

MENIHEK HIGH SCHOOL

MATHEMATICS 3204

Cumulative Exam January 2007

General Instructions:

1. Students are required to do ALL items.
2. This examination will consist of three parts as follows:

Part I:	35 Multiple Choice	1 Constructed Response	42 marks
Part II:	7 Constructed Response Questions	-	43 marks
3. Answers to multiple choice items are to be shaded on the answer sheet provided. All other items are to be answered on the paper.
4. For Parts II , students are reminded to show all necessary steps and calculations as credit may be given for incomplete or partially correct solutions. Correct answers without calculations will not merit full marks.

PART I

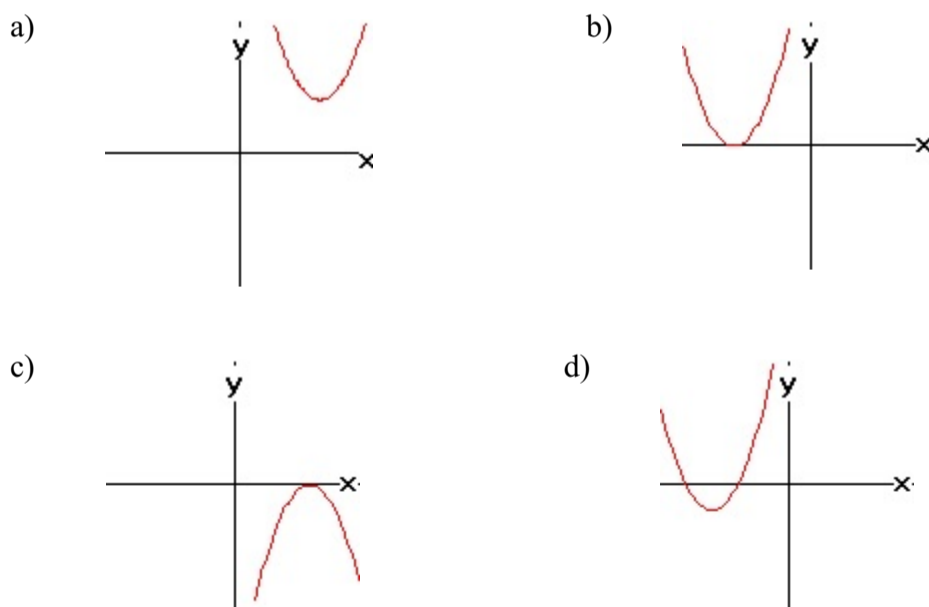
Total Value: 42 marks

1. Which transformation of $y = x^2$ produces $2(y + 3) = (x - 5)^2$?
- vertically stretched by a factor of 2, translated horizontally 5 units left, and translated vertically 3 units up
 - vertically stretched by a factor of 2, translated horizontally 5 units right, and translated vertically 3 units down
 - vertically stretched by a factor of $\frac{1}{2}$, translated horizontally 5 units left, and translated vertically 3 units up
 - vertically stretched by a factor of $\frac{1}{2}$, translated horizontally 5 units right, and translated vertically 3 units down

2. What is the range of $(y + 5) = (x + 2)^2$?

- $\{y \mid y \leq -5, y \in \mathbb{R}\}$
- $\{y \mid y \geq -5, y \in \mathbb{R}\}$
- $\{y \mid y \leq -2, y \in \mathbb{R}\}$
- $\{y \mid y \geq -2, y \in \mathbb{R}\}$

