Solving 3 X 3 Systems Algebraically

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A system of linear equations in 3 variables looks something like:

$$x + 3y - z = -11$$
$$2x + y + z = 1$$
$$5x - 2y + 3z = 21$$

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A <u>solution</u> is an ordered triple (x, y, z) that makes all 3 equations true.

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Is (1, 1, 1) a solution?

$$x + y + z = 3$$
$$2x - y + 4z = 5$$
$$x + 4y - 2z = 3$$

Yes

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Is (2, 1, 6) a solution?

$$x + y - z = -3$$
$$2x - y + z = 9$$
$$4x + y - z = 15$$

No

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Method #1: Substitution

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Substitution Method

$$\begin{cases} x + y + z = 24 \\ 5x + 3y + z = 56 \\ x + y = z \end{cases}$$

Since **x+y=z**, *substitute this for z* in the first two equations

$$\begin{cases} x + y + (x + y) = 24 \\ 5x + 3y + (x + y) = 56 \end{cases}$$

Simplify

$$\begin{cases} 2x + 2y = 24 \\ 6x + 4y = 56 \end{cases}$$

Finally, solve this linear system of two equations and two variables to get x = 4 and y = 8

Since z=x+y, z = 12. Our final solution is (4,8,12)

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1. Solve the system.

① $x + 3y - z = -11$ → $x = -3y + z - 11$

② $2x + y + z = 1$

③ $5x - 2y + 3z = 21$

$2(-3y + z - 11) + y + z = 1$

$-6y + 2z - 22 + y + z = 1$

$-5y + 3z = 23$ (*) ✓

(2, -4, 1)

$5(-3y + z - 11) - 2y + 3z = 21$

$-15y + 5z - 55 - 2y + 3z = 21$

$-17y + 8z = 76$ (*) ✓

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2. Solve the system.

1. $x + y + z = 5$
2. $2x - y + z = 4$
3. $3x - y + 2z = 8$

(1, 1, 3)

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4. Solve the system.

1. $2x + y - z = -7$
2. $-2x - y + 3z = 17$
3. $2x + 3y - 2z = -12$

(-1, 0, 5)

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1. $2x + y - z = -7 \rightarrow y = -2x + z - 7$
2. $-2x - y + 3z = 17$ ①
3. $2x + 3y - 2z = -12$ ②

$-2x - (-2x + z - 7) + 3z = 17$

$-2x + 2x - z + 7 + 3z = 17$

$0x + 2z = 10$ (*) ✓

$2x + 3(-2x + z - 7) - 2z = -12$

$2x - 6x + 3z - 21 - 2z = -12$

$-4x + 1z = 9$ (*) ✓

$0x + 2z = 10$

$-4x + 1z = 9 \rightarrow z = 4x + 9$

$0x + 2(4x + 9) = 10$

$0x + 8x + 18 = 10$

$8x = -8$

$x = -1$

$0(-1) + 2z = 10$

$0 + 2z = 10$

$2z = 10$

$z = 5$

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$x + 2y - 3z = 50$ (−2)

$2x + y + 2z = 3$ (−1)

$2x - 5y + 4z = -79$

$-2x - 4y + 6z = -100$

$2x + y + 2z = 3$

$-3y + 8z = -97$ ✓

$-2x - y - 2z = -3$

$2x - 5y + 4z = -79$

$-6y + 2z = -82$ ✓

$-3y + 8z = -97$ (−2)

$-6y + 2z = -82$

$6y - 16z = 194$

$-6y + 2z = -82$

$-14z = 112$

$z = -8$

$-3y + 8z = -97$

$-3y + 8(-8) = -97$

$-3y - 64 = -97$

$-3y = -33$

$y = 11$

$2x + y + 2z = 3$

$2x + 11 + 2(-8) = 3$

$2x + 11 - 16 = 3$

$2x - 5 = 3$

$2x = 8$

$x = 4$

$\therefore (x, y, z) = (4, 11, -8)$

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Method #2: Elimination

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Steps to solving a system of equations in 3 Variables

- Ensure that the equations are in standard form:

  $Ax + By + Cz = D$
- Remove any decimals or fractions from the equations.
- Eliminate one of the variables using two of the three equations. Result will be a new equation with two variables.
- Eliminate the same variable using another set of two equations. Result will be a second equation in two variables.
- Solve the new system of two equations.
- Using the solution for the two variables, substitute the values into one of the original equations to solve for the third variable.

