Section 1.4: Surface Areas of Right Pyramids and Right Cones

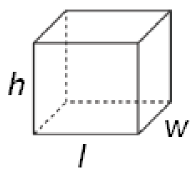
LESSON FOCUS Solve problems involving the surface areas of right pyramids and right cones.

Make Connections

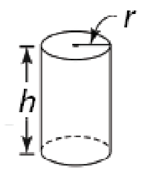
The ancient pyramids at Giza, Egypt, were built about 4500 years ago.

This pyramid has a square base with a side length of 755 feet. The original height of the pyramid was 481 feet. Archeologists believe that the pyramid was once covered with a white limestone casing. How could you calculate the area that was once covered with limestone?

Review
Surface Areas of Right Prisms and Cylinders



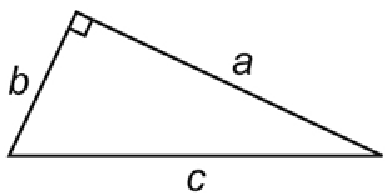
$SA = 2wl + 2hl + 2hw$



$SA = 2\pi r^2 + 2\pi r$

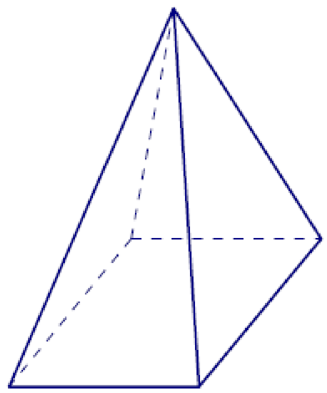
Review
The Pythagorean Theorem

In any right triangle, the sum of the squares of the two shorter sides is equal to the square of the longer side.



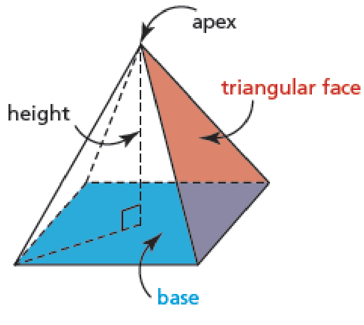
$$a^2 + b^2 = c^2$$

Consider the Right Square Pyramid below. Describe how you could estimate its surface area in both the imperial and metric system.



A **right pyramid** is a 3-Dimensional object that has **triangular faces** and a **base** that is a **polygon**. (A polygon is a closed shape that consists of line segments. Triangles and quadrilaterals are polygons.)

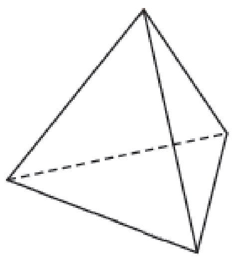
- The shape of the base determines the name of the pyramid.
- The triangular faces meet at a point called the apex.
- The **height** of the pyramid is the perpendicular distance from the apex to the centre of the base.



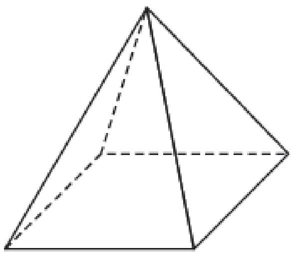
right square pyramid

(This diagram and the ones below can be found on page 27 of textbook.)

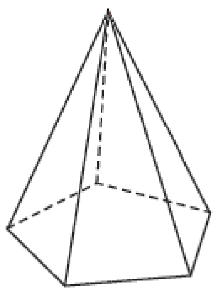
When the base of a right pyramid is a regular polygon, the triangular faces are congruent. Then the **slant height** of the right pyramid is the height of a triangular face.



regular tetrahedron



right square pyramid

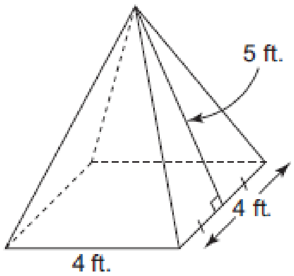


right pentagonal pyramid

The surface area of a right pyramid is the sum of the areas of the triangular faces and the base. Recall that the area, A, of a triangle with base, b, and height, h, is: $A = \frac{1}{2}bh$

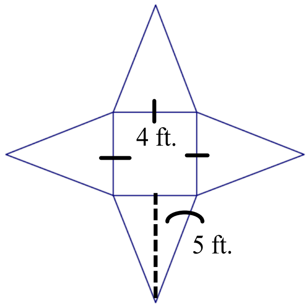
Example 1: Finding the Surface Area of a Square Pyramid

A square pyramid has a base with side length 4 ft. Each triangular face has height 5 ft.Find the surface area of the pyramid.