3. Which graph represents a quadratic function whose discriminant is less than zero?



4. If $y = x^2$ is transformed to $y = -4(x - 3)^2 - 2$, what is the vertical stretch factor?

- a) -4 b) $-\frac{1}{4}$ c) $\frac{1}{4}$ d) 4

5. For what value of x does the function $-3(y + 1) = (x - 2)^2$ have a maximum value?

- a) -2 b) -1 c) 1 d) 2

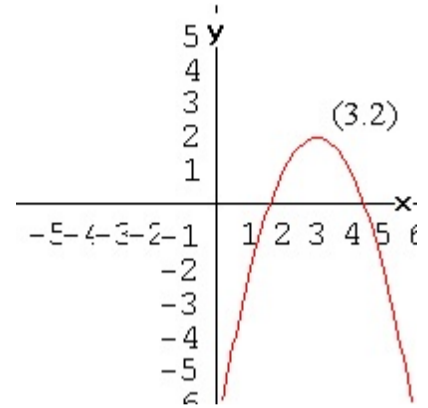
6. A quadratic function has a vertex $(-3, 0)$ and opens down. What is the nature of the roots?
- imaginary and equal
 - imaginary and unequal
 - real and equal
 - real and unequal
7. Which type of sequence is illustrated by $\{2, 5, 10, 17, 26, \dots\}$?
- arithmetic
 - cubic
 - geometric
 - quadratic
8. What is the transformational form of $y = x^2 + 12x - 3$?
- $y - 3 = (x + 6)^2$
 - $y - 3 = (x - 6)^2$
 - $y + 39 = (x + 6)^2$
 - $y + 39 = (x - 6)^2$
9. Which value of "c" makes $x^2 - 5x + c$ a perfect square?
- $-\frac{25}{4}$
 - $-\frac{25}{2}$
 - $\frac{25}{4}$
 - 25
10. What are the roots of $3x^2 + 27 = 0$?
- ± 3
 - $\pm 3i$
 - 3
 - $3i$
11. Which represents a quadratic relationship?
- | | | | | | |
|---|---|---|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 4 | 7 | 12 | 19 | 28 |
 - | | | | | | |
|---|----|---|---|---|----|
| x | 3 | 4 | 5 | 6 | 7 |
| y | -2 | 1 | 4 | 7 | 10 |
 - | | | | | | |
|---|---|---|----|-----|-----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 2 | 8 | 32 | 128 | 512 |
 - | | | | | | |
|---|---|----|-----|-----|------|
| x | 2 | 4 | 6 | 8 | 10 |
| y | 8 | 64 | 216 | 512 | 1000 |

12. What are the zeroes of $y = x^2 - 13x + 30$?

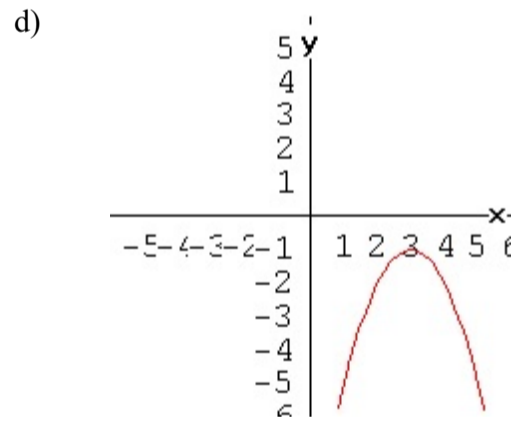
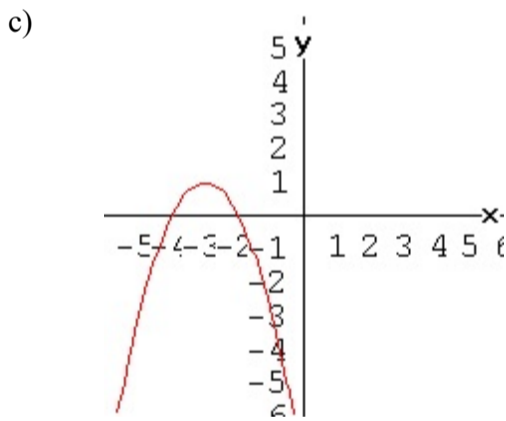
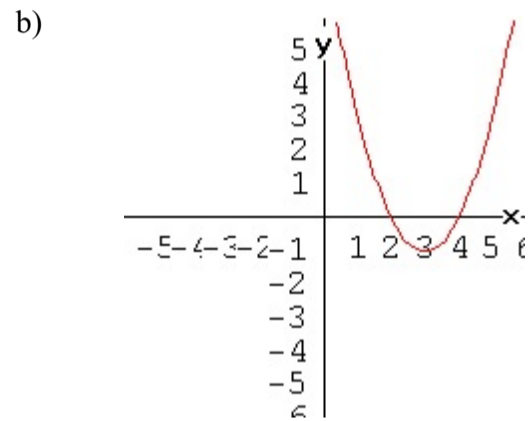
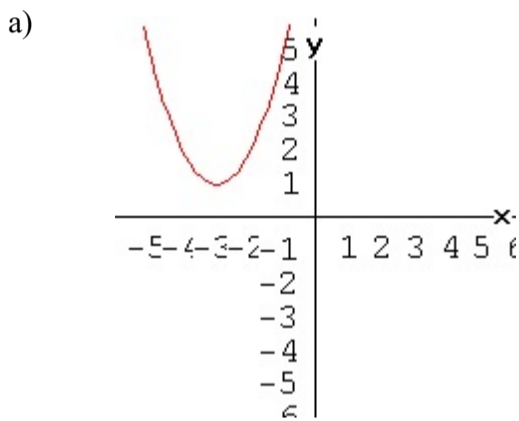
- a) -3, 10
- b) 3, 10
- c) -2, 15
- d) 2, -15

