

## Section 1.5: Volume of Right Prisms, Pyramids, Cylinders, and Cones

**Area:** the number of square units to cover a region (Done)

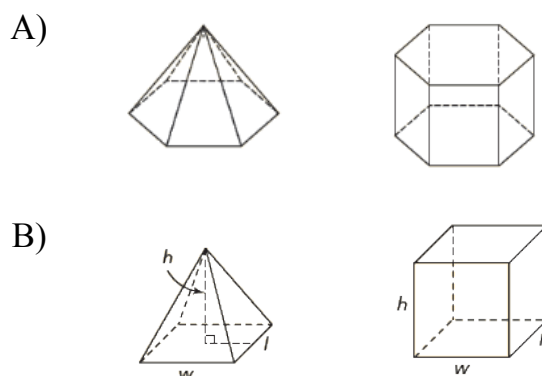
**Volume:** the amount of space an object occupies. It is measured in cubic units.

**Capacity:** The amount of material a container holds.

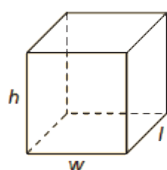
**Mathematically:** Volume=(base area)(height) = Ah

### 1. The Right Rectangular Prism and Right Pyramid

Predict how many full pyramids are needed to fill the prism. Note the prisms and pyramids have the same length, width, and height.



### Volume of Rectangular Prism

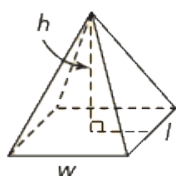


$$\text{Volume} = \text{Area of Base} \times \text{Height}$$

$$V = lwh$$

It takes 3 right pyramids to fill one rectangular prism. Both the prism and pyramid must have the same length, width and height.

Therefore, the **Volume of a Rectangular Pyramid** with Same Dimensions



$$V = \frac{1}{3}lwh$$

$$V = \frac{lwh}{3}$$

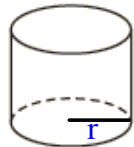
## 2. The Right Rectangular Cylinder and the Right Rectangular Cone

$$\left. \begin{aligned} SA_{cylinder} &= 2\pi rh + 2\pi r^2 \text{ (grade 8-9)} \\ SA_{cone} &= \pi rs + \pi r^2 \end{aligned} \right\} \text{ Done}$$

Predict how many full cones are needed to fill the cylinder. Note the cone and cylinder have the same radius and height.



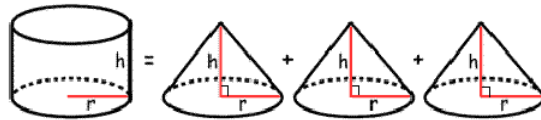
### Volume of a Right Cylinder (Grade 8)



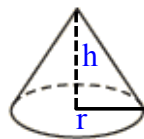
Volume = Area of base x Height

$$V = \pi r^2 h$$

The Right Cone: it takes 3 cones to fill one cylinder (both objects must have same height and radius)



Therefore, the Area of a Right Cone with the Same Dimensions as the cylinder



$$V = \frac{1}{3} \pi r^2 h$$

So:  $V_{pyramid} = \frac{1}{3} V_{prism}$

or

$$V_{prism} = 3 V_{pyramid}$$

$$V_{cone} = \frac{1}{3} V_{cylinder}$$

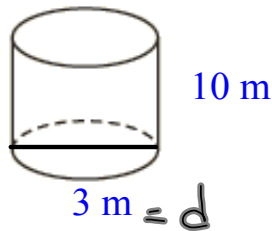
or

$$V_{cylinder} = 3 V_{cone}$$

**Examples**

1. Determine the volume of the following:

A)



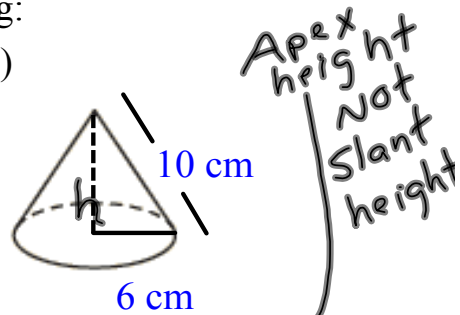
$$V = Ah$$

$$V = \pi r^2 h$$

$$V = \pi (1.5)^2 (10)$$

$$V = 70.7 \text{ m}^3$$

B)



$$V = \frac{1}{3} \pi r^2 h$$

Pythagorean Theorem to find "h".