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Steps for solving in 3 variables

1. Using the 1st 2 equations, cancel one of the variables.
2. Using the last 2 equations, cancel the same variable from step 1.
3. Use the results of steps 1 & 2 to solve for the 2 remaining variables.
4. Plug the results from step 3 into one of the original 3 equations and solve for the 3rd remaining variable.
5. Write the solution as an ordered triple (x,y,z).

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Eliminate One Variable

We have rewritten our three equations as follows:

1) $4x + y + 24z = 8$
2) $4x - y + 6z = 1$
3) $6x + y + 6z = 6$

Now, to solve, we need to first get two equations with two variables. To do so, we eliminate one of the variables from two of the equations. Then eliminate the same variable from another set of two equations.

1) $4x + y + 24z = 8$	2) $4x - y + 6z = 1$
2) $4x - y + 6z = 1$	3) $6x + y + 6z = 6$
A) $8x + 30z = 9$	B) $10x + 12z = 7$
In the first column, we eliminated y from equations	In the second column, we eliminated y from equations 2

Now solve the system of equations with two variables, using elimination.

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Solve this system

$$\begin{cases} 3x + 2y + 4z = 11 & \text{Equation 1} \\ 2x - y + 3z = 4 & \text{Equation 2} \\ 5x - 3y + 5z = -1 & \text{Equation 3} \end{cases}$$

(2) -3

Multiply Eq. 2 by 2 and add it to Eq. 1. Save this result

$$\begin{cases} 3x + 2y + 4z = 11 \\ 4x - 2y + 6z = 8 \end{cases}$$

7x +10z = 19

Now multiply Eq. 2 by -3 and add it to Eq. 3. Save this result.

$$\begin{cases} -6x + 3y - 9z = -12 \\ ...5x - 3y + 5z = -1 \end{cases}$$

-x -4z = -13

Solve this new system of linear equation in two variables.

Multiply the bottom eq. by -7 and add it to the top eq.

$$\begin{cases} 7x + 10z = 19 \\ -7x - 28z = -91 \end{cases}$$

-18z=-72 or z = 4

Substituting z=4 into either of the new equations will give x = -3......finally substituting these values into any of the original equations give y = 2.

Our final solution is (-3,2,4)

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x + 2y - 3z = 50

2x + y + 2z = 3

2x - 5y + 4 z = -79

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$$4x + 3y - 6z = -6$$

$$-5x + 8y - 6z = -53$$

$$4x - 2y + 7z = 27$$

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$$4x + 5y - 6z = -15$$

$$-6x + 5y - 4z = 35$$

$$2x + 3y - 3z = -7$$

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$$\begin{cases} x + 2y - 3z = -3 \\ 2x - 5y + 4z = 13 \\ 5x + 4y - z = 5 \end{cases}$$

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Solve this system

$$\begin{cases} 3x + 2y + 4z = 11 \\ 2x - y + 3z = 4 \\ 5x - 3y + 5z = -1 \end{cases}$$

Our strategy will be to use two of the equations to eliminate one of the variables.

We will then use two other equations to eliminate the same variable.

Once we have two equations with two variables, we can use the technique we learned in lesson 3.2

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$$\begin{aligned} 2x + y + z &= 4 \\ -3x + 2y - 2z &= -10 \\ x - 2y + 3z &= 7 \end{aligned}$$

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$$\begin{aligned} x + y + 2z &= 1 \\ 2x - y + z &= 2 \\ 4x + y + 5z &= 4 \end{aligned}$$

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