13. Which mapping rule was used to transform the graph of $y = x^2$ to the graph shown?

- a) $(x, y) \rightarrow (x - 3, -y - 2)$
- b) $(x, y) \rightarrow (-x - 3, y - 2)$
- c) $(x, y) \rightarrow (-x + 3, y + 2)$
- d) $(x, y) \rightarrow (x + 3, -y + 2)$



14. The graph of $y = x^2$ is transformed to $(y - 1) = (x + 3)^2$. Which graph results?



15. Which function describes the sequence $\{-8, -11, -14, -17, -20, \dots\}$?

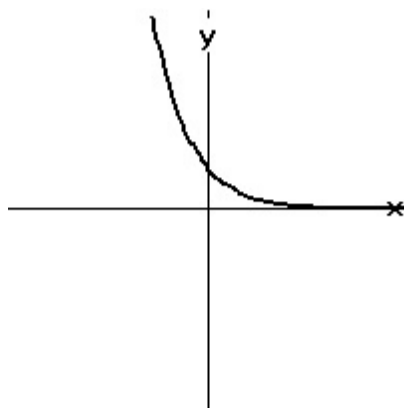
- a) $t_n = -3n - 8$
- b) $t_n = 3n - 11$
- c) $t_n = -3n - 5$
- d) $t_n = -8n - 3$

16. If $t_n = 2n - 3$, for what value of "n" is $t_n = 5$?

- a) -1
- b) 1
- c) 4
- d) 7

17. The graph of $y = b^x$ is shown. What value best represents "b"?

- a) -3
- b) $\frac{5}{3}$
- c) $\frac{1}{3}$
- d) 3



18. Which function represents a geometric sequence?

- a) $t_n = 2n + 3$
- b) $t_n = n^2 + 3$
- c) $t_n = 2^n$
- d) $t_n = \frac{n}{3}$