$$a^2 = c^2 - b^2$$

$$h^2 = 10^2 - 6^2$$

$$h^2 = 64$$

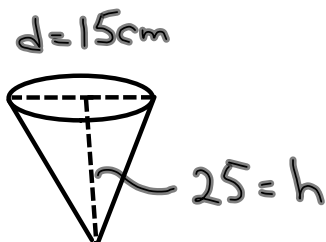
$$h = \sqrt{64}$$

$$\boxed{h = 8}$$

$$V = \frac{1}{3} \pi (6)^2 (8)$$

$$V = 301.6 \text{ cm}^3$$

2. A cake decorating bag is in the shape of a cone. To the nearest cubic centimetre, how much frosting will fit into the bag if the diameter is 15 cm and the height is 25 cm?



$$r = 7.5 \text{ cm}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (7.5)^2 (25)$$

$$= 1472.6 \text{ cm}^3$$

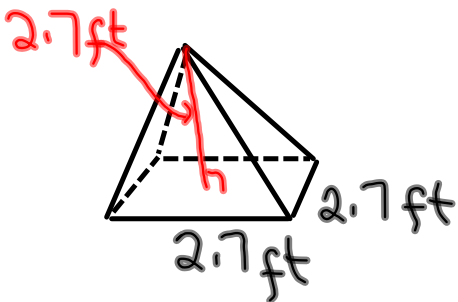
$$= 1472 \text{ cm}^3$$

3. A cone and a cylinder have the same height and the same base radius. If the volume of the cylinder is  $81 \text{ cm}^3$ , what is the volume of the cone in  $\text{cm}^3$ ? Explain.

DOE  
MC

$$\begin{aligned} V_{\text{cone}} &= \frac{1}{3} V_{\text{cylinder}} \\ &= \frac{1}{3}(81) \\ &= 27 \text{ cm}^3 \end{aligned}$$

4. Find the volume of a square based pyramid where the length of each side and the height measures 2.7 ft.



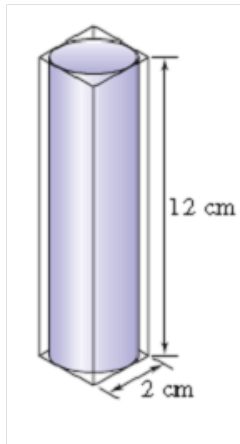
$$V = \frac{1}{3} lwh$$

$$V = \frac{1}{3} (2.7)(2.7)(2.7)$$

$$V = 6.6 \text{ ft}^3$$

$$V = \frac{1}{3} (2.7)^3$$

5. A closed cylindrical can is packed in a box. What is the volume of the empty space between the can and the box?



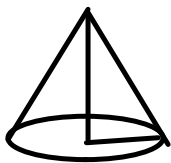
$$V = lwh = (2)(2)(12) = 48 \text{ cm}^3$$

$$V = \pi r^2 h = \pi (1)^2 (12) = 37.7 \text{ cm}^3$$

$$V_{\text{space}} = 48 - 37.7 = 10.7 \text{ cm}^3$$

**Determining the Missing Measurement given the volume of right cone, cylinder, prism or pyramid**

6. A cone has a volume of  $30 \text{ cm}^3$  and a base area of  $15 \text{ cm}^2$ . What is the height of the cone?



$$V = 30 \text{ cm}^3$$

$$A = 15 \text{ cm}^2$$

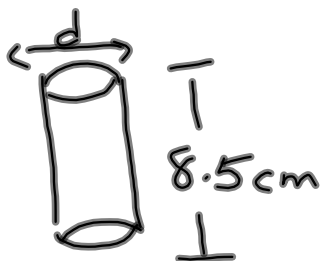
$$V = \frac{1}{3} Ah$$

$$30 = \frac{1}{3} (15) h$$

$$\frac{30}{5} = \frac{5h}{5}$$

$$\underline{6 \text{ cm} = h}$$

7. A cylinder has a volume of  $132.6 \text{ cm}^3$  and a height of  $8.5 \text{ cm}$ .  
What is the diameter of the cylinder?



$$V = 132.6$$

$$V = \pi r^2 h$$

$$132.6 = \pi r^2 (8.5)$$

$$\frac{132.6}{26.7} = \frac{26.7 r^2}{26.7}$$

$$4.966 = r^2$$

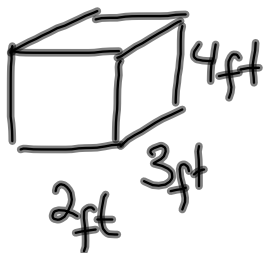
$$\sqrt{4.966} = r$$

$$2.23 \text{ cm} = r$$

$$\underline{2.23 \text{ cm} = r}$$

$$\text{Diameter} = 2 \times 2.23 = 4.46 \text{ cm}$$

8. A cord of firewood is 128 cubic feet. Jan has 3 storage bins for firewood that each measure 2 ft. by 3 ft. by 4 ft. Does she have enough storage space to hold a full cord of wood? Explain.



