cm}^3$

$V_{sphere} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(10)^3$

$= 4188.8\text{cm}^3$

$V_T = V_{water} + V_{sphere}$

$= 5340.7 + 4188.8$

$= 9529.5\text{cm}^3$

B) How high will the water rise once the sphere is completely under water. (Note: Once the sphere is dropped into the water, the water level will rise to a height that represents the volume of the water plus the volume of the sphere.)

$V_s + V_w = 9529.5\text{cm}^3$

$V = \pi r^2 h$

$9529.5 = \pi(10)^2 h$

$\frac{9529.5}{314.16} = \frac{314.16h}{314.16}$

$30.3\text{cm} = h$

Test