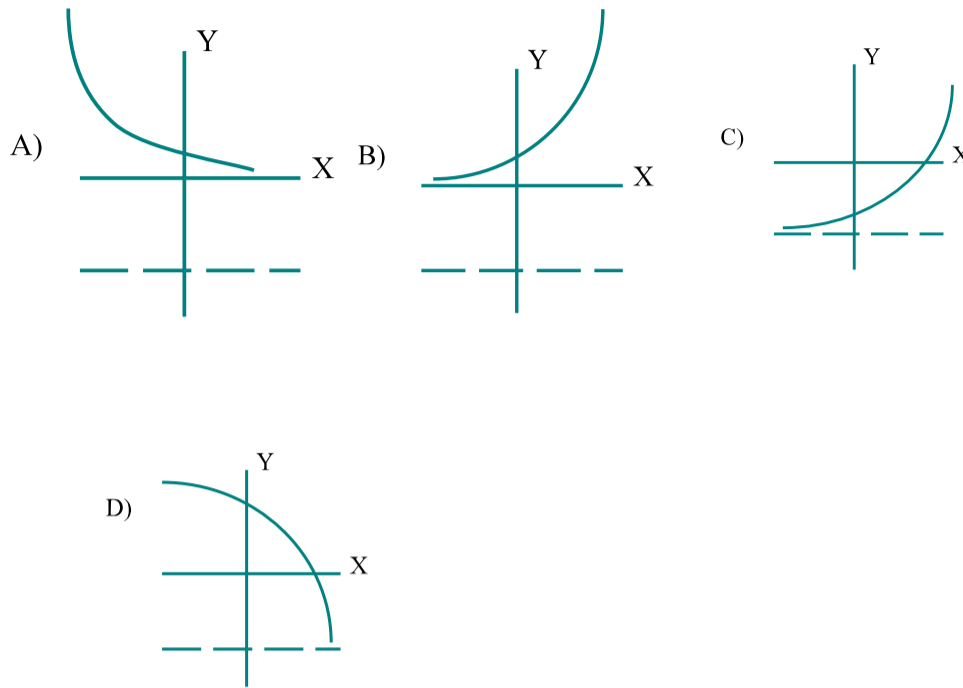
19. What is the range of $y = \left(\frac{1}{4}\right)^x$?

- a) $\{y \mid y > 0, y \in \mathbb{R}\}$
- b) $\{y \mid y < 0, y \in \mathbb{R}\}$
- c) $\{y \mid y > \frac{1}{4}, y \in \mathbb{R}\}$
- d) $\{y \mid y < \frac{1}{4}, y \in \mathbb{R}\}$

20. If the sequence $\left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots\right\}$ is represented by the equation $y = a \cdot b^x$, what is the value of "b"?

- a) $\frac{1}{3}$
- b) $\frac{1}{2}$
- c) 2
- d) 3

21. Which graph represents an increasing exponential function with asymptote $y = -3$?



22. Solve: $8^x = 32$

- a) $\frac{3}{5}$
- b) $\frac{5}{3}$
- c) 4
- d) 24

23. A \$15,000 car depreciates to in value by 22% annually. If it is worth \$5552 four years later, which equation best models its depreciation?

- A) $A = 5552(1.22)^t$
- B) $A = 15,000(1.22)^t$
- C) $A = 5552(.78)^{\frac{t}{4}}$
- D) $A = 15,000(.78)^t$

24. Which function describes this data?

x	2	4	6	8
y	15	45	135	405

a) $y = 5(3)^{\frac{x}{2}}$

b) $y = 5(3)^{2x}$

c) $y = 15(3)^{\frac{x}{2}}$

d) $y = 15(3)^{2x}$

25. Bob invests \$2000 into an account that has a 7% annual interest rate. What equation best models the amount in Bob's account after "t" years?

a) $A = 2000(0.07)^t$

b) $A = 2000(0.93)^t$

c) $A = 2000(1.07)^t$

d) $A = 2000(1.7)^t$

26. Evaluate: $(5^{-2} + 4^0)^{-1}$.

a) -29

b) $\frac{25}{26}$

c) $\frac{26}{25}$

d) 26

27. What is the simplified form of $\frac{b \cdot b^n}{(b^3)^n}$?

A) $\frac{1}{b^n}$ B) $\frac{1}{b^2}$ C) b^{1-2n} D) b^n

28. Express $2 \log B - \log C + \log D$ as a single logarithm.

A) $2 \log \frac{BD}{C}$ B) $\log \frac{B^2 D}{C}$ C) $\log \frac{2BD}{C}$ D) $2 \log(BD - C)$

29. Roy used the following steps to solve the equation $\log_x 9 = \frac{3}{2}$. Which step contains a mathematical error?