Solution

This net shows the faces and base of the pyramid.



The area, A, of each triangular face is:

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4)(5)$

$A = 10 \text{ ft.}^2$

$A = \frac{bh}{2}$

$= \frac{(4)(5)}{2} = 10 \text{ ft}^2$

The area of the base, B, is

B = lw

B = (4)(4)

$B = 16 \text{ ft.}^2$

So the surface area, SA, of the pyramid is:

SA = 4(area △)+ area of the base

SA = 4A + B

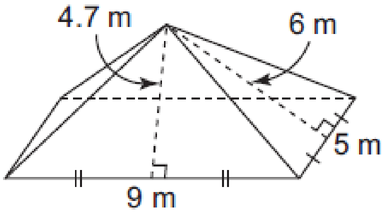
SA = 4(10) + 16

$SA = 56 \text{ ft.}^2$

The surface area of the triangular faces of the pyramid is called the lateral area.

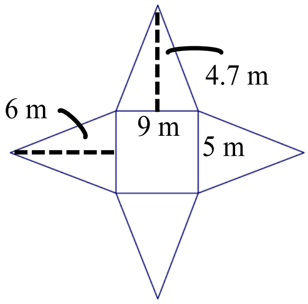
Example 2: Finding the Surface Area of a Rectangular Pyramid

Find the surface area of this rectangular pyramid.



Solution

This net shows the faces and base of the pyramid.



The base of the pyramid is a rectangle, so

$A = lw$
$B = (9)(5)$
$B = 45\text{ m}^2$

There are 2 triangular faces with base 9 m and height 4.7 m. The area of each face is

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(9)(4.7)$
$A = 21.15\text{ m}^2$

$A = \frac{bh}{2} = \frac{(9)(4.7)}{2} = 21.15\text{ m}^2$

There are 2 triangular faces with base 5 m and height 6 m. The area of each face is

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(5)(6)$
$A = 15\text{ m}^2$

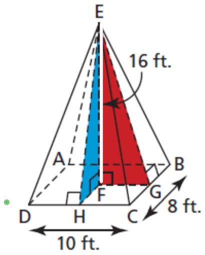
$A = \frac{bh}{2} = \frac{(5)(6)}{2} = 15\text{ m}^2$

So, the surface area of the pyramid is:

$SA = 2(\text{Area }\triangle\ 1) + 2(\text{Area }\triangle\ 2) + \text{Area }\blacksquare$
$= (2 \times 21.15) + (2 \times 15) + 45$
$= 117.3\text{ m}^2$

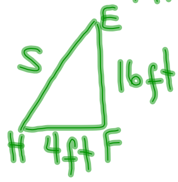
Example 3: Finding the Surface Area of a Rectangular Pyramid Given its height (not the slant height)

A right rectangular pyramid has base dimensions 8 ft. by 10 ft., and a height of 16 ft. Calculate the surface area of the pyramid to the nearest square foot.



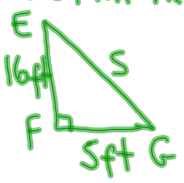
$\underline{\triangle EDC}$ - need Slant height

Look $\triangle EFH$



$c^2 = a^2 + b^2$
$S^2 = 4^2 + 16^2$
$S^2 = 16 + 256$
$S^2 = 272$
$S = \sqrt{272}$

Need slant height for $\triangle EBC$ - use $\triangle EFG$



$c^2 = a^2 + b^2$
$S^2 = 16^2 + 5^2$
$S^2 = 256 + 25$
$S^2 = 281$
$S = \sqrt{281}$