$$V_{\text{prism}} = lwh$$

$$= (2)(3)(4)$$

$$= 24 \text{ ft}^3$$

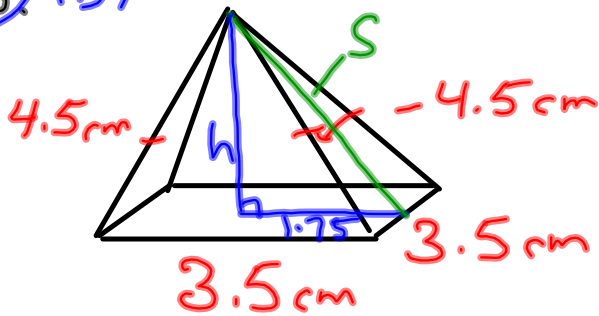
$$3 \times 24 \text{ ft}^3 = 72 \text{ ft}^3$$

Since  $72 \text{ ft}^3 < 128 \text{ ft}^3$ , she does not have enough room.

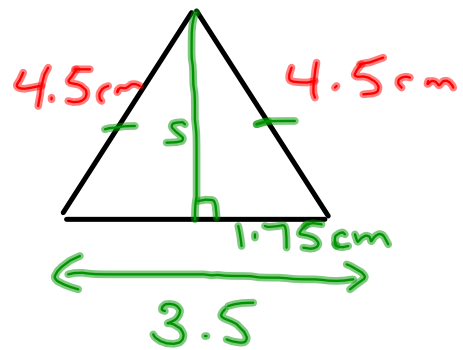
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Test

15. (b)

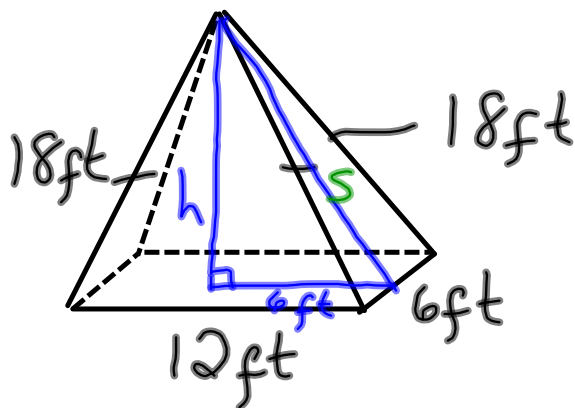


$$a^2 = c^2 - b^2$$

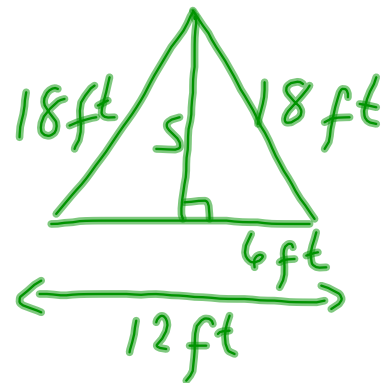


$$a^2 = c^2 - b^2$$

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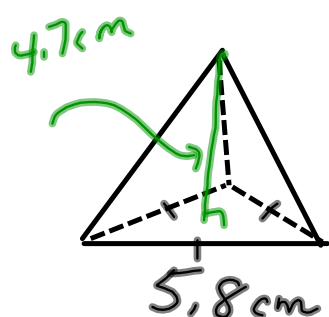


$$\begin{aligned} 6yd &= (6 \times 3)ft \\ &= 18ft. \end{aligned}$$

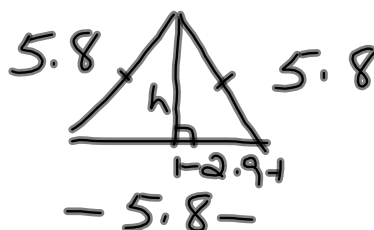


$$a^2 = c^2 - b^2$$

17.



a) Base



(b)

$$V = \frac{1}{3}Ah$$

$$a^2 = c^2 - b^2$$

$$h^2 = c^2 - b^2$$

$$A = \frac{1}{2}bh$$