Step 1 $x^{\frac{3}{2}} = 9$

Step 2 $x = 9^{\frac{3}{2}}$

Step 3 $x = (\sqrt{9})^3$

Step 4 $x = 27$

- A) step 1
B) step 2
C) step 3
D) step 4
30. Solve for x: $4^x = 213$
- A) $x = 0.259$
B) $x = 2.346$
C) $x = 3.867$
D) $x = 4.103$
31. What are the coordinates of the y-intercept of the graph of $y = -2(4)^x + 5$?
- A) (0,-2)
B) (0,3)
C) (0,4)
D) (0,5)
32. What is the exponential form of $\log_5 \frac{1}{25} = -2$?

A) $5 \bullet -2 = \frac{1}{25}$ B) $\frac{1}{25} = -2^5$ C) $5^{-2} = \frac{1}{25}$ D) $\left(\frac{1}{25}\right)^{-2} = 5$

- 33 What is the inverse of the function $y = 4^x$?
- A) $y = \log_4 x$ B) $x = \log y$ C) $x = \log_4 y$ D) $y = \log_x 4$

- 34 What is the value of x in $\log 3 + \log x = \log 15$?

- A) $\frac{1}{5}$ B) 5 C) 12 D) 18

- 35 Evaluate: $\left[27^{\frac{2}{3}} + \left(\frac{1}{2}\right)^0 \right]$

- A) 9
B) 10
C) 82
D) 730

Part II

1. Algebraically determine the **EXACT** roots in simplest form for: $2x = -3x^2 + 4$

(7 marks)

PART II

Total Value: 43 marks

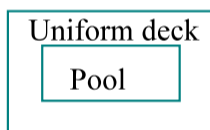
Answer ALL items in the space provided. Show ALL workings and formulas.

2. EITHER An object is fired upward vertically from the top of a building at a starting speed of 50m/s. If the building is 40 m high, the equation that gives the approximate height h of the object above the ground seconds after firing is $h(t) = -5t^2 + 50t + 40$. What is the maximum height reached by the object? At what time does this occur from the moment of its release? 8 marks

OR The Alexanders fenced the rectangular yard at their lake cottage on three sides and used the lake as the fourth side. If 120 m of fence was used, algebraically set up a quadratic function and use it to find the dimensions that will produce the maximum area.

3. A rectangular pool measures 10 m by 5m. A deck, of uniform width, is to be built all the around the pool such that the total area of the pool and deck will be 126m^2 . Set up and solve a quadratic equation to determine the width of the deck.?

(8 marks)



- 4 Solve for x: $3^{2x+1} = \sqrt[5]{3^x}$

(5 marks)

- 5 A bacterial culture doubles in size every 8 hours. If there are 5000 bacteria initially present, set up an equation to model this situation and determine how many bacterial will be present 2 days later algebraically. (4 marks)

- 6 You invested \$10,000 at 4.5% per annum. Set up an equation to model its growth and algebraically determine how long it will take for the \$10,000 to grow exponentially to \$40,000? (5)

- 7 Solve for x below:

$$\log_2(5x - 2) - \log_2 2 = \frac{1}{2} \log_2 36 + 2 \log_2 3 \quad (6 \text{ marks})$$

- 8 Hot chocolate cools exponentially over time after it is brought into a stadium and the cooling is described by the function given with temperature, T, in degrees Celsius and time in, m, in minutes.

$$T = 46(.95)^m + 4$$

- A) What is the initial temperature of the hot chocolate as well as the temperature of the air in the stadium? 2 marks
- B) Sketch a labeled graph for the first 100 minutes of cooling using increments of 20 minutes. Show at least two important features of the graph. 5 marks