Find Area of Triangles

$\underline{\triangle FDC}$

$A = \frac{1}{2}bh$
$= \frac{1}{2}(10)(\sqrt{272})$
$= 5\sqrt{272}$

$\underline{\triangle EBC}$

$A = \frac{1}{2}bh$
$= \frac{1}{2}(8)(\sqrt{281})$
$= 4\sqrt{281}$

$\underline{\text{Area of Base}}$: $A = lw$
$A = (10)(8)$
$= 80$

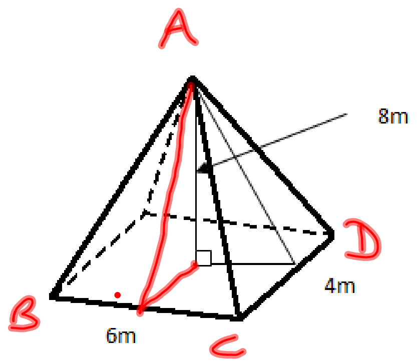
$SA = 2(A_\triangle) + 2(A_\triangle) + A_\square$

$SA = 2(5\sqrt{272}) + 2(4\sqrt{281}) + 80$

$SA = 164.924 + 134.104 + 80$
$= 379.028$
$SA = 379\ ft^2$

Example 4 (same as number 3)

A right rectangular pyramid has base dimensions 4 m by 6m and a height of 8m. Calculate the surface area of the pyramid to the nearest square metre.



Need Slant height for Δ ABC

$c^2 = a^2 + b^2$
$c^2 = 8^2 + 2^2$
$c^2 = 64 + 4$
$c^2 = 68$
$c = \sqrt{68}$

Need slant height for Δ ADC

$c^2 = a^2 + b^2$
$c^2 = 8^2 + 3^2$
$c^2 = 64 + 9$
$c^2 = 73$
$c = \sqrt{73}$

Area of ΔABC

$A = \frac{1}{2}bh$
$= \frac{1}{2}(6)(\sqrt{68})$
$= 3\sqrt{68}$

Area of ADC

$A = \frac{1}{2}bh$
$= \frac{1}{2}(4)(\sqrt{73})$
$= 2\sqrt{73}$

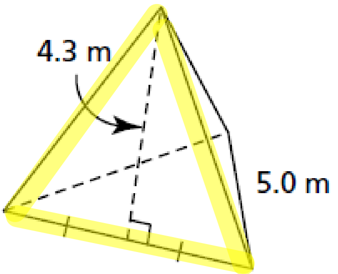
Area of Base

$A = lw$
$A = (6)(4)$
$A = 24$

$SA = 2A_{\Delta} + 2A_{\Delta} + A_{\square}$
$= 2(3\sqrt{68}) + 2(2\sqrt{73}) + 24$
$= 107.65 m^2$
$\doteq 108 m^2$

Example 5: Determining the Surface Area of a Regular Tetrahedron Given Its Slant Height

Calculate the surface area of this regular tetrahedron to the nearest square metre.



The figure has 4 congruent faces.

$A = \frac{bh}{2}$

$A = \frac{(5)(4.3)}{2}$

$A = 10.75$

$SA = 4A_{\triangle}$

$= 4(10.75)$

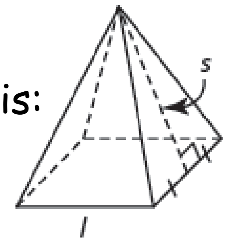
$= 43\text{ m}^2$

We can determine a formula for the surface area of any right pyramid with a regular polygon base. Consider this right square pyramid. Each triangular face has base ℓ and height s.

The area, A, of each triangular face is:

$A = \frac{1}{2}bh$

$A = \frac{1}{2}\ell s$



So the area of 4 triangular faces is

$4(\frac{1}{2}\ell s) = 4(\frac{1}{2})\ell s$

$= (\frac{1}{2}s)(4\ell)$

The area of the triangular faces of a pyramid is called the lateral area, denoted A_L.

So $A_L = (\frac{1}{2}s)(4\ell)$

What does 4ℓ represent?

The perimeter of the base of the pyramid.

Pull

Therefore, the surface area of the pyramid is:

SA = $\frac{1}{2}$(slant height)(Perimeter of Base) + Area of Base

Since any right pyramid with a regular polygon base has congruent triangular faces, the same formula is true for any of these pyramids.